B.E. International Program

Faculty of Economics, Thammasat University

Semester: 2/2014
EE325 Introductory Econometrics (section 046402)
Homework#2

For the coming midterm, at least 1 question will be on the exam.

1. Although we can simply study about Y (dependent variabe) by using its mean value to explain, we also have to concern regression analysis, why? Explain.

2. Phillip's Curve shows inverse relationship between unemployment and inflation rate, $\pi_t = a - bU_t$. There is an argument that Phillip's Curve is not true because some points (π_t, U_t) in scatter diagram are not on the curve. Unemployment level and price level could go in the same direction sometimes. Therefore, we can conclude that unemployment rate is not a function of inflation. Provide some comments on the above argument. Do you agree or disagree?

3. Answer question 3.1 to 3.8 from this table

X_i	Y_i	$X_i - \bar{X}$	$(X_i - \bar{X})^2$	$Y_i - \bar{Y}$	$(X_i - \bar{X})(Y_i - \bar{Y})$
0	6				
1	5				
2	3				
3	1				
4	0				
$\sum X_i =$	$\sum Y_i =$	$\sum(X_i - \bar{X}) =$	$\sum(X_i - \bar{X})^2 =$	$\sum(Y_i - \bar{Y})=$	$\sum(X_i - \bar{X})(Y_i - \bar{Y}) =$

3.1) Fill in the table above

3.2) Consider the two-variable model: $Y_i = \beta_1 + \beta_2 X_i + u_i, \quad u_i \sim NIID(0, \sigma^2)$.

Use ordinary least squares(OLS) method to find estimators of β_1 and β_2. Interpret the meaning.

3.3) Predict value of Y using the estimators from (4.2) and fill in the following table.

X_i	Y_i	$(Y_i - \bar{Y})^2$	$\hat{Y}_i$	$\hat{u}_i$	$\hat{u}_i^2$	$X_i \hat{u}_i$
0	6					
1	5					
2	3					
3	1					
4	0					
		$\sum(Y_i - \bar{Y})^2 =$	$\sum \hat{Y}_i =$	$\sum \hat{u}_i =$	$\sum \hat{u}_i^2 =$	$\sum X_i \hat{u}_i =$

3.4) Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?

3.5) Find estimated variance of error and variance of the slope.

3.6) Find value of R^2 and interpret.

3.7) Test the hypothesis that X is statistically significant variable.

3.8) Establish a 95 percent confident interval for β_2 .

4. Given this information

$n = 10$	$\sum_{i=1}^{n} X_i = 33$	$\sum_{i=1}^{n} Y_i = 32$	$\overline{X} = 3.3$	$\overline{Y} = 3.2$
$\sum_{i=1}^{n} (X_i)^2 = 141$	$\sum_{i=1}^{n} X_i Y_i = 93$	$\sum_{i=1}^{n} \left(X_i - \overline{X}\right)^2 = 32.1$		
$\sum_{i=1}^{n} (Y_i - \overline{Y})^2 = 211.6$	$\sum_{i=1}^{n} (X_i - \overline{X})(Y_i - \overline{Y}) = -12.6$			

Answer the following questions

4.1) From regression model: $Y_i = \beta_1 + \beta_2 X_i + u_i,\ u_i \sim NIID\left(0, \sigma^2\right)$, find the estimators of β_1 and β_2 with OLS method and explain the meaning of the model.

4.2) Given $\sum_{i=1}^{n} \hat{u}_i^2 = 206.6$, find R^2 and explain its meaning.

4.3) If $X_i = 5$, estimate the value of $\hat{Y}_i$ and explain its meaning.

4.4) Find the estimators of $\text{var}(u_i)$, $\text{var}\left(\hat{\beta}_1\right)$ and $\text{var}\left(\hat{\beta}_2\right)$

4.5) Test the hypothesis whether coefficients are different from zero at 0.05 level of significance.

5. Consider whether each of the following properties violate the assumptions of classical linear regression model.

5.1) E(u_i) = 0 for all i

5.2) $\text{var}(u_i) \neq \text{var}(u_j)$ for some $i \neq j$

6. Show that:

6.1) $\sum (X_i - \overline{X})^2 = \sum X_i^2 - n\overline{X}^2$

6.2) $\sum (X_i - \overline{X})(Y_i - \overline{Y}) = \sum X_i Y_i - n\overline{X}\,\overline{Y}$

7. Answer the following questions

7.1) Given $\sum_{i=1}^{10} (Y_i - \bar{Y})^2 = 600$ and $\sum_{i=1}^{10} \hat{u}_i^2 = 180$. Find R^2 .

7.2) Given $n = 20,\ \sum_{i=1}^{20} Y_i^2 = 6000, \bar{Y} = 16$ and Explained Sum of Squares (ESS) is 680. Find R^2 .

7.3) Given $R^2 = 0.7911$, Total Sum of Squares (TSS) = 552 and $n = 20$, find $\hat{\sigma}^2$.

