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EE431 Economics of Financial Markets and Institutions
Problem Set 6

Please submit at the BE office, 5th floor department of Economics building.

Deadline of submission : Tuesday 7 April, 2015, before 15.00 hrs.

PART I. Capital Asset Pricing Model (CAPM) and Arbitrage Pricing Theory (APT)

1. Use the following information to answer all parts. Assume that all assumptions of CAPM hold

Assets	Expected Return (%)	Standard Deviation (%)
Market Portfolio	7	10
Risk-free Asset	1	

- **Market portfolio consists of 0.7 of risky asset X and 0.3 of risky asset Y.**

(a) Suppose Stefan has 200US$ and he wants 4% rate of return. How much should the investor invest his fund on risk free asset, asset X and asset Y ?

Solution.

$$\begin{aligned} ER &= w_f R^f + w_m E(R^m), \\ &= w_f(0.01) + w_m(0.07). \\ 0.04 &= 0.01 w_f + w_m 0.07. \end{aligned}$$

As weights sum up to 1,

$$\begin{aligned} w_f + w_m &= 1. \\ w_m &= 1 - w_f. \end{aligned}$$

Substitute $w_m = 1 - w_f$ into the equation,

$$\begin{aligned} 0.04 &= 0.01 w_f + w_m 0.07. \\ 0.04 &= 0.01 w_f + 0.07(1 - w_f) \\ &= 0.01 w_f + 0.07 - 0.07 w_f \\ 0.06 w_f &= 0.07 - 0.04 \\ w_f &= \frac{0.03}{0.06} \\ &= 0.5 \end{aligned}$$

He will invests 0.5 of his weatlth (0.5(200) = US \$100) in risk free asset and the rest 0.5 of his wealth (0.5(200) = US \$100) in the market portfolio.

Since the market portfolio consists of 0.7 of risky asset X and 0.3 of risky asset Y.

- Stefan will invest $0.7 \times 100 = 70$ dollars of his wealth in risky asset X and
- Stefan will invest $0.3 \times 100 = 30$ dollars of his wealth in risky asset Y.

(b) What is the standard deviation of the portfolio in quation (a)?

Solution.

Standard deviation of portfolio in question a $= w_m \sigma_m = 0.5 \times 10\% = 5\%$.

(c) Suppose Stefanie has 100US$ and she wants 10% rate of return. How much should she invest her fund on risk free asset, asset X and asset Y ?

$$
\begin{aligned}
ER &= w_f R^f + w_m E(R^m), \\
&= w_f(0.01) + w_m(0.07). \\
0.10 &= 0.01 w_f + w_m 0.07.
\end{aligned}
$$

As weights sum up to 1,

$$
\begin{aligned}
w_f + w_m &= 1. \\
w_m &= 1 - w_f.
\end{aligned}
$$

Substitute $w_m = 1 - w_f$ into the equation,

$$
\begin{aligned}
0.10 &= 0.01 w_f + w_m 0.07. \\
0.10 &= 0.01 w_f + 0.07(1 - w_f) \\
&= 0.01 w_f + 0.07 - 0.07 w_f \\
0.06 w_f &= 0.07 - 0.10 \\
w_f &= \frac{-0.03}{0.06} \\
&= -0.5
\end{aligned}
$$

Shee will invests -0.5 of his weatlth (-0.5(100) = -US \$50) in risk free asset. She will borrow \$US 50 and invest $1.5 \times 100 = 150$ US\$ in the market portfolio.

Since the market portfolio consists of 0.7 of risky asset X and 0.3 of risky asset Y.

- Stefan will invest $0.7 \times 150 = 105$ dollars of his wealth in risky asset X and
- Stefan will invest $0.3 \times 150 = 45$ dollars of his wealth in risky asset Y.

(d) What is the standard deviation of the portfolio in quation (c)?

