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EE325 Section 1 HW 2 Due Thursday February 20th (23:00 hr.),2020

Use 4 decimal places for numerical answers

1. In Table 1.a. X_i is total microeconomics exam point (total points are 100) and Y_i is GPA of each student.

Table 1.a

Student	Y_i	X_i	$x_i-\bar{x}$	$y_i-\bar{y}$	$(x_i-\bar{x})(y_i-\bar{y})$	$(x_i-\bar{x})^2$	$\hat{y}$	$\hat{u}$
1	2.8	63	-14.625	-0.4125	6.0328	213.8906	2.7143	0.0857
2	3.4	72	-5.625	0.1875	-1.0547	31.6406	3.0209	0.3791
3	3	78	0.375	-0.2125	-0.0797	0.1406	3.2253	-0.2253
4	3.5	81	3.375	0.2875	0.9703	11.3906	3.3275	0.1725
5	3.6	87	9.375	0.3875	3.6328	87.8906	3.5319	0.0681
6	3.0	75	-2.625	-0.2125	0.5578	6.8906	3.1231	-0.1231
7	2.7	75	-2.625	-0.5125	1.3453	6.8906	3.1231	-0.4231
8	3.7	90	12.375	0.4875	6.0328	153.1406	3.6341	0.0659

$\bar{y} = 3.2125$ $\bar{x} = 77.625$ $\Sigma = 17.4374$ $\Sigma = 511.875$

1.1 Now consider the two-variable $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Use OLS to find the estimator of β_0 and β_1. (Note: *NIID* = Normally, Identically, and Independently Distributed).

$$\boxed{\begin{aligned}\hat{\beta}_0 &= \bar{y} - \hat{\beta}_1\bar{x} \\ \hat{\beta}_1 &= \frac{\Sigma(x_i-\bar{x})(y_i-\bar{y})}{\Sigma(x_i-\bar{x})^2}\end{aligned}}$$

$$\hat{\beta}_1 = \frac{17.4374}{511.875} = 0.0341$$

$$\hat{\beta}_0 = 3.2125 - (0.0341)77.625$$

$$= 0.5655$$

1.2 For each observation i, find $\widehat{Y}_i$ and $\widehat{u}_i$. Show that $\sum_{i=0}^{N} \hat{u}_i = 0$.

$$\boxed{\begin{aligned}\hat{y}_i &= \hat{\beta}_0 + \hat{\beta}_1 x_i \\ \hat{u}_i &= y_i - \hat{y}_i\end{aligned}}$$

y	x	$\hat{y}$	$\hat{u}$
2.8	63	2.7143	0.0857
3.4	72	3.0209	0.3791
3	78	3.2253	-0.2253
3.5	81	3.3275	0.1725
3.6	87	3.5319	0.0681
3	75	3.1231	-0.1231
2.7	75	3.1231	-0.4231
3.7	90	3.6341	0.0659

$\Sigma_{i=0}^{0} \hat{u}_i = 0$

1.3 Find $var(\widehat{u_i}), var(\widehat{\beta_0}), var(\widehat{\beta_1})$

$$Var(\hat{u}_i) = \frac{0.4347}{6} = 0.0725$$

$$var(\hat{\beta}_1) = \frac{\hat{\sigma}^2}{SSE} = \frac{\hat{\sigma}^2}{\Sigma(x_i-\bar{x})^2} = \frac{0.0725}{511.875} = 0.0001$$

$$var(\hat{\beta}_0) = \frac{\Sigma x_i^2 \hat{\sigma}^2}{n\Sigma(x_i-\bar{x})^2} = \frac{46.717(0.0001)}{(8)\,511.875} = 0.0119$$

$$\hat{y}_i = -8.81 + (0.8955)(x_i)$$

2. Data is listed in the table

$\bar{x} = 20$ $\bar{y} = 9.1$

X_i	Y_i	$x_i-\bar{x}$	$y_i-\bar{y}$	$(x_i-\bar{x})(y_i-\bar{y})$	$(x_i-\bar{x})^2$	$\hat{y}_i$	$\hat{u}_i$
10	0	-10	-9.1	91	100	0.145	-0.145
12	2	-8	-7.1	56.8	64	1.936	0.064
14	5	-6	-4.1	24.6	36	3.727	1.273
16	6	-4	-3.1	12.4	16	5.518	0.482
18	7	-2	-2.1	4.2	4	7.309	-0.309
22	10	2	0.9	1.8	4	10.891	-0.891
24	10	4	0.9	3.6	16	12.682	-2.682
26	15	6	5.9	35.4	36	14.473	0.527
28	16	8	6.9	55.2	64	16.264	-0.264
30	20	10	10.9	109	100	18.055	1.945
				394	440		

2.1 From the simple regression model $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Find estimators of β_0 and β_1 from the OLS method and interpret the meaning.

