A Review of Some Statistical Concepts

Summation and Product Operators

The Greek capital letter $\sum$ (sigma) is used to indicate summation. Thus,

$$\sum_{i=1}^{n} x_i = x_1 + x_2 + \dots + x_n$$

Some of the important properties of the summation operator $\sum$ are

1. $\sum_{i=1}^{n} k = nk$, where k is constant.

$$\sum_{i=1}^{n} k = \underbrace{k + k + \dots + k}_{n}$$

$$= nk$$

e.g. $\sum_{i=1}^{4} 3 = (4)(3) = 12$

$= 3 + 3 + 3 + 3 = 12$

2. $\sum_{i=1}^{n} kx_i = k\sum_{i=1}^{n} x_i$, where k is constant.

$$\sum_{i=1}^{n} kx_i = kx_1 + kx_2 + \dots + kx_n$$

$$= k\underbrace{(x_1 + x_2 + \dots x_n)}_{\sum_{i=1}^{n} x_i}$$

3. $\sum_{i=1}^{n}(a + bx_i) = na + b\sum_{i=1}^{n} x_i$, where a and b are constants and where use is made of properties 1 and 2 above.

$$\sum_{i=1}^{n}(a+bx_i) = \underbrace{na}_{1^{st}\ \text{property}} + \underbrace{b\sum_{i=1}^{n} x_i}_{2^{nd}\ \text{property}}$$

$$= na + b(x_1 + x_2 + x_3 + \dots x_n)$$

4. $\sum_{i=1}^{n}(x_i + y_i) = \sum_{i=1}^{n} x_i + \sum_{i=1}^{n} y_i$

Additivity

n term

$$= (x_1+y_1) + (x_2+y_2) + \dots + (x_n+y_n)$$

$$= (x_1+x_2+\dots+x_n) + (y_1+y_2+\dots+y_n)$$

$$= \sum_{i=1}^{n} x_i + \sum_{i=1}^{n} y_i$$

Example

x_i	5	8	6	9
y_i	4	6	8	9

1 $\sum_{i=1}^{4} x_i =$

2 $\sum_{i=1}^{4} x_i^2 =$

3 $\sum_{i=1}^{4} x_i y_i =$

4 $\sum_{i=1}^{4} (x_i + y_i) =$

5 $\sum_{i=1}^{4} (3x_i + 5y_i) =$

Example

x_i	5	8	6	9
y_i	4	6	8	9

6. $\sum_{i=1}^{10} 5 =$

7. $\sum_{i=1}^{4} (x_i + y_i)^2 =$

Example

x_i	5	8	6	9
y_i	4	6	8	9

1. $\sum_{i=1}^{4} x_i = x_1 + x_2 + x_3 + x_4 = 28$

2. $\sum_{i=1}^{4} x_i^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2 = 206$

3. $\sum_{i=1}^{4} x_i y_i = x_1 y_1 + x_2 y_2 + x_3 y_3 + x_4 y_4$
$= 197$

4. $\sum_{i=1}^{4} (x_i + y_i) = (x_1 + y_1) + \dots + (x_4 + y_4)$
$= \sum_{i=1}^{4} x_i + \sum_{i=1}^{4} y_i = 55$

5. $\sum_{i=1}^{4} (3x_i + 5y_i) = n\sum_{i=1}^{n} x_i + n\sum_{i=1}^{n} y_i$
$= 21$

Example

x_i	5	8	6	9
y_i	4	6	8	9

6. $\sum_{i=1}^{10} 5 = 5 \times 10 = 50$

7. $\sum_{i=1}^{4} (x_i + y_i)^2 = \sum_{i=1}^{4} (x_i^2 + 2x_i y_i + y_i^2$

$(a+b)^2 = a^2 + 2ab + b^2$

$= \sum_{i=1}^{n} x_i^2 + 2\sum_{i=1}^{4} x_i y_i + \sum y_i^2$

$= 797$

The summation operator can also be extended to multiple sums. This, $\sum\sum$, The double summation operator, is defined as

$$\sum_{i=1}^{n}\sum_{j=1}^{m} x_{ij} = \sum_{i=1}^{n}(x_{i1} + x_{i2} + \cdots + x_{im})$$

$$=(x_{11} + x_{21} + \cdots + x_{n1}) + (x_{12} + x_{22} + \cdots + x_{n2}) + \cdots + (x_{1m} + x_{2m} + \cdots + x_{nm})$$

Some of the properties of $\sum\sum$ are

1. $\sum_{i=1}^{n}\sum_{j=1}^{m} x_{ij} = \sum_{j=1}^{m}\sum_{i=1}^{n} x_{ij}$; that is, the order in which the double summation is performed is interchangeable.

2. $\sum_{i=1}^{n}\sum_{j=1}^{m} x_i y_j = \sum_{i=1}^{n} x_i + \sum_{j=1}^{m} y_j$.

The finite double series can be written as a product of series

$: x_1y_1 + x_2y_1 + \ldots + x_1y_2 + \ldots + x_ny_n$

$= (x_1 + x_2 + \ldots x_n) y_1 + (x_1 + x_2 + \ldots x_n) y_2 + \ldots$

$= \sum_{i=1}^{n} x_i (y_1 + y_2 + \ldots y_n) = \sum_{i=1}^{n} x_i \sum_{j=1}^{n} y_j$

3. $\sum_{i=1}^{n} \sum_{j=1}^{m} (x_{ij} + y_{ij}) = \sum_{i=1}^{n} \sum_{j=1}^{m} x_{ij} + \sum_{i=1}^{n} \sum_{j=1}^{m} y_{ij}$

$$= \sum_{j=1}^{m} (x_{1j} + y_{1j}) + \sum_{j=1}^{m} (x_{2j} + y_{2j}) + \dots + \sum_{j=1}^{m} (x_{nj} + y_{nj})$$

$$= \sum_{j=1}^{m} (\underbrace{x_{1j} + x_{2j} + x_{3j} + \dots + x_{nj}}_{\sum_{i=1}^{n} x_{ij}}) + \sum_{j=1}^{m} (\underbrace{y_{1j} + y_{2j} + \dots y_{nj}}_{\sum_{i=1}^{n} y_{ij}})$$

$$= \sum_{j=1}^{m} \sum_{i=1}^{n} x_{ij} + \sum_{j=1}^{m} \sum_{i=1}^{n} y_{ij}$$