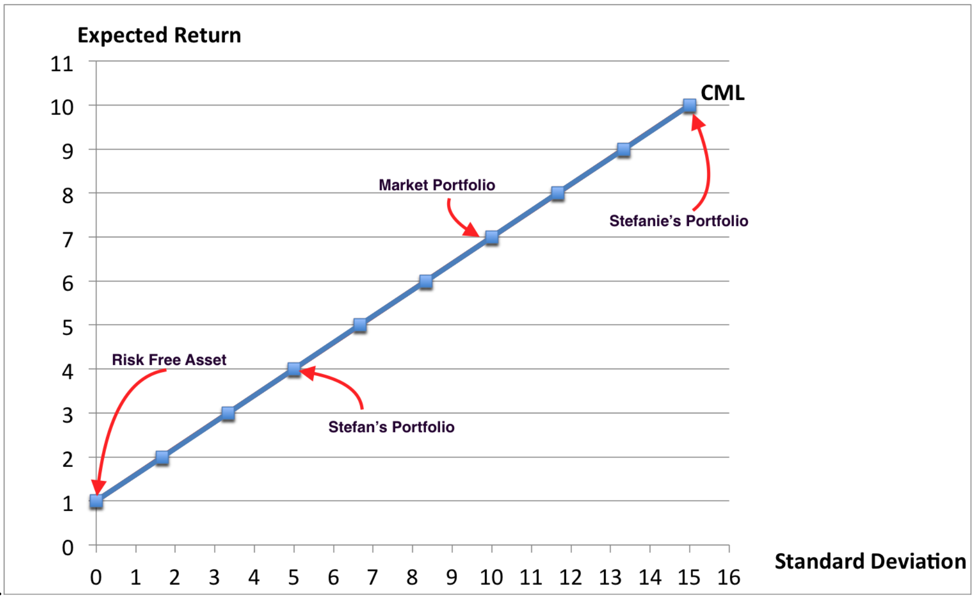
Solution.

Standard deviation of portfolio in question a $= w_m \sigma_m = 1.5 \times 10\% = 15\%$.

(e) Write down CML equation.

$$
CML : ER_p = R^f + \left[\frac{ER^m - R^f}{\sigma_m}\right]\sigma_p = 0.01 + \left[\frac{0.07 - 0.01}{0.10}\right]\sigma_p = 0.01 + 0.006\sigma_p
$$

(f) Indicate Stefan's choice and Stefanie's choice, riskfree asset and market portfolio on CML. Grapically illustrate.



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2. $ER_A = 0.05 + 0.5F_1$; asset A has $\beta_{A1} = 0.5$.

$ER_B = 0.02 - 0.25F_1$; asset B has $\beta_{B1} = -0.25$.

Notation: β_{j1} and β_{j1} for j = A; B denote the responses of the rates of return on assets A and B to the factor 1 (F_1).

(a) Determine the portfolio weights you need to place on A and B in order to construct a portfolio which has zero exposure to the factor 1 (F_1) (a risk-free portfolio). What is the expected return of the risk-free portfolio?

Solution.

Let w_A be the weight of asset A and w_B be the weight of asset B in a portfolio. Then, expected return of the portfolio can be written as follows.

$$\begin{aligned} E(R_p) &= aE(X) + bE(Y), \\ E(R_p) &= w_A E(R_A) + w_B E(R_B), \\ &= w_A(0.05 + 0.5F_1) + w_B(0.02 - 0.25F_1), \\ &= (0.05w_A + 0.02w_B) + (0.5w_A - 0.25w_B)F_1. \end{aligned}$$

A riskless portfolio must have beta equal to zero. Therefore,

$0.5w_A - 0.25w_B = 0 \Rightarrow$ equation (1).

The sum of the weights must always be 1.

$w_A + w_B = 1 \Rightarrow$ equation (2),

Solving the system of equations (equation (1) and equation (2)) for w_A and w_B and get $w_A = \dfrac{1}{3}$ and $w_B = \dfrac{2}{3}$.

