Chapter 15 Production

Production Problem: How a firm employs inputs (labor L and capital K) to

1. Maximize output Q for a given cost level C_0.

2. Minimize cost C to produce a give level of output Q_0.

- The understanding of this production problem is crucial to understanding the behavior of a firm in perfect competition and monopoly—and other markets to be discussed in EE311.

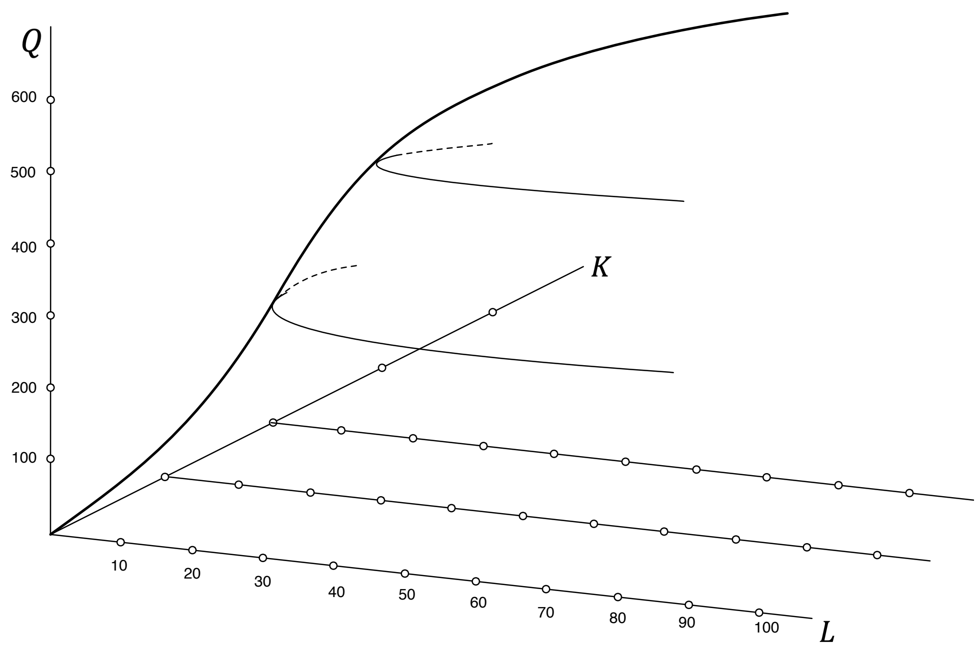
Production Function $Q = f(L, K)$ is a relationship between the highest output Q achievable from using labor $= L$ and capital $= K$ as efficiently as possible with the given technology

Example: $Q = f(L, K) = 10L^{0.5}K^{0.5}$,

$$\left.\begin{array}{l} L = 400 \\ K = 900 \end{array}\right\} \Rightarrow Q = 6{,}000\ units$$

$$\left.\begin{array}{l} L = 100 \\ K = 900 \end{array}\right\} \Rightarrow Q = 3{,}000\ units$$

Graph of a Production Function in 3D



Short-Run Production: Time frame that there is at least ***one input*** that cannot be changed (fixed factor)

- Usually in Short-Run, capital is assumed to be the fixed factor at $K = K_0$
- Fixed factor does not change with the quantity Q
  $\Longrightarrow$ Fixed Cost.

Long-Run Production: Time frame that is long enough for the firm to vary every input
$\Longrightarrow$ no fixed factor $\Longrightarrow$ no fixed cost.

<u>Short-Run Production</u> With production function

$$Q = f(L, K) = 10L^{0.5}K^{0.5}$$

Assume capital K is fixed at $K_0 = 900$, we have

$$\begin{aligned} Q &= 10L^{0.5}K_0^{0.5} \\ &= 300L^{0.5} \end{aligned}$$

- we can change output Q by changing L in the short run
- This is called the Total Product is the output that can be produced at various labor level L, with capital fixed at K_0,

$$TP_{K_0}(L) = f(L, K_0)$$

Total, Average, and Marginal

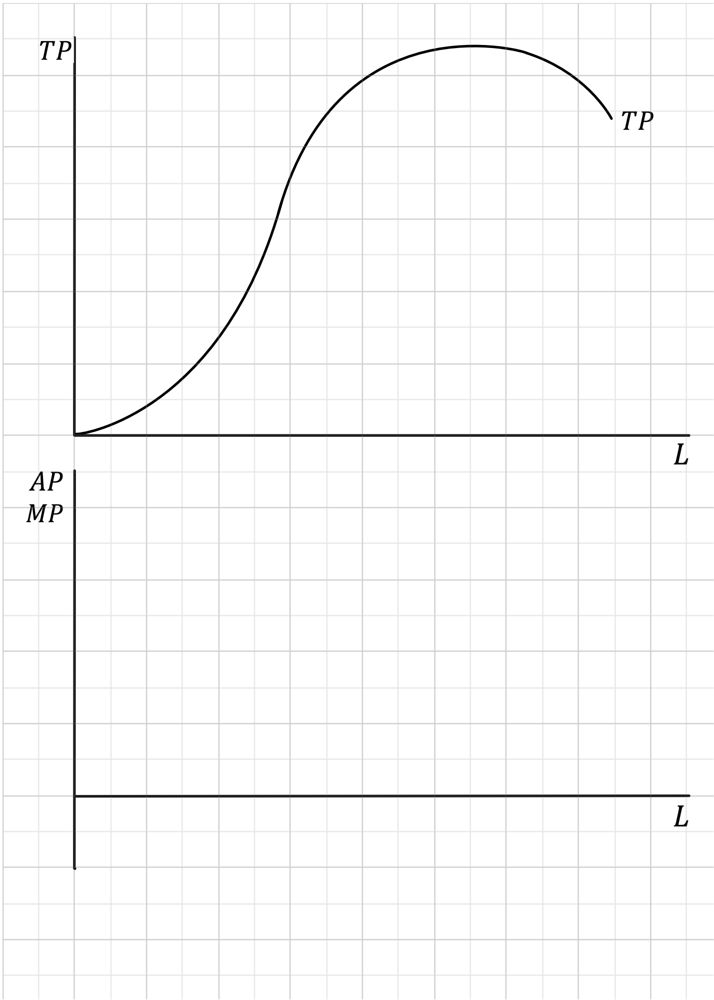
Given a Total, we can find Average and Marginal. This is the first T-A-M relation to be discussed in this class.

Given the Total Product $TP(L)$, (with subscript K_0 suppressed)

- Average Product $AP(L) = \frac{TP(L)}{L}$,
- Marginal Product $MP(L) = \frac{dTP(L)}{dL}$,

Marginal Product $MP(L)$ is the rate of change of the output $TP(L)$. That is, $MP(L)$ is the slope of $TP(L)$

The relationship of TP, AP and MP is demonstrated by the following graphs.



Total	Average	Marginal

The relationship between Average and Marginal can also be verified by calculus. By definition,

$$TP(L) = AP(L) \cdot L$$