Q1:

1.1. Linear in parameters but not in variables

Linear regression

1.2. Linear in parameters but not in variable

Linear regression

1.3. linear in parameters but not in variables (but if we define $Y_i^{new} = \ln Y_i$, then it is linear in variables)

Linear regression

1.4. linear in parameters but not in variables

but if we define

$$Y_i^{new} = \ln Y_i$$

$$X_i^{new} = \ln X_i$$

$$Y_i^{new} = \ln \beta_1 + \beta_2 X_i^{new} + \mu_i$$

Then, it is linear in variables.

Linear regression

1.5 linear in parameters but not in variable

Since X^{-1} (with power -1)

Linear regression

Q_2

① $Y_i = \dfrac{1}{\beta_1 + \beta_2 X_i}$

$$\beta_1 Y_i + \beta_2 X_i Y_i = 1$$

$$\therefore \quad \frac{1}{Y_i} = \beta_1 + \beta_2 X_i$$

∴ It is linear in parameters → linear regression model.

② $Y_i = \dfrac{X_i}{\beta_1 + \beta_2 X_i}$

$$\beta_1 Y_i + \beta_2 X_i Y_i = X_i$$

$$\frac{X_i}{Y_i} = \beta_1 + \beta_2 X_i$$

∴ It is linear in parameters → linear regression model.

③ $$Y_i + Y_i \exp(-\beta_1 - \beta_2 X_i) = 1$$

$$\frac{1 - Y_i}{Y_i} = \exp(-\beta_1 - \beta_2 X_i)$$

$$\ln\left[\frac{1 - Y_i}{Y_i}\right] = -\beta_1 - \beta_2 X_i$$

∴ It is linear in parameters & → linear regression model.