

THE SOCIAL DISCOUNT RATE

EE465/EE463 Project Evaluation

Semester 2/2015

Based on Chapter 10 of *Cost-Benefit Analysis: Concepts and Practice*, Boardman et al., 2011.

Topics

- Social discount rate: Introduction
- Does the choice of discount rate matter?
- The theory behind the appropriate SDR
- Alternatives in deriving SDR from market rates
- The shadow price of capital

Social Discount Rate (1)

- When evaluating government policies or projects, analysts must decide on the appropriate weights to apply to policy impacts that occur in different years.
- Given these weights, denoted by w_t , and estimates of the real annual net social benefits, NB_t , the estimated net present value (NPV) of a project is given by

$$NPV = \sum_{t=0}^n w_t NB_t$$

- Selection of the appropriate **social discount rate (SDR)** is equivalent to deciding on the appropriate *set of weights* for the above problem.
- These weights are sometimes referred to as *social discount factors*.

Social Discount Rate (2)

- Discounting reflects the idea that a given amount of real resources in the future is *worth less today* than the same amount is worth now because:
 - 1) Via investment, one can transform resources that are currently available into a greater amount in the future.
 - 2) People prefer to consume a given amount of resources now, rather than in the future.
- Thus, it is generally accepted that the social discount weights *decline over time*:

$$0 \leq w_n \leq w_{n-1} \leq \dots \leq w_1 \leq w_0 = 1.$$

Social Discount Rate (3)

- Three unresolved issues:
 - 1) Whether market interest rates can be used to determine the weights.
 - 2) Whether to include unborn future generations in determining the weights and, if so, what weight they should have.
 - 3) Whether society values a unit of investment the same as a unit of consumption.
- Different assumptions about these issues lead to different approaches towards determining the SDR, which, in turn, lead to different discount weights

Does the Choice of Discount rate Matter?

Example: Annual Net Benefits and NPV for 3 Alternative Projects

<i>Year</i>	<i>Project A</i>	<i>Project B</i>	<i>Project C</i>
0	−80,000	−80,000	−80,000
1	25,000	80,000	0
2	25,000	10,000	0
3	25,000	10,000	0
4	25,000	10,000	0
5	25,000	10,000	140,000
<i>NPV</i> (<i>i</i> = 2%)	37,836	35,762	46,802
<i>NPV</i> (<i>i</i> = 10%)	14,770	21,544	6,929

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In general,

- low SDRs favor projects with the highest total benefits,
- high SDRs favor projects where the benefits are front-end loaded.

THE THEORY BEHIND THE APPROPRIATE SOCIAL DISCOUNT RATE (SDR)

An Individual's Marginal Rate of Time Preference (MRTP)

- An individual's **MRTP** is the proportion of additional consumption that an individual requires in order to be willing to postpone (a small amount of) consumption for one year.

Example:

- Your rich grandma offers to give you either \$1000 this year or \$1200 next year.
- Suppose you are indifferent between the two options.
 - ➔ Your MRTP is _____

Equality of Discount Rates in Perfect Markets

- In a perfectly competitive capital market, an individual's MRTP (p) equals the market interest rate (i).
- This result is obtained from solving consumer's U-max problem:

