Utility Maximization : 2 goods: X & Y

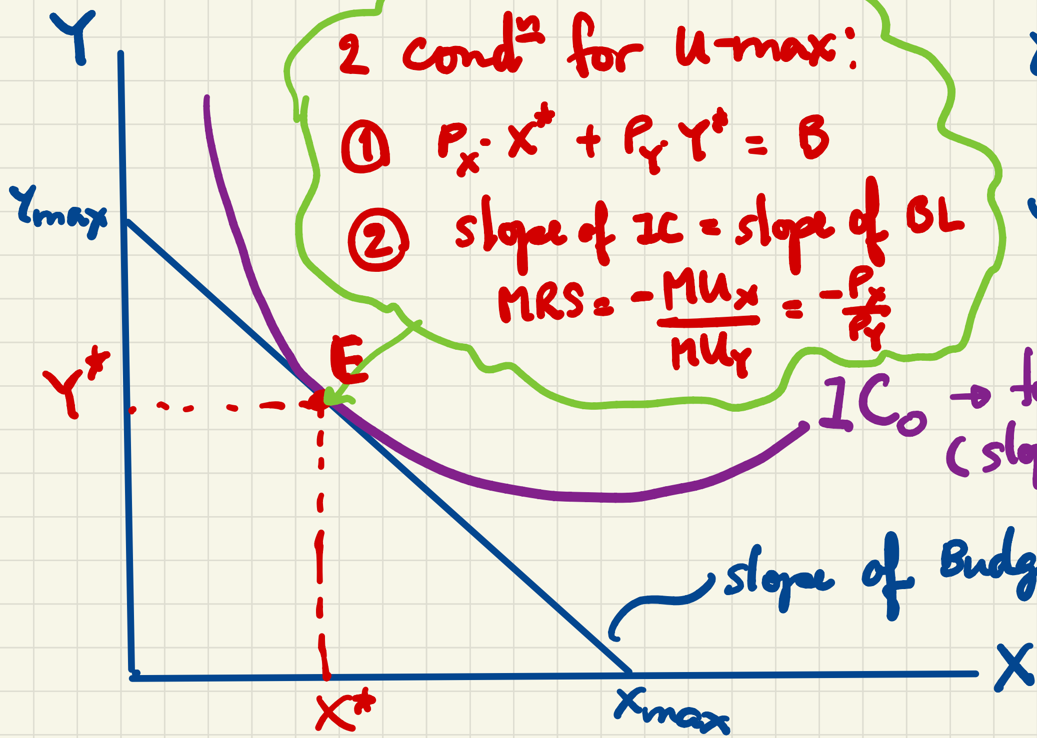
Budget = B

Max $U(X, Y)$

subject to $P_x \cdot X + P_Y \cdot Y = B$

$X_{max} = \dfrac{B}{P_x}$

$Y_{max} = \dfrac{B}{P_Y}$



2 condⁿ for U-max:

① $P_x \cdot X^* + P_Y Y^* = B$

② slope of IC = slope of BL

$MRS = -\dfrac{MU_x}{MU_Y} = \dfrac{-P_x}{P_Y}$

$IC_0 \rightarrow$ tells $U(X, Y)$

(slope of IC = MRS)

