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EE 325 Section 1 (Aj.Wanwiphang) Homework Assignment 1

Due date: 31 January 2020 before 11pm

*** Please submit this assignment on Moodle. For those who work on paper, please scan or submit the pictures of your work.***

1. Find the answers following questions (please also show your calculation)

   a. $\sum_{i=1}^{5}(a+bx_i)$

   $5a + bx_1 + bx_2 + bx_3 + bx_4 + bx_5$

   b. $\sum_{y=0}^{5} f(x+y)$

   $f(x) + f(x+1) + f(x+2) + f(x+3) + f(x+4) + f(x+5)$

   c. $\sum_{i=1}^{10} i^2$

   $1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2$

   $= 385$

   d. $\sum_{x=1}^{2}\sum_{y=2}^{3}(2x+y)$

   $\sum_{x=1}^{2}\left(\sum_{y=2}^{3}(2x+y)\right)$ $= \sum_{x=1}^{2}(2x+2+2x+3)$

   $= 2(1)+2+2(2)+3$

   $= 22$

2. Given X is discrete random variable. The probability distribution function (PDF) of this variable is shown in the table

0.25 0.1875

X	-2	-1	0	1	2	3	4
$f(x)$	0.5b	b	2.25b	2b	1.5b	0.5b	0.25b

** when b is constant number

   a. Find the value of b

   $0.5b + b + 2.25b + 2b + 1.5b + 0.5b + 0.25b = 1$

   $8b = 1$

   $b = 0.125$

   b. Find the answer for $P(X \le 2)$

   $1-(0.5(0.125) + 0.25(0.125))$

   $1 - 0.09 = 0.90625$

   c. Find the answer for $P(-2 \le X \le 3)$

   $1 - 0.25(0.125) = 0.96875$

   d. Find the answer for $P(X \ge 1)$

   $1 - P(X \le 2) + P(X=2) + P(X=1) = 0.53125$

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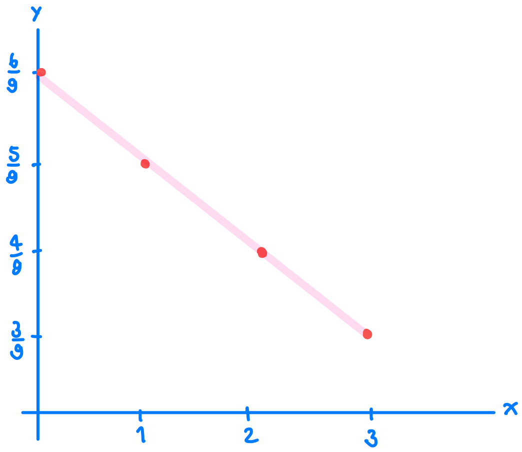
3. Given X is continuous random variable. The probability distribution function (PDF) of this variable is

$$f(x) = -\frac{1}{9}x + \frac{6}{9}, 0 \le x \le 3$$

a. Plot graph for $f(x)$



if $x = 0$

$-\frac{1}{9}(0) + \frac{6}{9} = y$

$\frac{2}{3} = y$

if $x = 2$

$-\frac{1}{9}(2) + \frac{6}{9} = y$

$\frac{4}{9} = y$

if $x = 1$

$-\frac{1}{9}(1) + \frac{6}{9} = y$

$\frac{5}{9} = y$

if $x = 3$

$-\frac{1}{9}(3) + \frac{6}{9} = y$

$\frac{3}{9} = y$

b. Find the answer for $P(1 \le X \le 3)$

$$\int_1^3 -\frac{1}{9}x + \frac{6}{9}\, dx$$

$$P(1 \le x \le 3) = \left. \frac{-\frac{1}{9}x^2}{2} + \frac{6}{9}x \right|_1^3$$

$$= \frac{-\frac{1}{9}(3)^2}{2} + \frac{6}{9}(3) + \frac{\frac{1}{3}(1)^2}{2} - \frac{6}{9}(1)$$