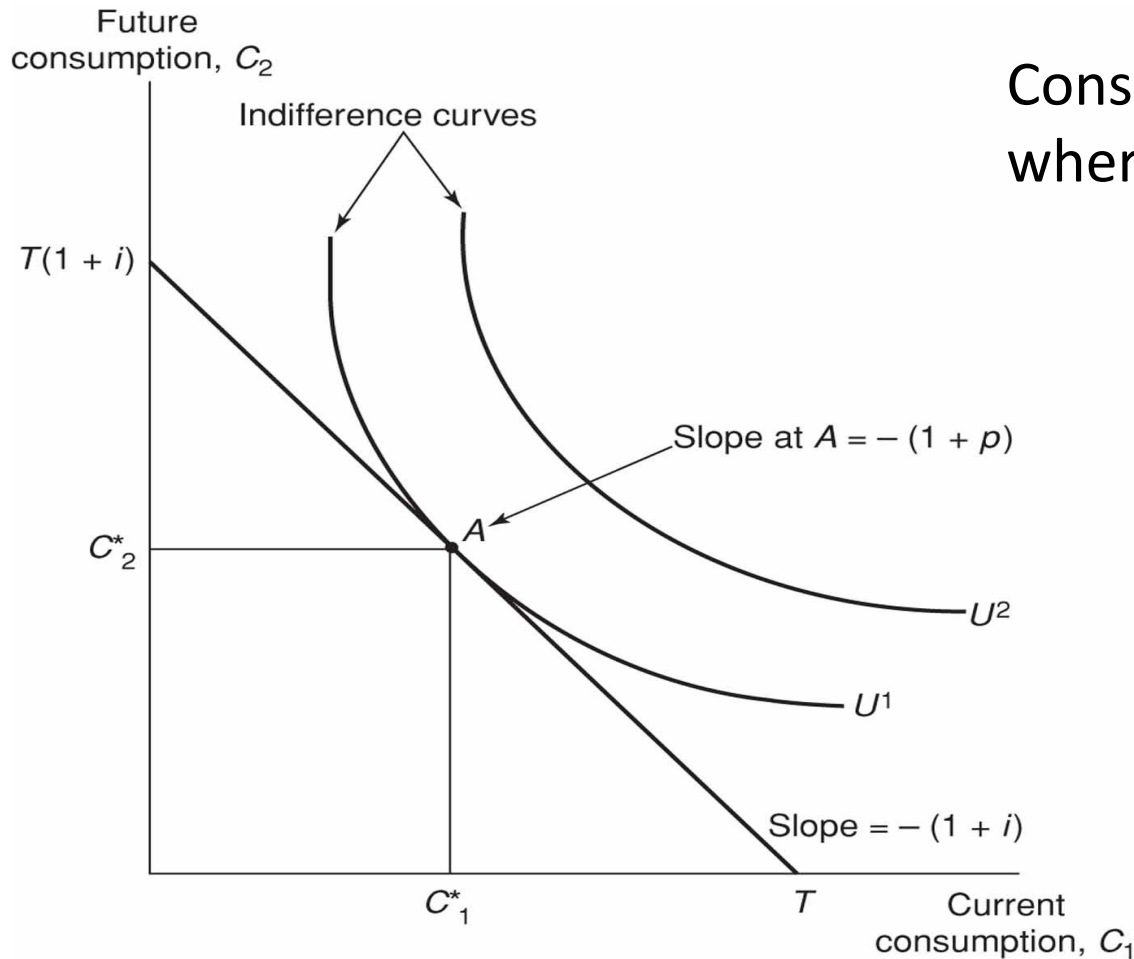
$$\max_{C_1, C_2} U(C_1, C_2)$$

$$\text{subject to } C_1 + \frac{C_2}{1+i} = T$$

→ Rewrite BC: $C_2 =$ _____

→ Slope of BC is _____

MRTP in a Two-Period Model



Consumption is maximized at A
where: $MRS = \text{slope of BC}$

$MRS = -(1+p)$ &
 $\text{slope of BC} = -(1+i)$.



Here, the market interest rate can be used as the social discount rate.