Substitute $w_A = \dfrac{1}{3}$ and $w_B = \dfrac{2}{3}$ in to $E(R_p) = (0.05w_A + 0.02w_B) + (0.5w_A - 0.25w_B)F_1$ and get $E(R_p) = \dfrac{0.05 + 2(0.04)}{3} = 0.03$.

Hence, to construct a riskless portfolio, the weight placed on asset A must be equal to $\dfrac{1}{3}$ and the weight placed on asset B must be equal to $\dfrac{2}{3}$. The return of the risk-free portfolio is then equal to 0.03 or 3%.

(a) From (a), is there any abritrage opportunity exist if the market risk-free interest rate is 1%? If yes, explain how an investor can make an arbitrage profit.

Solution.

The risk-free portfolio in (a) and the risk-free asset avaiable in the market (which pays the market risk-free interest rate) are both riskless. Their returns are certain, regardless of the value of F_1. They have the same value of beta. They should yield the same rate of return. Otherwise, everyone would want to buy the security or the portfolio with the lower price (higher yields) and to sell the security or the portfolio with the higher price (lower yields). The rate of return of the risk-free asset porfolio in (a) is equal to 0.03. The rate of return of the risk-free asset porfolio in (a) is *higher* than tje market risk-free interest rate (for example, the interest rate on treasury bill). $3\% > 1\%$. $0.03 > 0.01$. We should *short-sell* the one that *pays the lower rate of retur*n, the risk-free asset available in the market. In other words, we borrow from the market and pay the market risk-free interest rate. After that, we use the proceeds to *buy* the one that *pays the higher rate of return, risk-free portfolio in (a).* We get 3% rate of return from investing in a risk-free portfolio in (a). We would realize a profit of $(3\% - 1\%) = 2\%$. This arbitrage opportunity is inconsistent with market equilibrium.

(b) Determine the portfolio weights you need to place on A and B in order to construct a portfolio which has a unit exposure to the factor 1 (F_1). What is the expected return of the portfolio which has a unit exposure to the factor 1?

Solution.

Let w_A be the weight of asset A and w_B be the weight of asset B in a portfolio. Then, expected return of the portfolio can be written as follows.

$$\begin{aligned} E(R_p) &= aE(X) + bE(Y), \\ E(R_p) &= w_A E(R_A) + w_B E(R_B), \\ &= w_A(0.05 + 0.5F_1) + w_B(0.02 - 0.25F_1), \\ &= (0.05w_A + 0.02w_B) + (0.5w_A - 0.25w_B)F_1. \end{aligned}$$

Given that the portfolio must have beta equal to one. Therefore,

$0.5w_A - 0.25w_B = 1 \Rightarrow$ equation (1).

The sum of the weights must always be 1.

$w_A + w_B = 1 \Rightarrow$ equation (2),

Solving the system of equations (equation (1) and equation (2)) for w_A and w_B and get $w_A = \frac{5}{3}$ and $w_B = -\frac{2}{3}$.

Substitute $w_A = \frac{5}{3}$ and $w_B = -\frac{2}{3}$ in to $E(R_p) = (0.05w_A + 0.02w_B) + (0.5w_A - 0.25w_B)F_1$ and get $E(R_p) = \left(\frac{0.05(5) + (0.02)(-2)}{3}\right) + \left(\frac{0.5(5) - 0.25(-2)}{3}\right) F_1 = 0.07 + F_1$.

Hence, to construct a portfolio which has a unit exposure to the factor 1 (F_1), the weight placed on asset A must be equal to $\frac{5}{3}$ and the weight placed on asset B must be equal to $-\frac{2}{3}$ (we have to short sell asset B.). The expected return of the portfolio which has a unit exposure to the factor 1 (F_1) is then equal to $0.07 + F_1$.

3. Derive an equation for the equilibrium expected rate of return of an asset K which has $\beta_{K1} = b_K$.

Solution.

