EE325 Section 1 HW 2 Due Thursday February 20th (23:00 hr.),2020

Use 4 decimal places for numerical answers

1. In Table 1.a. X_i is total microeconomics exam point (total points are 100) and Y_i is GPA of each student.

Table 1.a

Student	Y_i	X_i
1	2.8	63
2	3.4	72
3	3	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

$\bar{Y} = \frac{1}{N}\sum_{i=1}^{n} Y_i = \frac{1}{8}(25.7) = 3.2125$

$\bar{X} = 77.625$

1.1 Now consider the two-variable $Y_i = \beta_0 + \beta_1 X_i + u_i, u_i \sim NIID(0, \sigma^2)$. Use OLS to find the estimator of β_0 and β_1. (Note: *NIID* = Normally, Identically, and Independently Distributed).

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} (Y-\bar{Y})(X-\bar{X})}{\sum_{i=1}^{n} (X-\bar{X})^2}$$

$$= \frac{17.4375}{511.875} = 0.0341$$

$$\hat{\beta}_0 = \bar{Y} - \beta_1 \bar{X}$$
$$= 3.2125 - 0.0341(77.625)$$
$$= 0.5681$$

1.2 For each observation i, find $\widehat{Y}_i$ and $\widehat{u}_i$. Show that $\sum_{i=0}^{N} \hat{u}_i = 0$.

$\hat{y} = 0.0341X + 0.5681$

$\hat{u}_i$

1.3 Find $var(\hat{u}_i)$, $var(\widehat{\beta_0})$, $var(\widehat{\beta_1})$

$$\frac{1}{n-2}\sum_{i=1}^{n}\hat{u}_i^2 = \frac{SSR}{n-2} = 0$$

$$var(\hat{\beta}_0) = \frac{\sigma^2 n^{-1}\sum_{i=1}^{n} x_i^2}{\sum_{i=1}^{n}(x_i-\bar{x})} = \frac{0.3586 \times 8^{-1} \times 48717}{511.875} = 4.2662$$

$$var(\hat{\beta}_1) = \frac{\sigma^2}{\sum_{i=1}^{n}(x_i-\bar{x})} = \frac{\sigma^2}{SST} = \frac{0.3586}{511.875} = 0.000$$

2. Data is listed in the table

X_i	Y_i
10	0
12	2
14	5
16	6
18	7
22	10
24	10
26	15
28	16
30	20

$n = 10$

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = 20$$

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i = 9.1$$

2.1 From the simple regression model $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Find estimators of β_0 and β_1 from the OLS method and interpret the meaning.

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n}(y-\bar{y})(x-\bar{x})}{\sum_{i=1}^{n}(x-\bar{x})^2}$$

$\hat{\beta}_1 = 0.8955$

$$\hat{\beta}_0 = \bar{y} - \beta_1\bar{x}$$

$= 9.1 - 0.8955(20)$

$= -8.8091$

The slope $\hat{\beta}_1$ is 0.8955, meaning that if x increases by 1 unit, y increases by approximately 0.8955

The y intercept $\hat{\beta}_0$ is -8.8091

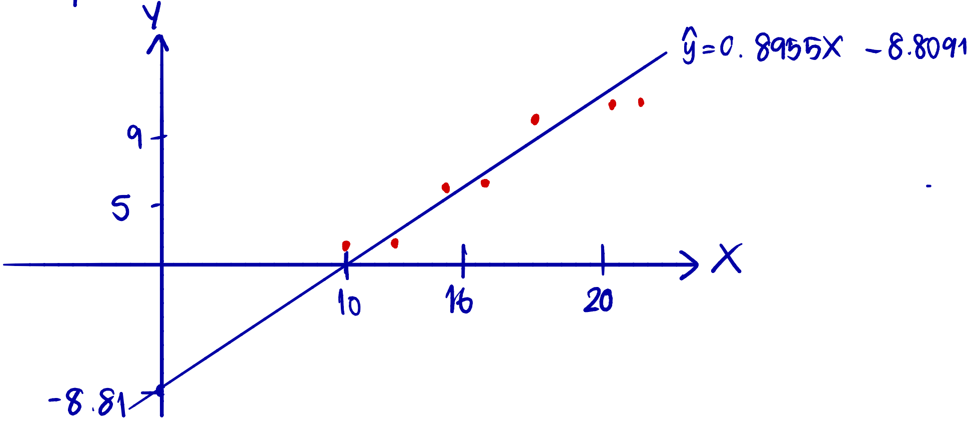
2.2 Find the value of $\widehat{Y}_i$ and $\widehat{u}_i$. Show that $\sum_{i=0}^{N} \hat{u}_i = 0$.

$\hat{y} = 0.8955X - 8.8091$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?

$\hat{y} = 0.8955X - 8.8091$

$\hat{y} = 0.8955(20) - 8.8091 = 9.1$, the regression line passes $(\bar{x}, \bar{y})$



2.4 If X_i =16, what is the predicted Y?

$\hat{y} = 0.8955(16) - 8.8091 = 5.5189$

2.5 Find $var(\widehat{u}_i), var(\widehat{\beta_0}), var(\widehat{\beta_1})$

$$var(\hat{u}_i) = \frac{1}{n-2} \sum_{i=1}^{n} \hat{u}_i^2 = \frac{SSR}{n-2} = \frac{0}{10-2} = 0 \#$$

$$var(\hat{\beta}_0) = \frac{\sigma^2 n^{-1} \sum_{i=1}^{n} X_i^2}{\sum_{i=1}^{n} (X_i - \bar{X})^2}$$

$$var(\hat{\beta}_1) = \frac{\sigma^2}{\sum_{i=1}^{n} (X_i - \bar{X})^2} = \frac{\sigma^2}{SST}$$

$\sigma^2 = Var(u|X) = Var(y|X)$

$Var(y|X) = 6.0572$

$$Var(\hat{\beta}_0) = \frac{6.0572 \times 10^{-1} \times 4440}{-240} = -11.2058$$

$$Var(\hat{\beta}_1) = \frac{6.0572}{-240} = -0.02524$$

3. Consider the below regression function:

$$Y_i = \beta_1 X_i + u_i \,,$$

Where $u_i \sim NIID(0, \sigma^2)$. Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator. Please state the SLR assumption(s) when used.

$$\hat{\beta} = \sum_{i=1}^{n} \frac{(X_i - \bar{X}) y_i}{SST_X} = \frac{\sum_{i=1}^{n} (X - \bar{X})(\beta_0 + \beta_1 X_1 + u_1)}{SST}$$

$$(X - \bar{X}) = 0 \qquad \hat{\beta}_1 = \beta_1 + \sum_{i=1}^{n} \frac{(X_i - \bar{X}) u_i}{SST}$$

$$= \beta_1 + \frac{1}{SST} \sum_{i=1}^{n} d_i u_i$$

$$E(\hat{\beta}_1) = \beta_1 + E(1/SST) \sum_{i=1}^{n} d_i u_i = \beta_1 (1/SST_X) E(d_i u_i)$$

$$= \beta_1 + (1/SST) \sum_{i=1}^{n} d_i E(u_i)$$

$$= \beta_1 + (1/SST) \sum_{i=1}^{n} d_i \times 0$$

$$= \beta_1$$