EE325 Section 1 HW 2 Due Thursday February 20th (23:00 hr.),2020

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Use 4 decimal places for numerical answers

1. In Table 1.a. X_i is total microeconomics exam point (total points are 100) and Y_i is GPA of each student.

Table 1.a

Student	Y_i	X_i	$y-\bar{y}$	$x_i-\bar{x}$
1	2.8	63	-0.4125	-14.625
2	3.4	72	0.1875	-5.625
3	3	78	-0.2125	0.375
4	3.5	81	0.2875	3.375
5	3.6	87	0.3875	9.375
6	3.0	75	-0.2125	-2.625
7	2.7	75	-0.5125	-2.625
8	3.7	90	0.4875	12.375

$\bar{x} = \dfrac{63+72+78+81+87+75+75+90}{8}$

$= 77.625$

$\bar{y} = \dfrac{2.8+3.4+3+3.5+3.6+3+2.7+3.7}{8}$

$= 3.2125$

1.1 Now consider the two-variable $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Use OLS to find the estimator of β_0 and β_1. (Note: *NIID* = Normally, Identically, and Independently Distributed).

$\hat{\beta}_1 = \dfrac{\Sigma_i (x_i-\bar{x})(y_i-\bar{y})}{\Sigma_i (x-\bar{x})^2} = \dfrac{(63-77.625)(2.8-3.2125)+(72-77.625)(3.4-3.2125)+\dots+(90-77.625)(3.7-3.2125)}{(-14.625)^2+(-5.625)^2+\dots+(12.375)^2}$

$= \dfrac{17.4375}{511.875} = 0.0341$

$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} = 3.2125-(0.0341)(77.625)$

$\hat{\beta}_0 = 0.5659$

1.2 For each observation i, find $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum_{i=0}^{N} \hat{u}_i = 0$.

$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i = 0.5655 + 0.0341 x_i$

$\hat{u}_i = y_i - \hat{y}_i = y_i - 0.5655 - 0.0341 x_i$

$\Sigma_{i=0}^{10} \hat{u}_i = (2.8-0.5681-0.0341(63)) + (3.4-0.5681-0.0341(72)) + \dots +$

$(3.7-0.5681-0.0341(90))$

$= -0.0002$

1.3 Find $var(\hat{u}_i)$, $var(\widehat{\beta_0})$, $var(\widehat{\beta_1})$

$SST = \sum_i (x_i - \bar{x})^2 = 551.875$

$\sum x_i^2 = 48{,}717$

$SSR = \sum (\hat{y} - \bar{y})^2 = 1.02875$

$\sigma^2 = var(u_i|x_i) = E[(u_i - E(u_i))^2]$

$= E(u_i^2)$

$var(\hat{u}_i) = \sigma^2 = \frac{SSR}{n-2} = \frac{1.02875}{8-2} = 0.1715$

$var(\hat{\beta}_0) = \frac{\sigma^2 n^{-1} \sum_{i=1}^{n} x_i^2}{SST}$

$= \frac{(0.1715)(48717)}{8(551.875)}$

$= 2.0398$

$var(\hat{\beta}_1) = \frac{\sigma^2}{SST} = \frac{\sigma^2}{551.875} = 0.003$

2. Data is listed in the table

X_i	Y_i	$x_i - \bar{x}$	$y_i - \bar{y}$	$\hat{u}_i$
10	0	-10	-9.1	-0.1455
12	2	-8	-7.1	0.0636
14	5	-6	-4.1	1.2727
16	6	-4	-3.1	0.4818
18	7	-2	-2.1	-0.3091
22	10	2	0.9	-0.8909
24	10	4	0.9	-2.6818
26	15	6	5.9	0.5273
28	16	8	6.9	-0.2636
30	20	10	10.9	1.9455

2.1 From the simple regression model $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Find estimators of β_0 and β_1 from the OLS method and interpret the meaning.