A Simple Two-Period Model with Production

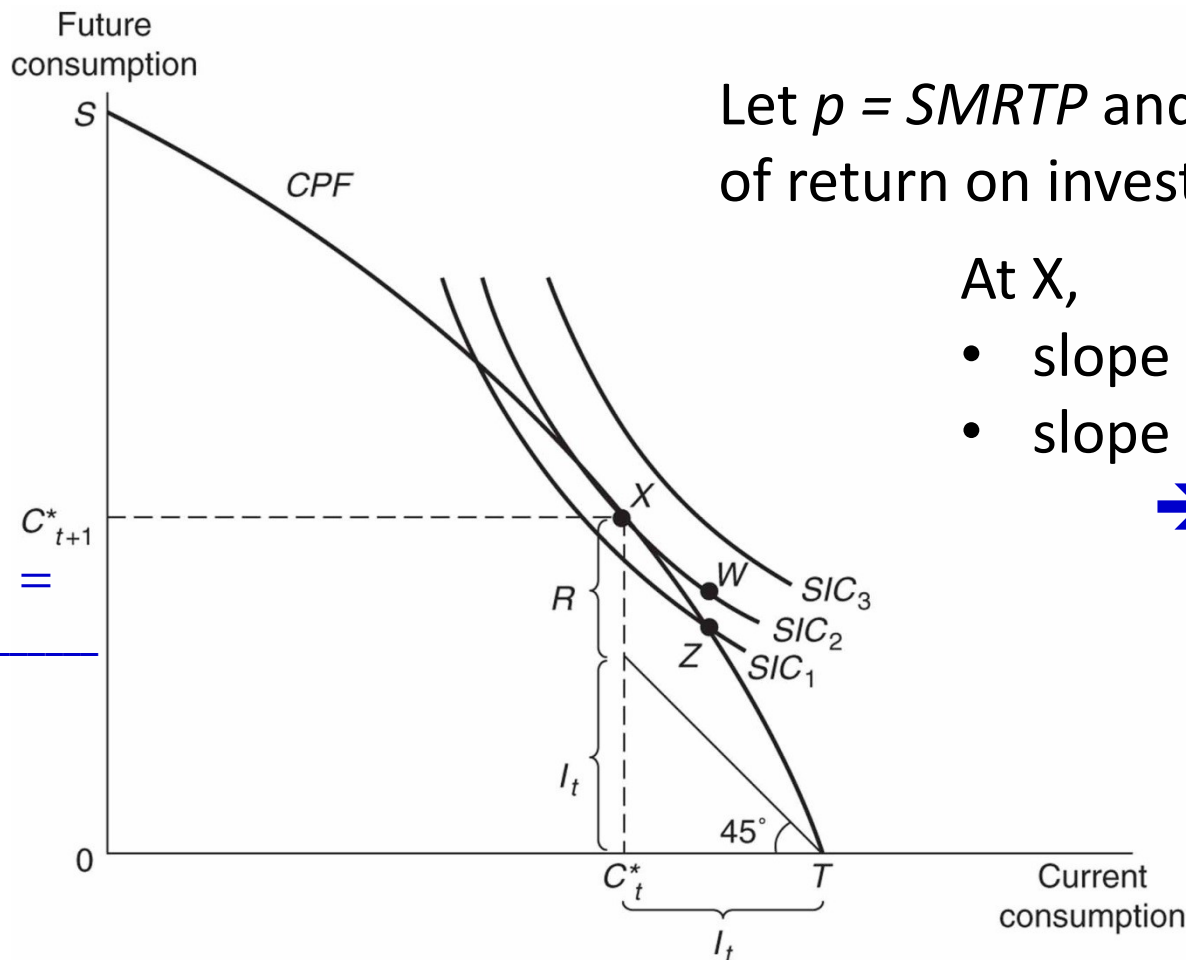
- Consider a more general, multi-individual, two-period model with production of a hypothetical country.
- Assume: no tax, no transaction cost, no market failures, etc.
- Trade-off between current and future consumptions are represented by *consumption possibility frontier (CPF)*, which is a concave curve.
- Society's preference is represented by *social indifference curve (SIC)*, and its slope is the *social marginal rate of time preference (SMRTP)*.
- SMRTP is the extra amount of future consumption that society requires as a compensation for giving up one unit of current consumption.

The Optimal Levels of Consumption and Investment in a Two-Period Model

Let $p = SMRTP$ and r_x be the marginal rate of return on investment, at point X.

