

EE 325 Section 1 (Aj.Wanwiphang) Homework Assignment 1

Due date: 31 January 2020 before 11pm

**** Please submit this assignment on Moodle. For those who work on paper, please scan or submit the pictures of your work. ****

1. Find the answers following questions (please also show your calculation)

$$a. \sum_{i=1}^5 (a + bx_i) = \sum_{i=1}^5 a + \sum_{i=1}^5 bx_i = 5a + b \sum_{i=1}^5 x_i$$

$$b. \sum_{y=0}^5 f(x+y) = f(x+0) + f(x+1) + f(x+2) + f(x+3) + f(x+4) + f(x+5)$$

$$c. \sum_{i=1}^{10} i^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2 = 385$$

$$d. \sum_{x=1}^2 \sum_{y=2}^3 (2x+y) = \sum_{x=1}^2 (2x+2) + (2x+3) = [(2)(1)+2] + [(2)(2)+3] = 11$$

2. Given X is discrete random variable. The probability distribution function (PDF) of this variable is shown in the table

X	-2	-1	0	1	2	3	4
$f(x)$	0.5b 0.0625	b 0.125	2.25b 0.28125	2b 0.25	1.5b 0.1875	0.5b 0.0625	0.25b 0.03125

** when b is constant number

- a. Find the value of b

$$\sum f(x) = 1$$

$$0.5b + b + 2.25b + 2b + 1.5b + 0.5b + 0.25b = 1 \rightarrow b = \frac{1}{8} \text{ or } 0.125$$

- b. Find the answer for $P(X \leq 2)$

$$0.0625 + 0.125 + 0.28125 + 0.25 + 0.1875 = 0.90625$$

- c. Find the answer for $P(-2 \leq X \leq 3)$

$$1 - P(X = 4) = 1 - 0.03125 = 0.96875$$

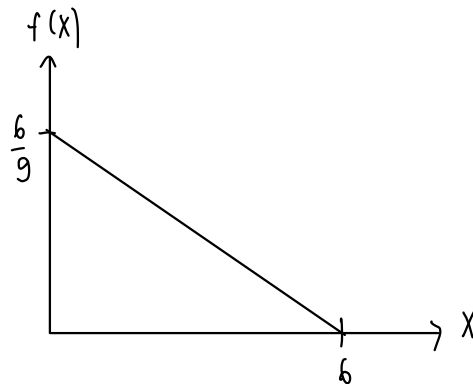
- d. Find the answer for $P(X \geq 1)$

$$0.25 + 0.1875 + 0.0625 + 0.03125 = 0.53125$$

3. Given X is continuous random variable. The probability distribution function (PDF) of this variable is

$$f(x) = -\frac{1}{9}x + \frac{6}{9}, 0 \leq x \leq 3$$

- a. Plot graph for $f(x)$



- b. Find the answer for $P(1 \leq X \leq 3)$

$$\begin{aligned} \int_1^3 -\frac{1}{9}x + \frac{6}{9} dx &= \left[-\frac{1}{18}x^2 + \frac{6x}{9} \right]_1^3 = \left(-\frac{1}{18}(3)^2 + \frac{6(3)}{9} \right) - \left(-\frac{1}{18}(1)^2 + \frac{6(1)}{9} \right) \\ &= \frac{3}{2} - \frac{11}{18} \\ &= \frac{8}{9} \end{aligned}$$

- c. Find the answer for $P(X \geq 2)$

$$\begin{aligned} \int_2^3 -\frac{1}{9}x + \frac{6}{9} dx &= \left[-\frac{1}{18}x^2 + \frac{6x}{9} \right]_2^3 = \left(-\frac{1}{18}(3)^2 + \frac{6(3)}{9} \right) - \left(-\frac{1}{18}(2)^2 + \frac{6(2)}{9} \right) \\ &= \frac{3}{2} - \frac{20}{18} \\ &= \frac{7}{18} \end{aligned}$$

- d. Find the expected value of X

$$\begin{aligned} E(X) &= \int_0^3 x f(x) dx \\ &= \int_0^3 -\frac{1}{9}x^2 + \frac{6}{9}x dx \\ &= \left[-\frac{1}{27}x^3 + \frac{6}{18}x^2 \right]_0^3 \\ &= -\frac{27}{27} + \frac{54}{18} \\ &= 2 \end{aligned}$$

4. Let random variable X be the outcome of throwing one dice and random variable Y be the outcome of tossing one coin. Coin has two sided that has valued 1 and 0.

- a. Construct the joint probability distribution function (PDF) table of X and Y

X/Y	1	2	3	4	5	6	
0	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{6}{12}$
1	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{6}{12}$
	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	

- b. Find the marginal probability distribution function (PDF) of X

$$\frac{2}{12}$$

- c. Find the marginal probability distribution function (PDF) of Y

$$\frac{6}{12}$$

- d. Find the conditional probability distribution function (PDF) of

X given Y is equal to 1

$$P(X|Y=1) = \frac{P(Y=X, Y=1)}{P(Y=1)} = \frac{P(X=1, Y=1)}{P(Y=1)} + \frac{P(X=2, Y=1)}{P(Y=1)} + \dots + \frac{P(X=6, Y=1)}{P(Y=1)}$$

$$= \left(\frac{1}{12} \times 2 \right) 6 = 1$$

- e. Find the expected value of X given Y is equal to 1

$$E(X|Y=1) = \sum x f(x) = \frac{(X)(P(X=x, Y=1))}{P(Y=1)} = \frac{(1)(P(X=1, Y=1))}{P(Y=1)} + \frac{(2)(P(X=2, Y=1))}{P(Y=1)} + \dots + \frac{(6)(P(X=6, Y=1))}{P(Y=1)}$$

$$= \frac{1}{6} + \frac{1}{3} + \frac{1}{2} + \frac{2}{3} + \frac{5}{6} + 1 = \frac{21}{6}$$