$\bar{x} = (10+12+14+16+18+22+24+26+28+30)/10 = 20$

$\bar{y} = (0+2+5+6+7+10+10+15+16+20)/10 = 9.1$

$\hat{\beta}_1 = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sum_i (x - \bar{x})^2} = \frac{(-10)(-9.1) + (-8)(-7.1) + \ldots + (10)(10.9)}{(-10)^2 + (-8)^2 + \ldots + (10)^2} = \frac{394}{440} = 0.8955$

$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} = 9.1 - 0.8955(20) = -8.61$

2.2 Find the value of $\widehat{Y}_i$ and $\widehat{u}_i$. Show that $\sum_{i=0}^{N} \hat{u}_i = 0$.

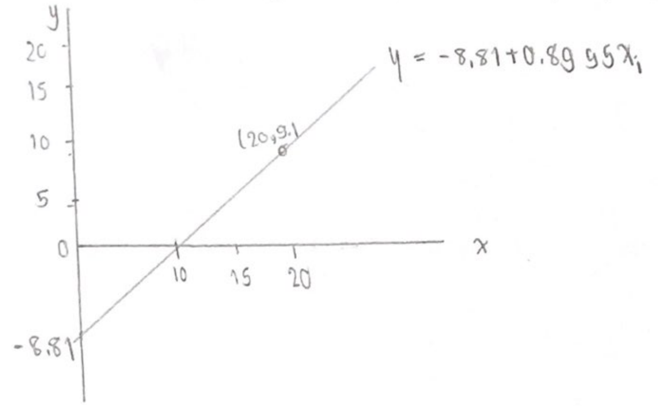
$\hat{Y}_i = \beta_0 + \beta_1 X_1 = -8.81 + 0.8955 X_1$

$u_i = y_i - \hat{y}_i = y_i + 8.81 - 0.8955 x_i$

$\sum_{i=0}^{N} \hat{u}_i = (0 + 8.81 - 0.8955(10)) + (2 + 8.81 - 0.8955(12)) + \ldots + (20 + 8.81 - 0.8955(30))$

$= 0$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?



yes, it pass $(\bar{x}, \bar{y})$

2.4 If X_i =16, what is the predicted Y?

$E(y) = -8.81 + 0.8955(16)$

$= 5.518$

2.5 Find $var(\hat{u}_i)$, $var(\widehat{\beta_0})$, $var(\widehat{\beta_1})$

$SSR = \Sigma(y - \bar{y})^2 = 366.9$

$SST = \Sigma(x - \bar{x})^2 = 440$

$var(\hat{u}_i) = \frac{SSR}{n-2} = \frac{366.9}{10-2} = 45.8625$

$var(\hat{\beta}_0) = \frac{\sigma^2 n^{-1} \sum_{i=1}^{n} x_i^2}{SST}$

$= (45.8625)(4490) / 10(440)$

$= 46.7994$

$var(\hat{\beta}_1) = \frac{\sigma^2}{SST}$

$= 46.8625 / 440$

$= 0.104233$

3. Consider the below regression function:

$$Y_i = \beta_1 X_i + u_i \,,$$

Where $u_i \sim NIID(0, \sigma^2)$. Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator. Please state the SLR assumption(s) when used.

$y_i = \beta_1 x_i + u_i$

$\text{argmin } \sum_i (y_i - \tilde{\beta}_1 x_i)^2$

F.O.C. w.r.t. $\tilde{\beta}_1$

$$\frac{\partial \sum_i (y_i - \tilde{\beta}_1 x_i)^2}{\partial \tilde{\beta}_1} = 0$$

$$\sum_i 2(y_i - \tilde{\beta}_1 x_i)(-x_i) = 0$$

$$\sum_i x_i (y_i - \tilde{\beta}_1 x_i) = 0(-2)$$

$$\sum_i x_i y_i - \sum_i \tilde{\beta}_1 x_i = 0$$

$$\sum_i x_i y_i = \sum_i \tilde{\beta}_1 x_i^2$$

$$\tilde{\beta}_1 = \frac{\sum_i x_i y_i}{\sum_i x_i^2}$$

Substitute $y_i = \beta_1 x_i + u_i$

$$\hat{\beta}_1 = \frac{\sum_i x_i (\beta_1 x_i + u_i)}{\sum_i x_i^2}$$

$$\hat{\beta}_1 = \frac{\sum_i \beta_1 x_i^2}{\sum_i x_i^2} + \frac{\sum_i u_i x_i}{\sum_i x_i^2}$$

$$E(\hat{\beta}_1) = E(\beta_1) + E\left(\frac{\sum u_i x_i}{\sum_i x_i^2}\right)$$ by SLR 4 $E(u_i|x_i) = 0$

$$E(\hat{\beta}_1) = \beta_1$$