Lecture 6
CAPM and Single Index Models

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FN 312 – INVESTMENTS

Portfolio Theory in Practice

- To create an optimal portfolio, we need to estimate the variance/efficient frontier, MVE portfolio and the best CAL
- Step 1: Get the inputs

  ↪ For a portfolio of N (100) securities we need:

σ_i's	N	100
$E(r_i)$'s	N	100
$Cov(r_i, r_j)$'s	$\frac{1}{2}N(N-1)$	4950
Total	$\frac{1}{2}N(N+3)$	5150

A Remedy

- Use CAPM to determine what the market believes expected returns should be
- Use a single factor model to calculate asset covariances
- Then we can combine our "views" with the CAPM-derived estimates to get portfolio weights

Getting Expected Returns

- To get the market's beliefs about expected returns, use the CAPM
- How to get beta? The standard way to estimate a beta is to estimate the following characteristic line equation

How to estimate beta?

- We typically use monthly data
  - Can we use weekly, daily, or intra day data?
- Betas may change over time, thus we use short windows of data (5-years)
- Possible reasons

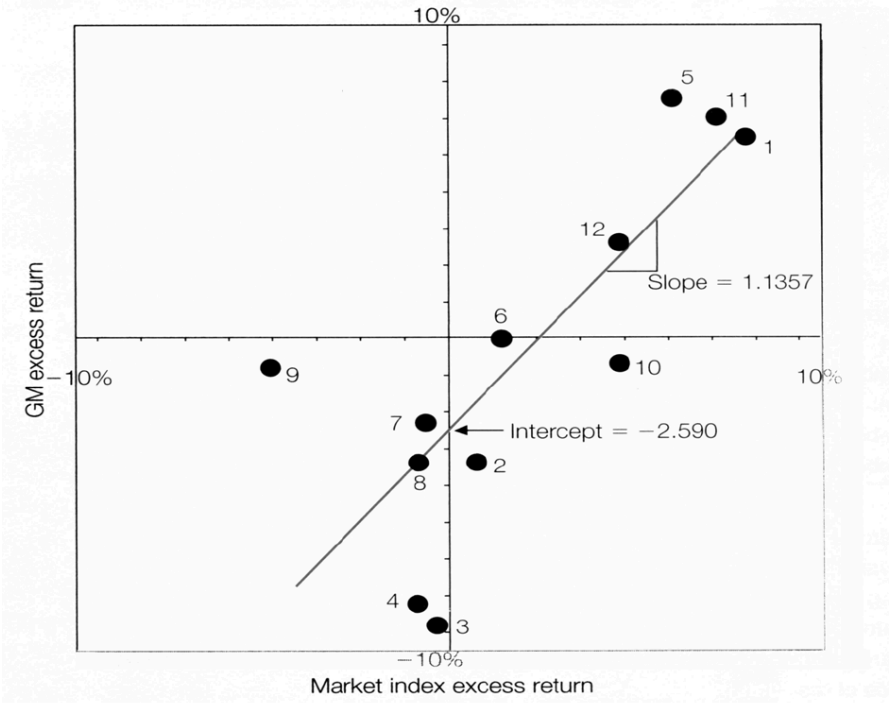
- We can use rolling window regressions to estimates the betas
  - At month t estimate beta using months t-60 through t-1
  - At month t+1 estimate beta using months t-59 through t
- Alternatively, use more sophisticated statistical models that allow for time-variation in beta

Month	GM Return	Market Return	Monthly T-Bill Rate	Excess GM Return	Excess Market Return
January	6.06	7.89	0.65	5.41	7.24
February	−2.86	1.51	0.58	−3.44	0.93
March	−8.18	0.23	0.62	−8.79	−0.38
April	−7.36	−0.29	0.72	−8.08	−1.01
May	7.76	5.58	0.66	7.10	4.92
June	0.52	1.73	0.55	−0.03	1.18
July	−1.74	−0.21	0.62	−2.36	−0.83
August	−3.00	−0.36	0.55	−3.55	−0.91
September	−0.56	−3.58	0.60	−1.16	−4.18
October	−0.37	4.62	0.65	−1.02	3.97
November	6.93	6 85	0.61	6.32	6.25
December	3.08	4.55	0.65	2.43	3.90
Mean	0.02	2.38	0.62	−0.60	1.75
Std Dev	4.97	3.33	0.05	4.97	3.32

Regression Results $r_{GM} - r_f = \alpha + \beta(r_M - r_f)$

	α	β
Estimated coefficient	−2.590	1.1357
Standard error of estimate	(1.547)	(0.309)

Variance of residuals = 12.601
Standard deviation of residuals = 3.550
R-SQR = 0.575



Total versus excess returns

- If you use total rather than excess returns

Getting the covariances

- We can estimate the covariances between securities by assuming that a single index model describes the returns

Getting the covariances (cont.)