slope of Budget line $= \dfrac{-P_x}{P_Y}$

Recall: 2 types of goods

1. Normal goods: Income $\uparrow \Rightarrow Q_D \uparrow$

   $\varepsilon_I > 0$

2. Inferior goods: Income $\uparrow \Rightarrow Q_D \downarrow$

   $\varepsilon_I < 0$

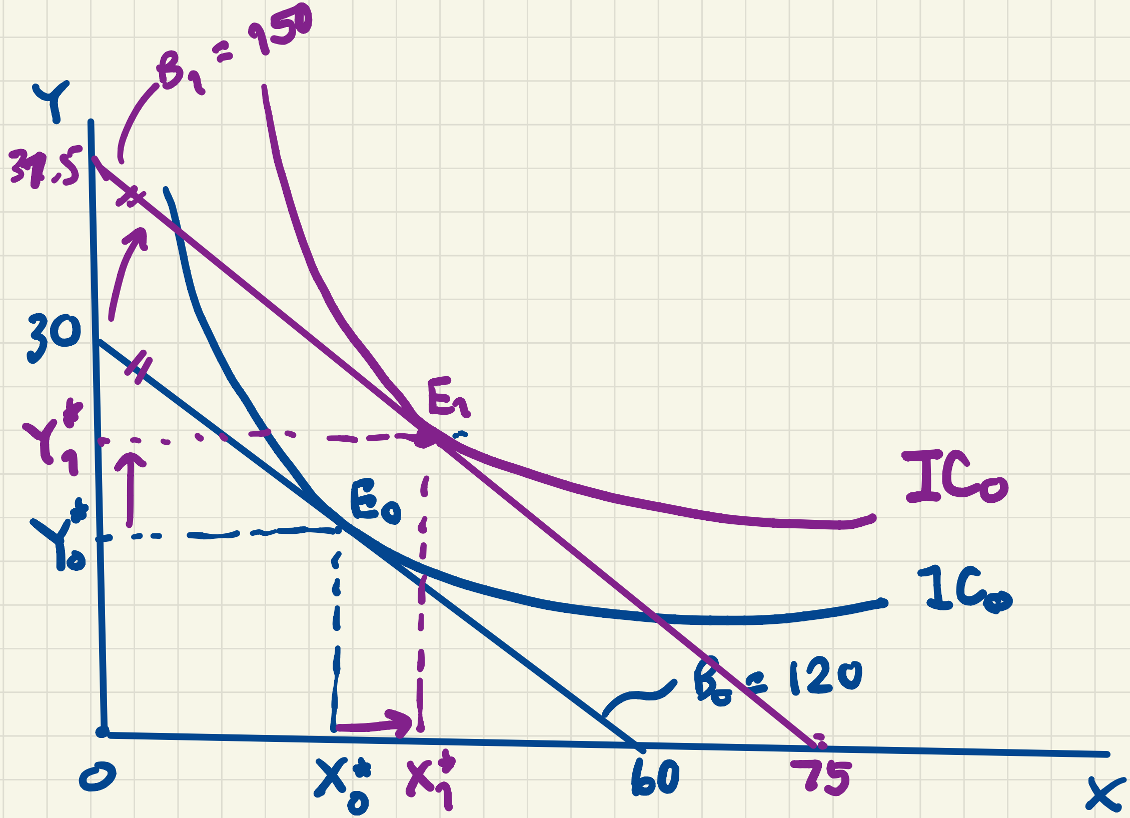
Question: How do we show the change in optimal bundles in U-max problem if:

Let income increase.

1) Both goods X & Y are normal goods

2) Good X is a normal good but Good Y is an inferior good.

Case 1 : Both goods are normal goods.