At X,

- slope of CPF = $1 + r_x$
- slope of SIC = $1 + p_x$

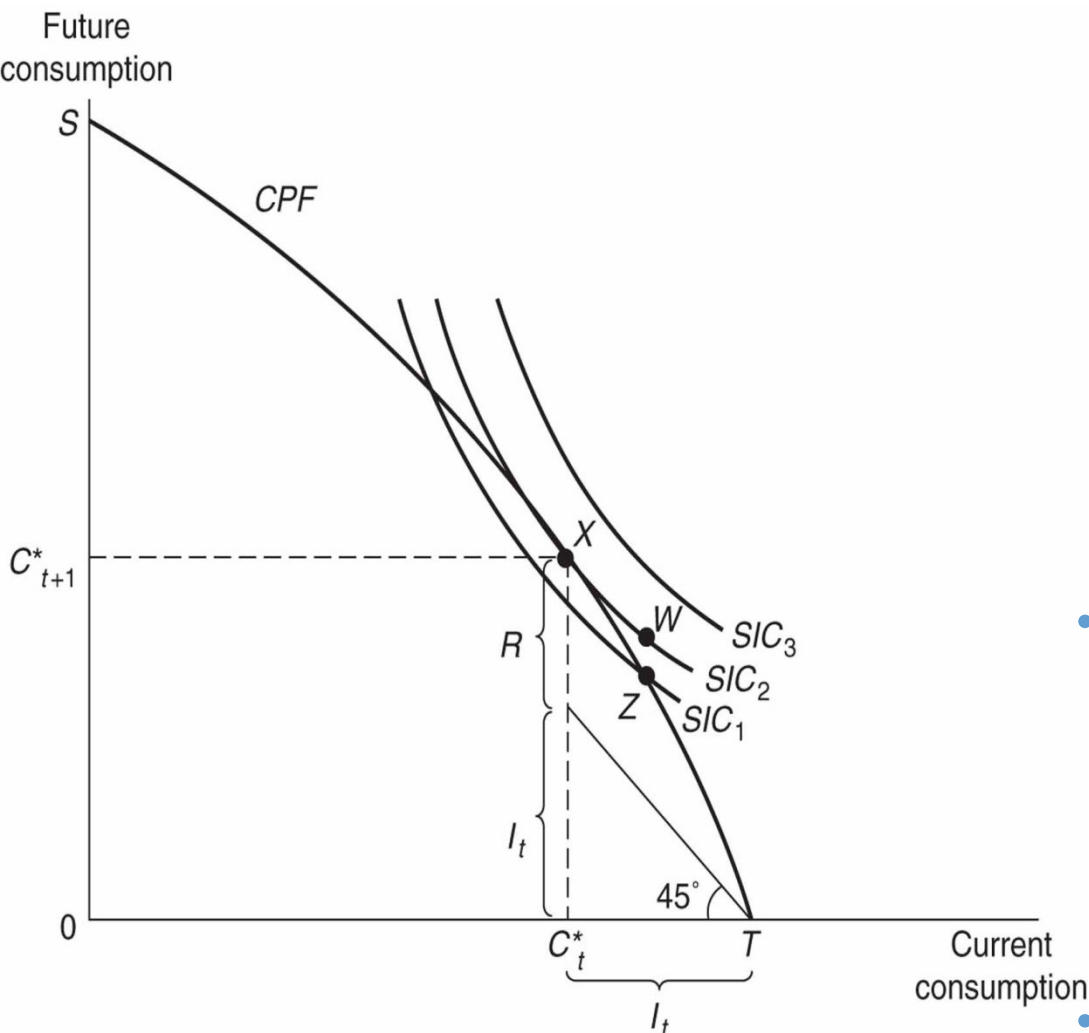


$$I_t = \underline{\hspace{2cm}}$$

The First-Best Solution in the Two-Period Model

- From previous slide, society achieves its first-best solution when the SMRTP will equal the marginal rate of return on investment (i.e. $p_x = r_x$).
- These rates would also equal the economy-wide market interest rate, i ($= r$).
- This implies that all individuals will have the same MRTP because if their $MRTP > i$ they would borrow at i and consume more in the current period until their $MRTP = i$; if $MRTP < i$ they would invest until their $MRTP = i$.
- Since everyone's MRTP equals i , it should be the unanimous choice for the social discount rate.

Problems with Real Economies: The Second Best



- An actual economy (with taxes, risk and transaction costs) would not operate at the optimal point X , but at a point such as Z , which is a **second-best outcome**. Here, society would *underinvest* and _____.
- Also, different people have different preferences, face different tax rates, numerous values exist for both MRTPs and $1+r$.
- **No obvious choice for SDR.**