Note: The Single Index Model vs CAPM

- The single index model is a statistical model
  - It specifies that all common movements between stocks can be captured by a single index (also called the single factor model)
  - Can be generalized to multiple common factors
  - It is a statistical technique designed to estimate large covariance matrices
- The CAPM is an economic model of expected returns
  - It specifies that the market portfolio captures systematic risk
  - However, it does allow securities i and j to be correlated on top of what their covariance with the market portfolio implies, for instance if they are in the same industry. The only requirement is that these correlations "wash out" as we add more stocks.

- Therefore, to construct efficient frontier based on the single index model (for 100) securities, we need estimates of the following

r_f	1	1
$E(r_m)$	1	1
σ_m^2	1	1
α_i	N	100
β_i	N	100
$\sigma_{\varepsilon,i}^2$	N	100
Total	$3N+3$	303

- This is considerably smaller than the 5150 we had before.

Once you have the inputs…

Step 2: Plug expected returns, return standard deviations, correlation matrix into Excel spreadsheet to get the weights that form the tangency portfolio

- However, there may be times when we think that the market is a little wrong along one or more dimensions

- How can we combine our views with what the market expects?

- What if I think the market has underestimated the earnings that a company will announce next month, eg. the expected return will be 2 percent higher than what market expects

- What if I think its risk (beta) is increasing?

Lessons Learnt

Far from everyone agrees…

"To suppose that safety-first consists in having a small gamble in a large number of different companies where I have no information to reach good judgment, as compared with a substantial stake in a company where one's information is adequate, strikes me as a travesty of investment policy"	"Diversification is an admission of not knowing what to do and striking an average"	"…and is the business understandable? Despite high regard for Microsoft, Mr. Buffet avoids its stock because the field puzzles him. Ignorance, he says, increases danger. This belief is a departure from the common wisdom of stock diversification. Owning many different stocks – good, bad and mediocre – depresses the returns a more selective portfolio would achieve, he believes, and makes it impossible to understand all that you own. Thus in 1987 his $2 billion portfolio had just three companies. Today nearly $15 billion is spread among just 10."
- J.M. Keynes 1939	**- G. Loeb, 1935**	**- W. Buffet, 1996**

Why?

Reconsider the assumptions for mean-variance analysis:

- Separation Theorem
- No market friction
- No heterogeneity in investors
- Static expected returns and variances
- Risk averse investor

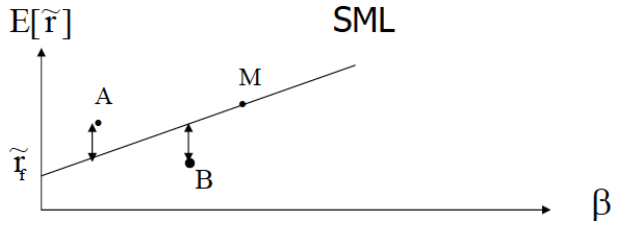
CAPM and Applications

- Whether the CAPM holds in the data is a hotly debated topic among academics
- Even if the CAPM is wrong empirically, there are several reasons we want to spend some time with it:
  - CAPM provides a benchmark rate of return for possible investments which allows you to measure and compare portfolio performance
  - Identify inefficient priced assets eg. is the forecast for the stock more or less than its "fair" return given risk?
  - Discount uncertain cash flow for pricing projects, IPO, non-traded assets
  - CAPM can help make an educated guess of expected return on assets not yet traded in the market place. What should be the IPO of a stock? How will a new investment project affect return required on a company's stock?

Performance Measurement

- Compare the performance of mutual funds and fund managers
  - Example: $E[\tilde{r}_m]=15\%$ and $r_f=5\%$. The average returns of fund A and B are 16% and 18%. Their betas are 1 and 1.4 Which fund manager has better information than the market?
  - Jensen's alpha measures the performance of a portfolio or security relative to the market

Identify inefficient securities



- All efficiently priced assets fall on the SML
- Which security should we buy/sell?

Compute NPV of Uncertain Cash Flows

- Calculating NPV for a project or IPO

Estimating beta for new companies

- In the absence of historical data, how would we estimate beta for new companies?
- Standard industry practice is to use "comparables"
  - Find a similar company and use their beta
  - If a comparable company can't be found, predict beta from a sample of companies with similar characteristics
- Which characteristics to use?

- Select a sample of companies to estimate the model
- Estimate the beta for these companies using historical return data
- Regress estimated betas on several characteristics that drive betas

Homework

- Ch 7: 4-10, 17-19
- Ch 9: 9a-d, 21