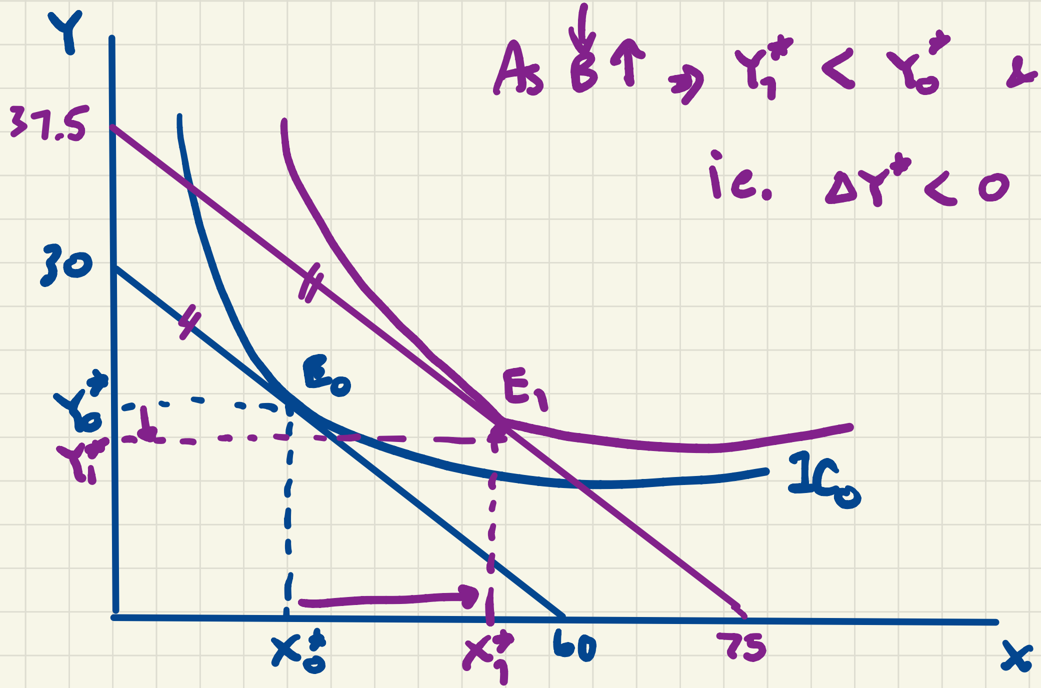
$P_x = 2$, $P_y = 4$, $B_0 = 120$ | Let $B_1 = 150$

Case 2: Good X is normal good, Good Y is inferior good.

Income

As $B \uparrow \Rightarrow Y_1^* < Y_0^*$ & $X_1^* > X_0^*$