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1\bar{x}$$

$$\hat{\beta}_1 = \frac{\Sigma(x_i-\bar{x})(y_i-\bar{y})}{\Sigma(x_i-\bar{x})^2}$$

$$\hat{\beta}_1 = \frac{394}{440} = 0.8955$$

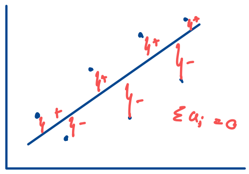
$$\hat{\beta}_0 = 9.1 - (0.8955)(20)$$

$$= -8.81$$

$$\bar{x} = \frac{10+12+14+16+18+22+24+26+28+30}{10} = 20$$

$$\bar{y} = \frac{0+2+5+6+7+10+10+15+16+20}{10} = 9.1$$

2.2 Find the value of $\widehat{Y}_i$ and $\widehat{u}_i$. Show that $\sum_{i=0}^{N} \hat{u}_i = 0$.



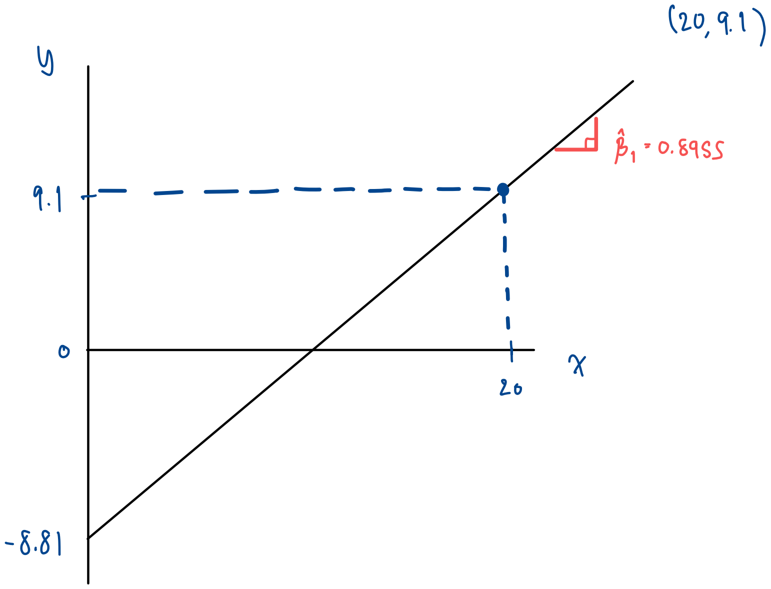
$\boxed{\begin{array}{l} \hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i \\ \hat{u}_i = y_i - \hat{y}_i \end{array}}$

$\sum_{i=0}^{N} (y_i - \hat{y}_i) = 0$

X_i	y_i	$\hat{y}_i$	$\hat{u}_i$
10	0	0.145	-0.145
12	2	1.936	0.064
14	5	3.727	1.273
16	6	5.518	0.482
18	7	7.309	-0.309
22	10	10.891	-0.891
24	10	12.682	-2.682
26	15	14.473	0.527
28	16	16.264	-0.264
30	20	18.055	1.945

$\sum_{i=0}^{N} \hat{u}_i = 0$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?



$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 X_i$

$\hat{y} = -8.81 + 0.8955 X_i$

(estimated y = $\hat{y}$)

2.4 If X_i =16, what is the predicted Y?

$X_i = 16 \quad \hat{y} = 5.518$

2.5 Find $var(\hat{u}_i)$, $var(\widehat{\beta_0})$, $var(\widehat{\beta_1})$

$var(\hat{u}_i) = \dfrac{\sum u_i^2}{n-2} = \dfrac{14.089}{8} = 1.7611$

$var(\hat{\beta}_0) = \dfrac{\sum X_i^2}{n \sum (X_i - \bar{X})^2} = 1.7771$

$var(\hat{\beta}_1) = 0.004$

3. Consider the below regression function:

$$Y_i = \beta_1 X_i + u_i \,,$$

Where $u_i \sim NIID(0, \sigma^2)$. Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator. Please state the SLR assumption(s) when used.

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} (y_i - \bar{y})(x_i - \bar{x})}{\sum_{i=1}^{n} (x_i - \bar{x})^2}$$

Suppose $x_i = x_i - \bar{x}$ $\quad k_i = \frac{x_i}{\sum_i x_i^2}$

$$\beta_1 = \sum_{i=1}^{n} (y_i - \bar{y}) k_i \quad \text{OR} \quad \hat{\beta}_1 = \frac{\sum_{i=1}^{n} (y_i - \bar{y})(x_i - \bar{x})}{\sum_{i=1}^{n} (x - \bar{x})^2}$$

$$= \sum_{i=1}^{n} (\hat{\beta}_0 - \beta_1 x_i + u_i - \beta_0 - \beta_1 \bar{x}) k_i$$

$$= \sum_{i=1}^{n} \beta_1 (x_i - \bar{x}) k_i + \sum_i u_i k_i$$

$$= \beta_1 \sum_{i=1}^{n} x_i \frac{x_i}{\sum_i x_i^2} + \sum_i u_i k_i$$

$$= \beta_1 \frac{\sum_i x_i^2}{\sum_i x_i^2} + \sum_i u_i k_i$$

$$E(\hat{\beta}_1) = E[\beta_1 + \sum_i k_i u_i]$$

SLR 4: $E(u_i | x_i) = 0$

$$E(\beta_1) = \beta_1 + \sum_i k_i \underbrace{E(u_i)}_{0}$$

$$\therefore E(\hat{\beta}_1) = \beta_1$$