# EE 320 Introductory Mathematical Economics

## Semester 1/2016

## Homework 3

**Due 3 November 2016 (in class)**

### Question 1

Calculate $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$ and $\frac{\partial f}{\partial z}$ (if possible) of the following functions.

a. $f(x,y) = \frac{5xy^2}{x^2+y^2}$

b. $f(x,y) = \ln(x^2y + xy^2) - x^2 - y^2$

c. $f(x,y,z) = \mathrm{xz}^2 \ln(y) - \frac{y}{z^2+x-y}$

d. $f(x,y,z) = \mathrm{e}^{x+\ln(z)} - \ln(x^2)y^2z^3$

### Question 2

The optimal profit function of a firm can be given by,

$$\pi^*(p, w_1, w_2, A) = A * p^{1/(1-\beta)}(w_1^{\gamma} + w_2^{\gamma})^{\beta/(\gamma(\beta-1))}$$

where $0 < \beta < 1$ and $\gamma < 1$, A is the level of technology, P is price of output, $w_1$ is the factor price of capital, and $w_2$ is the factor price of labor.

Consider the following problem

a. Use the partial derivative to conclude about the relationship between price and the level of profit.
b. How does the technical progress affect the level of profit? Show your result by using the partial derivative.
c. How does the level of factor price of inputs affect the level of profit? Show your result for both types of input, using the partial derivative. Then, explain the intuition of your result in economics.
d. Show that the profit function is convex in factor price of inputs. That is, the second-order partial derivatives of profit with respect to factor price of both capital and labor are greater than zero.

## Question 3

Consider a simple macroeconomic model given below,

$$Y = C + G$$
$$C = C_0 + c_1(Y - T)$$
$$T = T_0 + t_1 Y$$
$$G = G_0$$

where Y is national income, C is consumption, T is the amount of tax collected, G is the level of government expenditure.

Answer the following questions:

a. State all the endogenous variables. What are the parameters and exogenous variables in the model?
b. Derive the equilibrium solution of all the endogenous variables?
c. Use the partial derivative to show the effect change in $c_1$ and $G_0$ on the equilibrium of endogenous variables?
d. How does the marginal propensity to tax ( $t_1$ ) affect the equilibrium level of income (Y*) and consumption (C*)?

## Question 4

Write the Hessian Matrix for each of the following functions:

a. $U(x, y) = 7x^2 + 8xy + 3y^2$
b. $z(x, y) = 5(13x - 5y)^2$
c. $f(x, y) = 4x^3 - 11xy - 7y^5$
d. $Q(K, L) = (2K + 1)(3L^2 + 2)$

## Question 5

The demand for a product depends on the price $p_1$ of the product and on the price $p_2$ charged by a competing producer, and it is given by:

$$D(p_1, p_2) = 36 - \frac{8p_1}{\sqrt{p_2}}.$$

a. Find $\frac{\partial D}{\partial p_1}$ and $\frac{\partial D}{\partial p_2}$, and comment on the signs of the partial derivatives.
b. Calculate the own-price and cross-price elasticities of demand when $p_1 = 3$ and $p_2 = 4$.

## Question 6

Suppose the production function $Q$ depends on the number of workers $L$ according to the formula:

$$Q = L * g\left(\frac{ln(L)}{L}\right)$$

where g(.) is a differentiable function. Find expressions for $\frac{dQ}{dL}$ and $\frac{d^2Q}{dL^2}$.