i.e. $\Delta Y^* < 0$, $\Delta X^* > 0$



$P_X = 2$, $P_Y = 4$, $B_0 = 120$, $B_1 = 150$

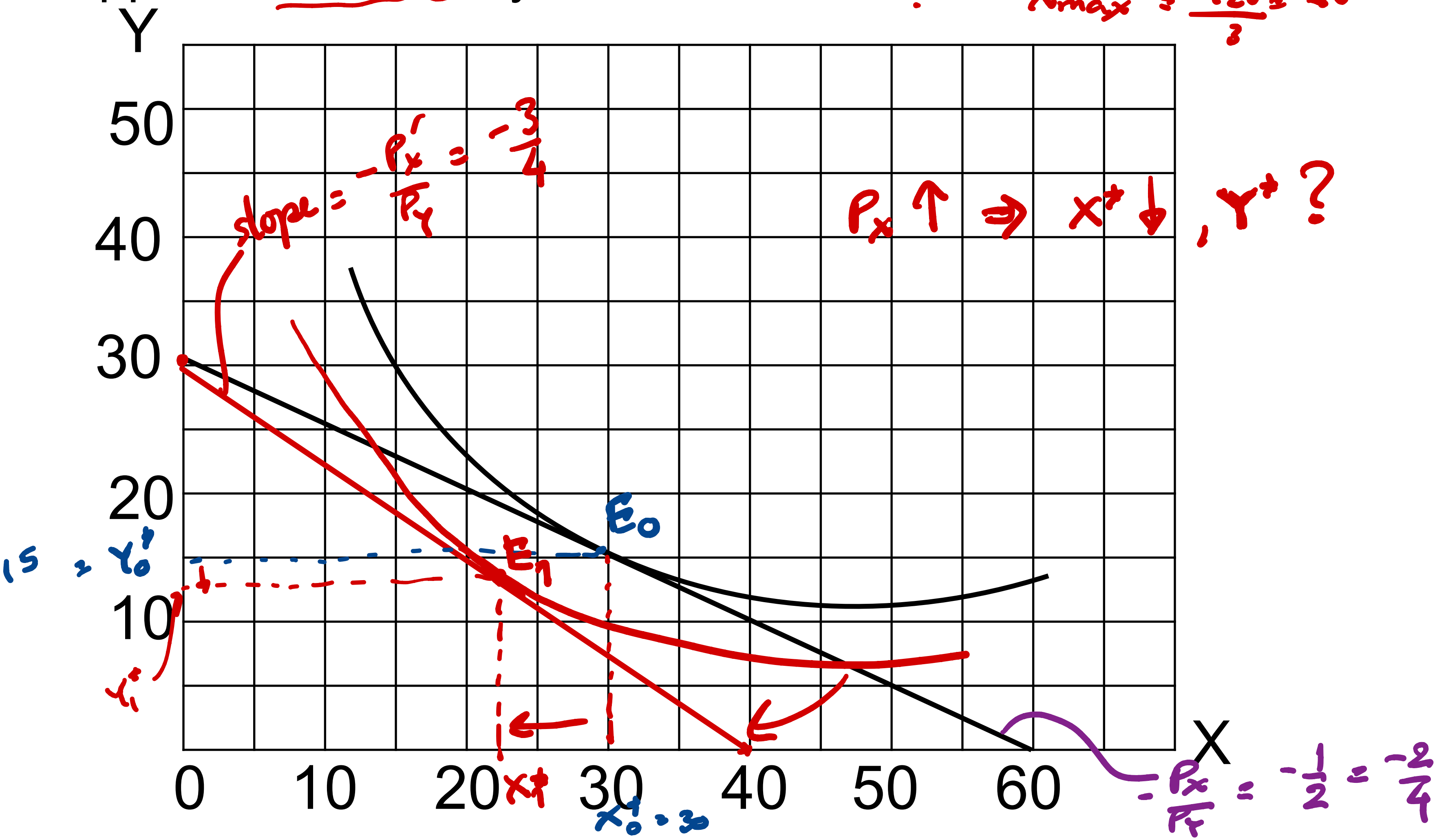
Example: Effect of Price Change (1)