$$= -\frac{1}{2} + \frac{18}{9} + \frac{1}{6} - \frac{6}{9}$$

$$= \frac{16}{18}$$

c. Find the answer for $P(X \ge 2)$

$$\int_1^2 -\frac{1}{9}x + \frac{6}{9}\, dx$$

$$P(X \ge 2) = \left. \frac{-\frac{1}{9}x^2}{2} + \frac{6}{9}x \right|_2^3$$

$$= \frac{-\frac{1}{9}(3)^2}{2} + \frac{6}{9}(3) + \frac{\frac{1}{3}(2)^2}{2} - \frac{6}{9}(2)$$

$$= \frac{7}{1}$$

d. Find the expected value of X

$$E(x) = \int x f(x)\, dx$$

$$= \int -\frac{1}{9}x^2 + \frac{6}{9}x\, dx$$

$$= \left(-1 + \frac{59}{18}\right) - 0$$

$$= 2$$

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4. Let random variable X be the outcome of throwing one dice and random variable Y be the outcome of tossing one coin. Coin has two sided that has valued 1 and 0. x and y are independent so. $P(x) \cdot P(y) = P(x,y)$

   a. Construct the joint probability distribution function (PDF) table of X and Y

Y \ X	1	2	3	4	5	6
0	$\frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$
1	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$

   b. Find the marginal probability distribution function (PDF) of X

$P(x=1) = \frac{1}{12} + \frac{1}{12} = \frac{2}{12} \approx 0.167$

x_i	$P(X=x_i)$
1	0.167
2	0.167
3	0.167
4	0.167
5	0.167
6	0.167

   c. Find the marginal probability distribution function (PDF) of Y

$P(Y=1) = \frac{1}{12} 6 = \frac{1}{2}$

y_i	$P(Y=y_i)$
0	$\frac{1}{2}$
1	$\frac{1}{2}$

   d. Find the conditional probability distribution function (PDF) of X given Y is equal to 1

$P(X=x_i \mid Y=1) = \frac{P(X=1, Y=1)}{P(Y=1)} = \frac{\frac{1}{12}}{\frac{1}{2}} = \frac{1}{6}$

$P(X=x_i \mid Y=1) = \frac{P(X=2, Y=1)}{P(Y=1)} = \frac{\frac{1}{12}}{\frac{1}{2}} = \frac{1}{6}$

$P(X=x_i \mid Y=1) = \frac{P(X=3, Y=1)}{P(Y=1)} = \frac{\frac{1}{12}}{\frac{1}{2}} = \frac{1}{6}$

$P(X=x_i \mid Y=1) = \frac{P(X=4\ Y=1)}{P(Y=1)} = \frac{\frac{1}{12}}{\frac{1}{2}} = \frac{1}{6}$

$P(X=x_i \mid Y=1) = \frac{P(X=5, Y=1)}{P(Y=1)} = \frac{\frac{1}{12}}{\frac{1}{2}} = \frac{1}{6}$

$P(X=x_i \mid Y=1) = \frac{P(X=6, Y=1)}{P(Y=1)} = \frac{\frac{1}{12}}{\frac{1}{2}} = \frac{1}{6}$

   e. Find the expected value of X given Y is equal to 1

$$E(X \mid Y=1) = \frac{\sum_i x_i P(X=x_i, Y=1)}{P(Y=1)}$$

$$= \frac{1}{P(Y=1)} \sum x_i P(X=x_i, Y=1)$$

$$= \frac{1}{\frac{1}{2}} \circ \left[(1)\frac{1}{12} + (2)\frac{1}{12} + (3)\frac{1}{12} + (4)\frac{1}{12} + (5)\frac{1}{12} + (6)\frac{1}{12}\right]$$

$$= 3.5$$

   f. Find the variance of X given Y is equal to 1

$var(X \mid Y = y_i)$

$= \sum_x [x - E(X \mid Y=y)]^2 f(x \mid y=y_i)$

$$var(X \mid Y=1) = \sum_x [x - E(X \mid Y=1)]^2 f(x \mid Y=1)$$

$$= \left[(1-3.5)^2 + (2-3.5)^2 + (3-3.5)^2 + (4-3.5)^2 + (5-3.5)^2 + (6-3.5)^2\right] \left(\frac{1}{6}\right)$$