8. Consider the PRF: The data relate to a sample of 42 rural households in India. The regressand (Y) is expenditure on food and the regressor (X) is income, both figures in thousand rupees. On the basis of the given data, we obtained the following regression:

$$\widehat{Y_i} = 94.2087 + 0.4368X_i$$
$$se = (50.8563) \quad (0.0783)$$
$$TSS = 375{,}916.4364, RSS = 236{,}893.6164$$

8.1) Test the hypothesis that income has negative influence on expenditure on food at α=1%

8.2) If the unit income changes from thousand rupees to rupee, what is the estimator for?

9. Suppose the indifferent curve from consumption of goods X and Y is as following:

Quantity consumed of X	1	2	3	4	5
Quantity consumed of Y	4	3.5	2.8	1.9	0.8

$$\ln(Y_i / X_i) = \beta_1 + \ln(\beta_2 X_i^{\beta_3})$$

9.1) How can we transform this model into linear regression model? (Hint: Y as dependent variable)

9.2) By using information from the table, estimate value of the parameters in the model from (9.1). Also interpret the value of slope coefficient and test hypothesis whether the slope coefficient is equal to zero at 95% confidence interval.

9.3) Find the 95% confidence interval of the slope coefficient from model in (9.1)

10. Consider the following regression from 1,000 households:

$$\hat{Y}_i = (a) + 0.5091X_i$$
$$se \quad 6.4138 \quad (b)$$
$$t \quad 3.8128 \quad 14.2605$$

Y_i = Household consumption expenditure per week

X_i = Household income per week

10.1) Find an estimator of the intercept coefficient and standard error of slope coefficient.

10.2) Interpret the meaning of slope coeffient. Does it show the expected sign ?

10.3) Establish 99%, 95% and 90% confidence interval for slope coefficient.

10.4) Test the hypothesis that slope coeffiecient = 0.2.

11. Consider the regression result:

$$\hat{Y} = 0.8345 + 1.9123X \qquad R^2 = 0.564$$
$$se \quad 3.233 \quad\quad 0.1417$$

How will the above result, including the values of se and R^2, change if;

11.1) X is divided by k

11.2) Y is divided by k

11.3) Both X and Y are divided by k, where k is a constant.

Show your method and explain.

12. Given the sample data of Y and X:

Y	X
3.5	1
-3.2	-1
-0.3	0
0.3	0

Suppose $Y_i = \beta X_i + u_i$ (regression through origin) where $E(u_i|X_i) = 0$ and $Var(u_i|X_i) = \sigma^2$

12.1) Plot the scatter diagram of the given data

12.2) Use Ordinary least square (OLS) method to find the value of β

12.3) Suppose that b is other estimators of β such that $\hat{u}_i = Y_i - bX_i$. Fill in the table below and plot the graph. Let X-axis be the values of b and Y-axis be the values of RSS (b). Observe whether the value of RSS (b) is the same as the value of $\widehat{\beta_{OLS}}$ in b? How?

b	$\hat{u}_1$	$\hat{u}_2$	$\hat{u}_3$	$\hat{u}_4$	RSS(b)
3.60					
3.50					
3.35					
3.15					
3.00					

RSS = residual sum of square

13. CAPM (Capital Asset Pricing Model) is estimated with this econometric model from n = 240:

$$ER_i - r_f = a + b(ER_m - r_f) + u_i$$

where

ER_i is expected return of security i

ER_m is expected return of the market

r_f is risk-free interest rate

u_i is error term with Classical Linear Regression Model assumptions

The result follows:

$$ER_i - r_f = - 0.447 + 1.171\ (ER_m - r_f)$$
$$\text{se} \qquad (0.363) \qquad (0.075)$$

As an econometrician, are there any problems in such model? If yes, what will your recommendation be?

14. From the data collection of 13 automobile-parts manufacturing companies, consider this regression result:

$$\hat{rd} = 0.7437 + (a) sales$$
$$se = \quad (b) \qquad (0.0664)$$
$$t = (0.8797) \qquad (c)$$

where $\dfrac{\sum_{i=1}^{13}(sales_i - \bar{sales})(rd_i - \bar{rd})}{\sum_{i=1}^{13}(sales_i - \bar{sales})^2} = 0.6416$

rd_i = R&D expenditure (million baht), and

$sales_i$ = Annual sale (million baht)

14.1) Find standard error of the slope coefficient and the intercept, and the t statistic value of the slope coefficient.

14.2) Interpret the meaning of the slope coefficient and the intercept.

14.3) Find the 99%, 95% and 90% interval of the slope coefficient and the intercept.

14.4) Test the hypothesis that the slope coefficient is more than or equal to 1 at 0.05 level of significance.

For Further Study (not in the midterm)

15. If u_i is normally distributed, $\sum \hat{u}_i^2$ = 9.8296, and number of observation (n) = 13

15.1) Find the 95% confidence interval of σ^2.

15.2) Test hypothesis that σ^2 = 0.6 at 0.05 level of significance

16. Consider the following statement whether they are true of false and explain.

16.1) If the p-value is higher than the significance level, we will reject the null hypotheis H_0 .

16.2) We and use t-test, F-test, and chi-square value in hypothesis testing that $H_0 : \beta_2 = 0$ where $\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i$

16.3) Type one error is the value of significance level (α) which we reject H_0 when it is true.