$P_x = \$2$

- Suppose P'x = \$3, Py = \$4, and B = 120.

$Y_{max} = \frac{120}{4} = 30$

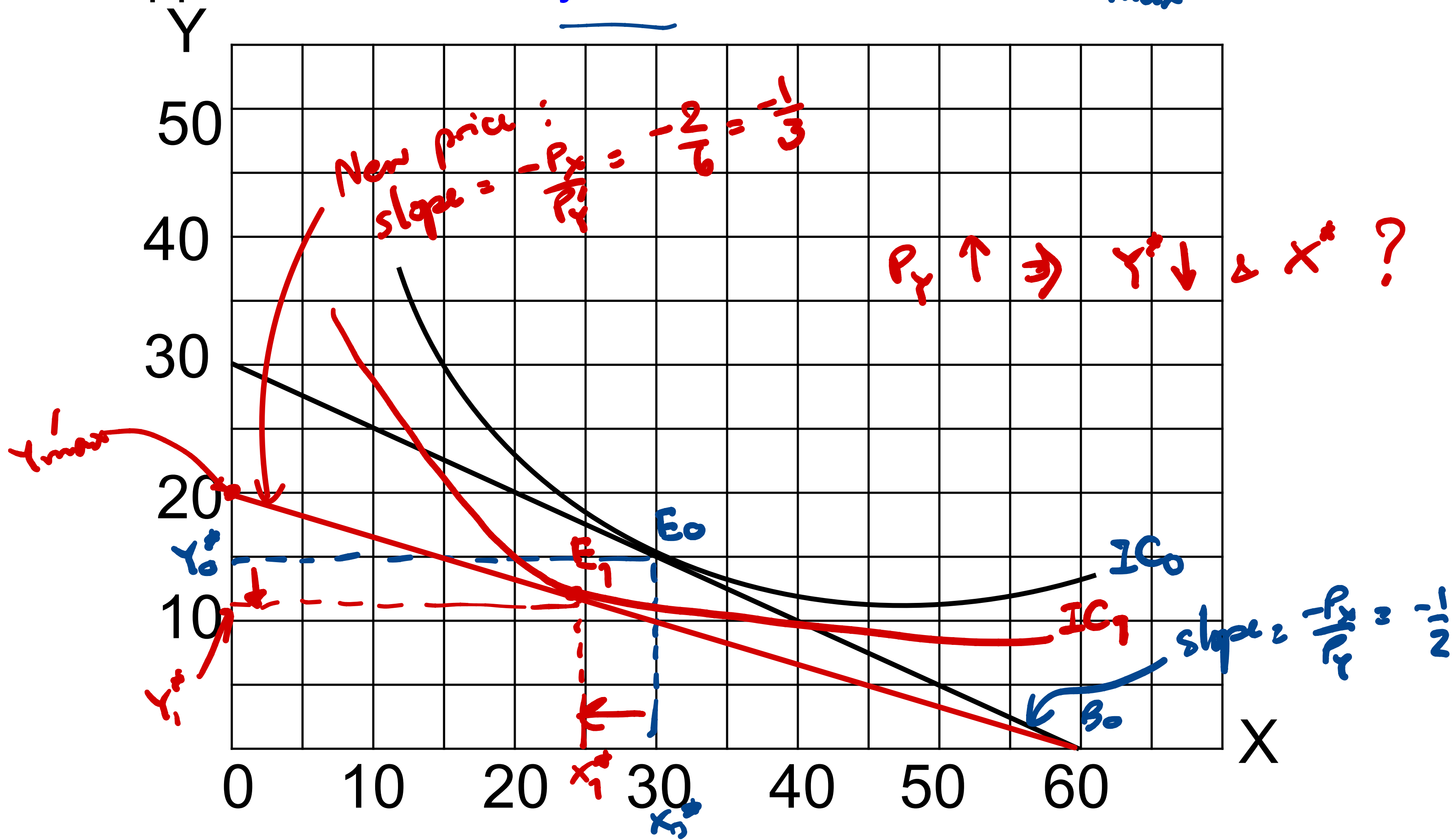
$X'_{max} = \frac{120}{3} = 40$



Example: Effect of Price Change (2)

$P_Y = \$4$ $\quad Y'_{max} = \frac{120}{6} = 20$ $\quad X_{max} = 60$

- Suppose Px = \$2, P'y = \$6, and B = 120.