DERIVING THE SDR FROM THE MARKET

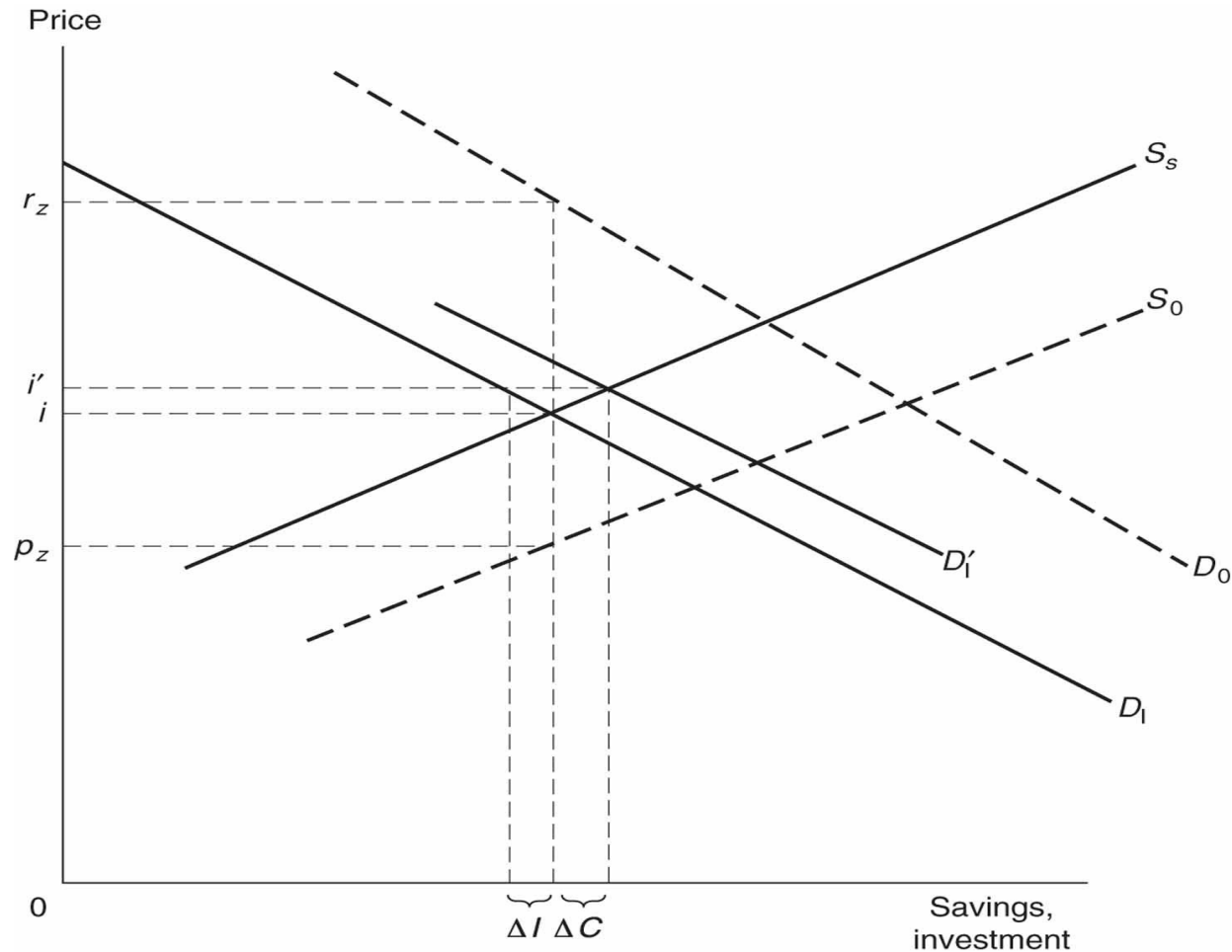
Alternative SDRs

- Assume for now that the social discount weights are constant over time.
- Four potential social discounting rates that are derived from rates observable in markets.
 1. Using the marginal rate of return on private investment (r_z)
 2. Using the Marginal Social Rate of Time Preference (p_z) method
 3. Using the government's borrowing rate (i)
 4. Using the weighted average approach

I. Marginal Rate of Return on Private Investment (r_z)

- Argument: Society should receive an equal rate of return in the public sector to what it would have received had the resources remained in the private sector.
- Arnold Harberger's paper:
 - Assume that a government project would be financed entirely by borrowing in a closed domestic financial market, which results in higher market rate of interest.
 - 2 effects:
 1. Private-sector investment falls (ΔI).
 2. Private savings increase $\sim$ Private consumption falls (ΔC).
 - i.e. the project would "crowd-out" both investment and consumption

The Effects of Taxes and Government Borrowing on Savings and Investment



Marginal Rate of Return on Private Investment (cont'd)

- Harberger suggests that the social discount rate should be obtained by weighting r_z and p_z by the relative contributions that I and C would make toward funding the project.
- Hence, SDR should be computed using the weighted average cost of capital:

$$WSOC = ar_z + bp_z$$

where $a = \underline{\hspace{2cm}}$ and $b = \underline{\hspace{2cm}}$

- Moreover, Harberger claims that savings are not very responsive to changes in interest rates, and hence $\Delta C \approx 0$. Thus, r_z is a good approximation of the SDR.
- Numerical value of r_z – the best proxy of r_z is the real, before-tax rate of return on corporate bonds.

II. Marginal Social Rate of Time Preference (p_z)

- Argument: SDR should be thought of as the rate at which individuals in society are willing to postpone a small amount of current consumption in exchange for additional future consumption (and vice versa); i.e. at the consumption rate of interest (CRI).
- In principle, p_z represents this rate.
- Numerical Values of p_z - In practice, the best return that most people can earn in exchange for postponing consumption is the real after-tax return on savings.
- Example: Suppose the nominal, pre-tax interest rate on government bonds and adjusting for taxes on savings and inflation yields estimates of p_z between 0.00 and 0.04.
 - This method suggests using a real SDR = 2% with sensitivity analysis at 0% and 4%.