$$= 2.91668$$

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5. If X_1, X_2, X_3 is a random sample from a population with mean u and variance σ^2. X_1, X_2, X_3 are not independent

$$Cov(X_1, X_2) = Cov(X_1, X_3) = Cov(X_2, X_3) = \frac{1}{4}\sigma^2$$

$\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{3}(X_1 + X_2 + X_3)$

Find $E(\bar{X})$ and $\text{var}(\bar{X})$

$E(\bar{x}) = E\left[\frac{1}{3}(x_1 + x_2 + x_3)\right]$

$= \frac{1}{3}E(x_1 + x_2 + x_3)$

$E(\bar{x}) = \frac{1}{3}(3\mu)$

$E(\bar{x}) = \mu$

$var(\bar{x}) = var\left[\frac{1}{3}(x_1 + x_2 + x_3)\right]$

$= \frac{1}{9} var(x_1 + x_2 + x_3)$

$= \frac{1}{9} var(var(x_1) + var(x_2) + var(x_3) + 2cov(x_1 x_2)$

$+ 2cov(x_2 x_3) + 2cov(x_1, x_3))$

$= \frac{1}{9}\left[3\sigma^2 + \frac{3}{4}\sigma^2\right]$

$var(\bar{x}) = \frac{\sigma^2}{2}$

6. Given X_1, X_2, X_3, X_4 are independent identically distributed random variables from population with mean u and variance σ^2. $\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{4}(X_1 + X_2 + X_3 + X_4)$

a. Find $E(\bar{X})$ and $\text{var}(\bar{X})$ in term of μ and σ

$E(\bar{x}) = E\left[\frac{1}{4}(x_1 + x_2 + x_3 + x_4)\right]$

$= \frac{1}{4}\left[\underbrace{E(x_1)}_{\mu} + \underbrace{E(x_2)}_{\mu} + \underbrace{E(x_3)}_{\mu} + \underbrace{E(x_4)}_{\mu}\right]$

$= \frac{1}{4}[4\mu]$

$= \mu$ #

$var(\bar{x}) = var\left[\frac{1}{4}(x_1 + x_2 + x_3 + x_4)\right]$

$= \frac{1}{16}\left[var(x_1) + var(x_2) + var(x_3) + var(x_4)\right]$

$= \frac{1}{16}[4\sigma^2]$

$= \frac{1}{4}\sigma^2$ #

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b. Given $\tilde{X} = \frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4$ is another estimator of μ. Show that $\tilde{X}$ is an unbiased estimator of u

$$E(\tilde{x}) = E\left[\frac{1}{8}x_1 + \frac{1}{4}x_2 + \frac{1}{8}x_3 + \frac{1}{2}x_4\right]$$

$$E(\tilde{x}) = \frac{1}{8}E(x_1) + \frac{1}{4}E(x_2) + \frac{1}{8}E(x_3) + \frac{1}{2}E(x_4)$$

$$E(\tilde{x}) = \frac{1}{8}\mu + \frac{1}{4}\mu + \frac{1}{8}\mu + \frac{1}{2}\mu$$

$$E(\tilde{x}) = \mu$$

$E(\tilde{x}) = \mu$, so $\tilde{x}$ is an unbiased estimator of μ.

c. Between $\bar{X}$ and $\tilde{X}$, which one is the better estimator for μ? Why?

$$var(\tilde{x}) = var\left[\frac{1}{8}x_1 + \frac{1}{4}x_2 + \frac{1}{8}x_3 + \frac{1}{2}x_4\right]$$

$$var(\tilde{x}) = \frac{1}{64}var(x_1) + \frac{1}{16}var(x_2) + \frac{1}{64}var(x_3) + \frac{1}{4}var(x_4)$$

$$var(\tilde{x}) = \frac{11}{32}\sigma^2$$

$var(\bar{x}) < var(\tilde{x})$, so $\tilde{x}$ is the better estimator.