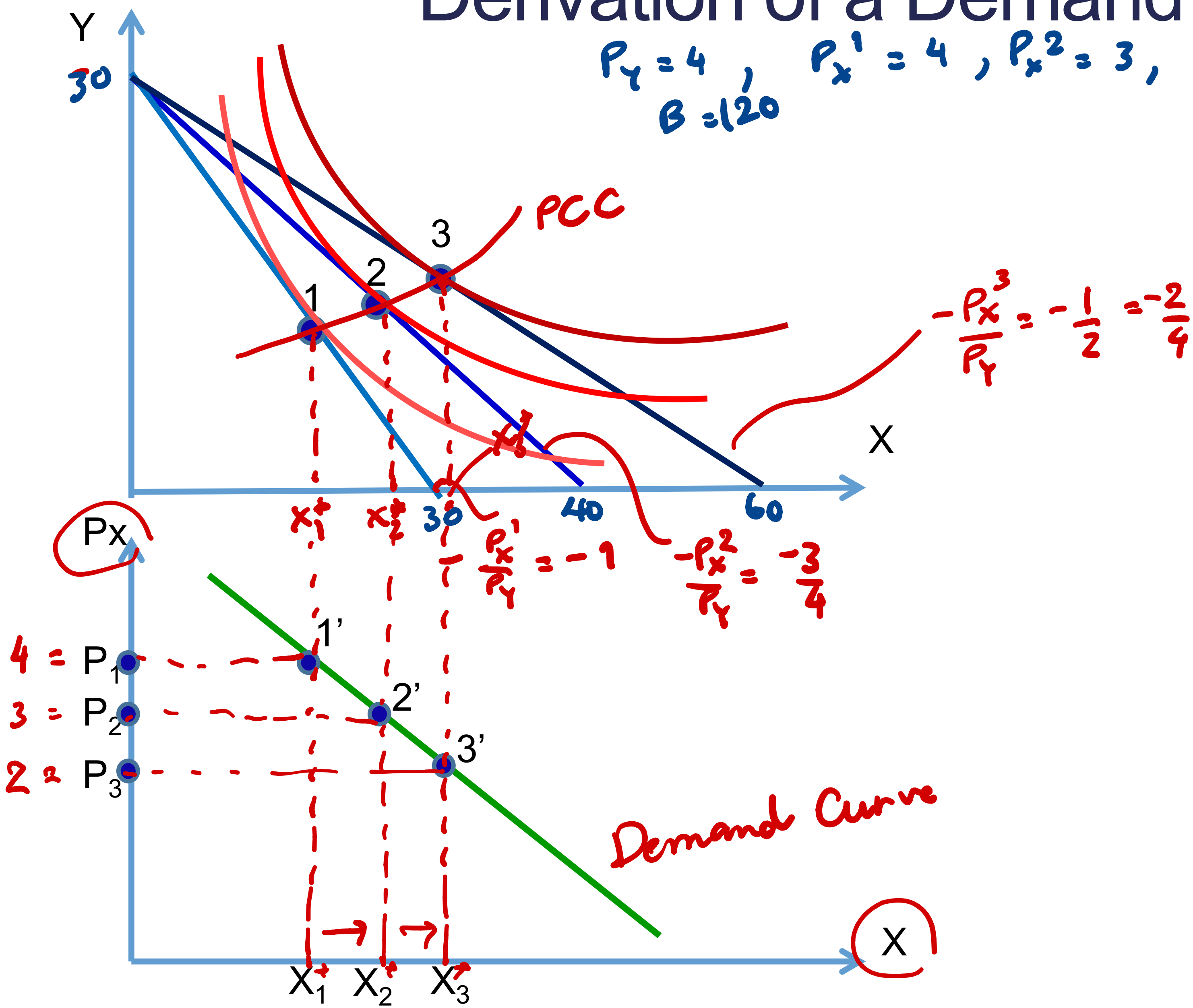


Price Consumption Curve

- A change in relative prices of two goods changes the slope of the budget constraint.
- Suppose Px changes while Py is constant. For each Px, there is a different utility-maximizing consumption bundle. (optimal)
- **Price Consumption Curve (PCC)** is the line that connects all the utility-maximizing points for different Px's, given income and Py constant.
- I.e. , PCC shows how the consumer's purchases react to a change in one price with income and other prices being held constant.

Derivation of a Demand Curve

$P_Y = 4$, $B = 120$, $P_x^1 = 4$, $P_x^2 = 3$, $P_x^3 = 2$



Income and Substitution Effects

- A fall in Px has two effects:

Substitution effect "Price effect"

- Change in X due to change in relative price with *real income* unchanged
- A fall in Px makes Y more expensive relative to X, causing consumer to buy more X and less Y

(Note: Real income is kept unchanged by staying on the original IC.) → "Hicksian Approach"