III. Government's Borrowing Rate (i)

- Argument: the government should discount projects using its long-term borrowing rate, i , it reflects the actual cost of financing a project.
- Numerical Values of i – the real rate of return on 10-year Treasury bonds (Adjusted by inflation)
- Example:
 - The average monthly yield on 10-year U.S. Treasury bonds for the period between April 1953 and December 2001 was 6.71% and CPI during that period is 3.89%.
 - A value for i of 2.7 %, with a plausible range for sensitivity analysis of 1.7% to 3.7%.

IV. Weighted Average Approach (WSOC)

- Argument: SDR should be calculated in terms of the social opportunity cost of the different sources weighted according to the relative contribution of each source.
- If a is the proportion of the project's resources that displaces private domestic investment, b is the proportion financed by borrowing from foreigners, and $1-a-b$ is the proportion that displaces domestic consumption, then SDR is the weighted social opportunity cost of capital (WSOC) :

$$WSOC = ar_z + bi + (1 - a - b)p_z$$

where i = the government's real, long-term borrowing rate.

Note that: As $p_z < i < r_z$, it follows that $p_z < WSOC < r_z$.

SHADOW PRICE OF CAPITAL

Shadow Price of Capital (SPC)

Argument:

- Due to market distortions, the rate at which individuals are willing to trade present for future consumption, p_z , differs from the rate of return on private investment, r_z .
- Thus, flows of investment should be treated differently from flows of consumption; i.e. investment flows should be weighted by some parameters.
- The **shadow price of capital (SPC)**, denoted by θ , is a parameter that converts investment gains or losses into *consumption equivalents*.
- In SPC discounting method, these consumption equivalents, like consumption flows themselves, are then discounted at p_z .

Shadow Price of Capital Method (1)

The shadow price of capital method requires that discounting be done in four steps:

- 1) Costs and benefits in each period are divided into those that affect consumption and those that affect investment.
- 2) Flows into and out of investment are multiplied by the SPC to convert them into consumption equivalents.
- 3) Changes in consumption are added to changes in consumption equivalents.
- 4) Resulting amounts are discounted at p_z .

Shadow Price of Capital Method (2)

- A general expression for the shadow price of capital is:

$$\theta = \frac{(r_z + \delta)(1 - f)}{p_z - r_z f + \delta(1 - f)}$$

where r_z is the net return on capital after depreciation,
 δ is the depreciation rate of the capital invested,
 f is the fraction of the gross return on capital reinvested,
and p_z is the marginal social rate of time preference.

- Note: In the absence of reinvestment and depreciation, SPC becomes:

Example: Shadow Price of Capital (2)

Example: Let $r_z=2\%$, $p_z=1.5\%$. $\rightarrow \theta = \underline{\hspace{2cm}}$

- Consider a project that is financed by a bond issue of \$3 million.
- Assume the bond yields real interest rate of 4% per year through five equal annual installments of \$673,854.
- The estimated benefits from the project are \$700,000 per year for five years.
- Assume the entire amount borrowed would displace private-sector investment (Harberger's crowd-out assumption).

Example: Shadow Price of Capital (2)

Year	Principal	Annual Installment	Interests	Loan Repayment	Loan Balance
1	3,000,000	673,854	120,000	553,854	2,446,146
2	2,446,146	673,854	97,846	576,008	1,870,138
3	1,870,138	673,854	74,806	599,048	1,271,089
4	1,271,089	673,854	50,844	623,010	648,080
5	648,080	673,854	25,923	648,080	0

Assume also:

- the *full* amount of each loan repayment will be reinvested.
- changes in consumption in each period include the project benefits, loan repayment, and interest on the loan.

Example: Shadow Price of Capital (2)

ΔI_0	-3,000,000
ΔI_1	553,854
ΔI_2	576,008
ΔI_3	599,048
ΔI_4	623,010
ΔI_5	648,080

ΔC_0	=	0
ΔC_1	$700,000 - 673,854 + 120,000 =$	146,146
ΔC_2	$700,000 - 673,854 + 97,846 =$	123,992
ΔC_3	$700,000 - 673,854 + 74,806 =$	100,952
ΔC_4	$700,000 - 673,854 + 50,844 =$	76,146
ΔC_5	$700,000 - 673,854 + 25,923 =$	52,146

- With SPC, NPV =
- Without SPC, NPV =