Making use $ER_i = R_f + (\lambda_1 - R_f)\beta_{i1}$, $R_f = 0.03$ (from question (a) and $E(R_p) = \lambda_1 = 0.07 + F_1$ (from question (c) , and get

$$\begin{aligned} E(R_i) &= 0.03 + ((0.07 + F_1) - 0.03)\,\beta_{i1}, \\ &= 0.03 + (0.04 + F_1)\,\beta_{i1}, \\ &= (0.03 + 0.04\beta_{i1}) + \beta_{i1}F_1. \end{aligned}$$

The equilibrium expected rate of return of the asset K is equal to $(0.03 + 0.04b_K) + b_K F_1$.

PART II. Financial Institutions

1. The bank you own has the following balance sheet:

Assets		Liabilities	
Reserves	\$75 million	Deposits	\$500 million
Loans	\$255 million	Bank Capital	\$100 million

   (a) If the bank suffers a deposits outflow of \$20 million with a required reserve ratio on deposits of 10%, does the bank has enough reserves to meet the reserve requirement?

   Solution.

   When there ia a deposit outflow of \$20 million, the bank deposits and bank reserves decline by \$20 million.
   Deposits declined from \$500 million to \$480 million (500 - 20) and reserves declined from \$75 to \$55 million (75-20).
   With \$480 million of deposits, the bank is required to hold \$48 million of reserves.
   The bank has \$55 million of reserves, which is more than than the required reserves of \$48 million.
   Therefore, the bank has enough reserves to meet the reserve requirement.

   (b) If the bank suffers a deposits outflow of \$50 million (in stead of \$20 million in question a) with a required reserve ratio on deposits of 10%, does the bank have enough reserves to meet the reserve requirement?

   Solution.

   When there ia a deposit outflow of \$50 million, the bank deposits and bank reserves decline by \$50 million.
   Deposits declined from \$500 million to \$450 million (500 - 50) and reserves declined from \$75 to \$25 million (75-50).
   With \$450 million of deposits, the bank is required to hold \$45 million of reserves.
   The bank has \$25 million of reserves, which is less than than the required reserves of \$45 million.
   Therefore, the bank does not have enough reserves to meet the reserve requirement.

2. Suppose that you are the manager of a bank whose \$75 million of asset have an average modified duration of 4, while its \$75 million of liabilities have an average modified duration of 6. Conduct a duration analysis for the bank and show what will happen to the net worth of the bank if interest rate rise by 2%.

   Solution.

   Percent change in market value of security = - percent change in interest rate $\times$ modified duration
   As the interest rate rise by 2%, the market value of the bank's asset falls by 8% (2 $\times$ 4).
   The bank's asset declines from \$75 million to \$69 million (75 - 8%(75)).
   As the interest rate rise by 2%, the market value of the bank's liability falls by 12% (2 $\times$ 4).
   The bank's liabilities delines from \$75 million to \$66 (75-12%(75)).
   The net result is that the net worth (the market value of asset minus the liabilities) is increased by \$3 million.

3. Suppose that you are the manager of a bank that has \$15 million of fixed-rate assets, \$30 million of rate-sensitive assets, \$25 million of fixed rate liabilities and \$20 million of rate-sensitive liabilities. Conduct a gap analysis for the bank, and show what will happen to bank profits if interest rate rises by 5%.

   Solution.

Suppose that interest rate rises by 5%. The income on asset rises by \$1.5 million (\$30 million of rate-sensitive assets $\times$ 5%).
The payments on the liabilities rise by \$1 million (\$20 million of rate-sensitive liabilities $\times$ 5%). The bank's profit now rises by \$0.5 million.

The gap is equal to the amount of rate-sentitive asset minus the amount of rate-sensitive liabilities. In this case, the gap is equal to 30 -20 = \$10 million.
The gap is positive. Therefore, the bank profits will rise when the interest rate increases.