- f. Find the variance of X given Y is equal to 1

$$\text{var}(X|Y=1) = E(X^2) - [E(X)]^2$$

$$= \sum x^2 f(x) - \left(\frac{21}{6} \right)^2$$

$$= (1)^2 \left(\frac{1}{12} \times 2 \right) + (2)^2 \left(\frac{1}{12} \times 2 \right) + (3)^2 \left(\frac{1}{12} \times 2 \right) + (4)^2 \left(\frac{1}{12} \times 2 \right) + (5)^2 \left(\frac{1}{12} \times 2 \right) + (6)^2 \left(\frac{1}{12} \times 2 \right) - \frac{441}{36}$$

$$= \left(\frac{1}{12} \times 2 \right) (91) - \frac{441}{36} = \frac{546}{36} - \frac{441}{36} = \frac{35}{12}$$

5. If X_1, X_2, X_3 is a random sample from a population with mean μ and variance σ^2 . X_1, X_2, X_3 are not independent *dependent*

$$\text{Cov}(X_1, X_2) = \text{Cov}(X_1, X_3) = \text{Cov}(X_2, X_3) = \frac{1}{4}\sigma^2$$

$\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{3}(X_1 + X_2 + X_3)$

Find $E(\bar{X})$ and $\text{var}(\bar{X})$

$$E(\bar{X}) = E\left(\frac{1}{3} \sum_{i=1}^3 X_i\right)$$

$$= E\left[\left(\frac{1}{3}\right) (\sum X_i)\right]$$

$$= E(X_i) = \mu$$

$$\text{var}(\bar{X}) = \text{var}\left(\frac{1}{3}X_1\right) + \text{var}\left(\frac{1}{3}X_2\right) + \text{var}\left(\frac{1}{3}X_3\right) - \text{cov}$$

$$= \frac{1}{9}\sigma^2 + \frac{1}{9}\sigma^2 + \frac{1}{9}\sigma^2 - \frac{1}{4}\sigma^2$$

$$= \frac{1}{3}\sigma^2 - \frac{1}{4}\sigma^2$$

6. Given X_1, X_2, X_3, X_4 are independent identically distributed random variables from population with mean μ and variance σ^2 . $\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{4}(X_1 + X_2 + X_3 + X_4)$

a. Find $E(\bar{X})$ and $\text{var}(\bar{X})$ in term of μ and σ

$$E(\bar{X}) = E\left(\frac{1}{4} \sum_{i=1}^4 X_i\right)$$

$$= E\left[\left(\frac{1}{4}\right) (\sum X_i)\right]$$

$$= E(X_i) = \mu$$

$$\begin{aligned}\text{var}(\bar{x}) &= \text{var} \left(\frac{1}{4} \sum_{i=1}^4 x_i \right) \\ &= \text{var} \left[\left(\frac{1}{4} \right) (x_i) \right] \\ &= \text{var}(x_i) = 6\end{aligned}$$

- b. Given $\tilde{X} = \frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4$ is another estimator of μ . Show that $\tilde{X}$ is an unbiased estimator of μ *check var $\neq 6$: unbiased*

$$\begin{aligned}E(\tilde{X}) &= E\left(\frac{1}{8}x_1 + \frac{1}{4}x_2 + \frac{1}{8}x_3 + \frac{1}{2}x_4\right) \\ &= E\left(\frac{1}{8}x_1\right) + E\left(\frac{1}{4}x_2\right) + E\left(\frac{1}{8}x_3\right) + E\left(\frac{1}{2}x_4\right) \\ &= \frac{1}{8}E(x_1) + \frac{1}{4}E(x_2) + \frac{1}{8}E(x_3) + \frac{1}{2}E(x_4) \\ &= \frac{1}{8}\mu + \frac{1}{4}\mu + \frac{1}{8}\mu + \frac{1}{2}\mu \\ &= \mu\end{aligned}$$

$\tilde{X}$ is an unbiased estimator of μ

- c. Between $\bar{X}$ and $\tilde{X}$, which one is the better estimator for μ ? Why?

$$\begin{aligned}\text{var}(\tilde{X}) &= \text{var}\left(\frac{1}{8}x_1 + \frac{1}{4}x_2 + \frac{1}{8}x_3 + \frac{1}{2}x_4\right) \\ &= \text{var}\left(\frac{1}{8}x_1\right) + \text{var}\left(\frac{1}{4}x_2\right) + \text{var}\left(\frac{1}{8}x_3\right) + \text{var}\left(\frac{1}{2}x_4\right) \\ &= \frac{1}{64}6 + \frac{1}{16}6 + \frac{1}{64}6 + \frac{1}{4}6 \\ &= \frac{38}{64}6\end{aligned}$$

$\bar{X}$ is the better estimator for μ