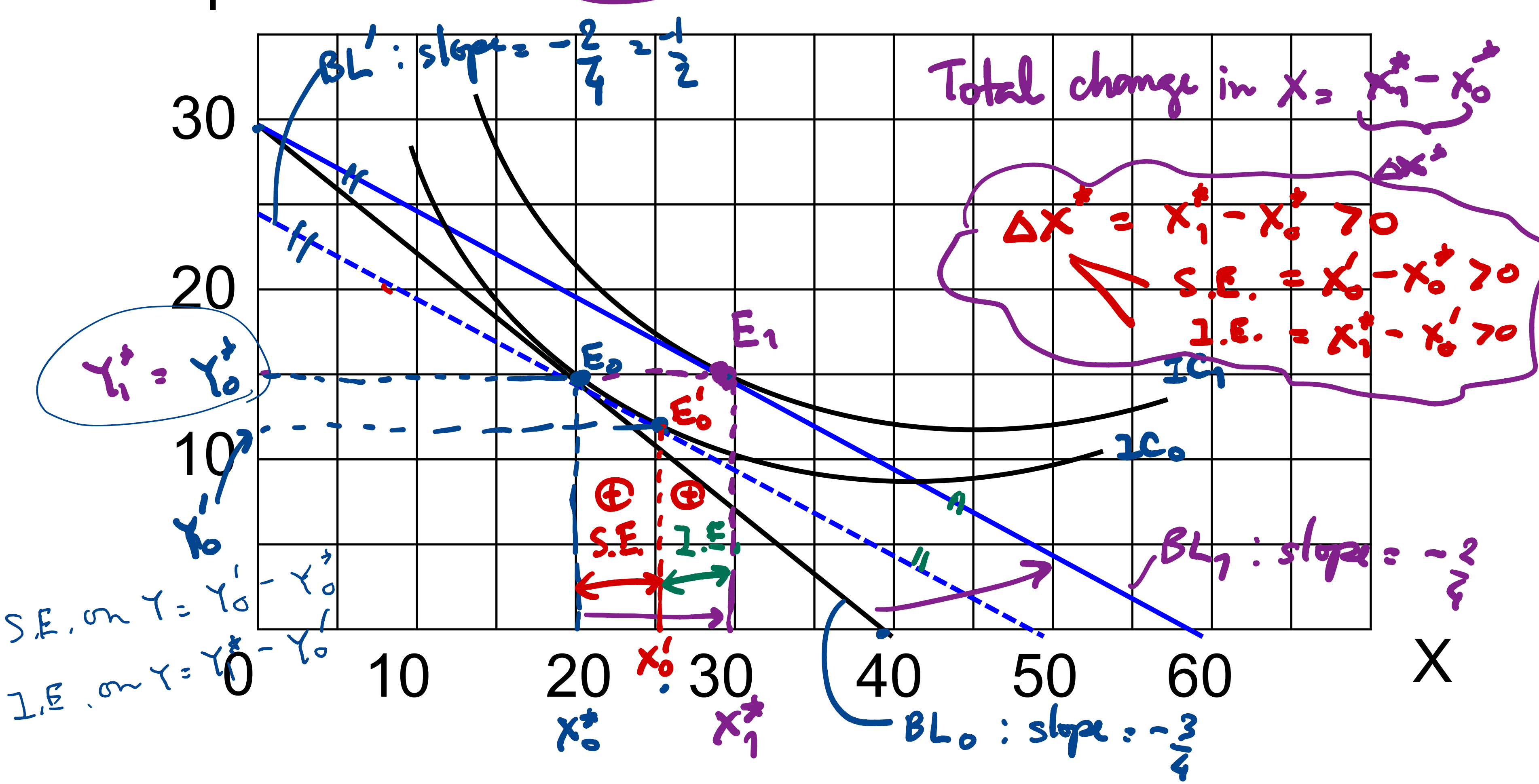
Income effect

- Change in X due to change in real income
- A fall in Px increases the consumer's purchasing power, allowing him to reach a higher IC

Graph

$X_{max} = 40$ $X'_{max} = 60$ $Y_{max} = 30$

- Suppose Px = \$3, P'x = \$2, Py = \$4, and B = 120.



Steps to decompose S.E. and I.E.

1. Start with the original BL & IC. Find the u-max bundle at E_0 (X_0^*, Y_0^*)

2. Determine the impact of the given change (e.g. $P_x \downarrow$) on the BL. (Call new BL: BL_1).

3. Identify the new u-max bundle. Call it E_1 or (X_1^*, Y_1^*).

4. Draw a hypothetical budget line that has the same slope as the new BL (ie. $-\frac{P_x'}{P_y}$) and tangent to the original IC.

→ Call the tangency of the hypothetical BL and the original IC (IC_0) E_0' (X_0', Y_0').

5. Substitution effect (S.E.) on $X = X_0' - X_0^*$

Income effect (I.E.) on $X = X_1^* - X_0'$

Total effect = S.E. + I.E. = $X_1^* - X_0^*$

Note : S.E. on $Y = Y_0' - Y_0^*$

I.E. on $Y = Y_1^* - Y_0'$

Total effect = $= Y_1^* - Y_0^*$