

EE320 (2/2012)

INTRODUCTORY MATHEMATICAL ECONOMICS

INTEGRATION AND ITS APPLICATION

(Part 2)

Topics

- Total revenue function from MR function
- Total cost function from MC function
- Profit function from MR-MC
- Utility function from MU function
- Consumption and saving functions from marginal propensity functions
- Capital formation and investment functions
- Consumer surplus, producer surplus and total surplus
- First degree price discrimination

Derivation of TR from MR

- $TR = \int MR(Q)dQ$
- Example: Let $MR = 10 - 2Q$, what is the TR function?

Derivation of TC from MC

- $TC = \int MC(Q)dQ$
- Example: Let $MC = C'(Q) = 2e^{0.2Q}$.

If the fixed cost is $C_F = 90$, what is the TC function?

Derivation of Profit Function from MR-MC

- $\pi' = MP = MR - MC. \rightarrow \pi = \int \pi'(Q)dQ$
- Example: Let $MR = 50 - 2Q$ and $MC = 10 + Q$. Find total profit when $Q = 10$. Assume that there is no fixed cost.

Derivation of Utility Function from MU

- $U(x) = \int MU(x) dx$
- Example: Let $MU(x) = \frac{5}{3\sqrt{x}}$. Find the utility function.

Derivation of Consumption/Saving from Marginal Propensity Function

- Suppose the marginal propensity to save (MPS) function is:
 $S'(Y) = 0.3 - 0.1Y^{1/2}$, and $S = 0$ when $Y = \$81$.
Find the saving and consumption functions.

Capital Formation and Investment functions (1)

- Definitions:

- $K(t)$ = capital stock at time t
- dK/dt = the rate of capital formation
- $I(t)$ = the rate of net investment flow at time t

- Relationship between capital stock and net investment:

$$\frac{dK}{dt} = \dot{K} \equiv I(t) \quad \Rightarrow \quad K(t) = \int I(t)dt = \int \frac{dK}{dt} dt = \int dK$$

- **Gross investment:** $I_g(t) = I(t) + \delta K(t)$

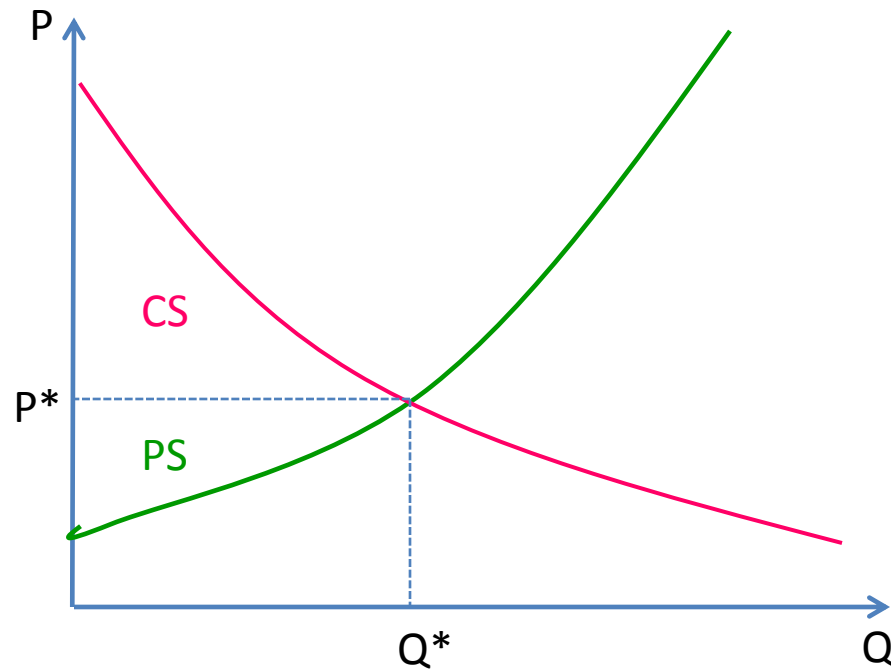
- **Capital formation** during a time interval $[a, b]$:

$$\int_a^b I(t)dt = K(t) \Big|_a^b = K(b) - K(a)$$

Capital Formation and Investment functions (2)

- Example: Suppose the net investment flow is $I(t) = 3t^{1/2}$ and the initial capital stock at time $t = 0$ is $K(0) = 25$.
 1. What is the time path of capital K ?
 2. What is the capital formation during the time interval $[1, 4]$?

Consumer Surplus, Producer Surplus, & Total Surplus



$$CS = \int_0^{Q^*} [D(Q) - P^*] dQ$$

$$PS = \int_0^{Q^*} [P^* - S(Q)] dQ$$

Example: Consumer Surplus & Producer Surplus

- Given a supply function $S(P) = -\frac{1}{2} + \frac{1}{2}P$ and a demand function $D(P) = \frac{25}{2} - \frac{1}{2}P$. Calculate: 1) producer and consumer surplus and 2) total welfare.

Welfare Effects of Price Change

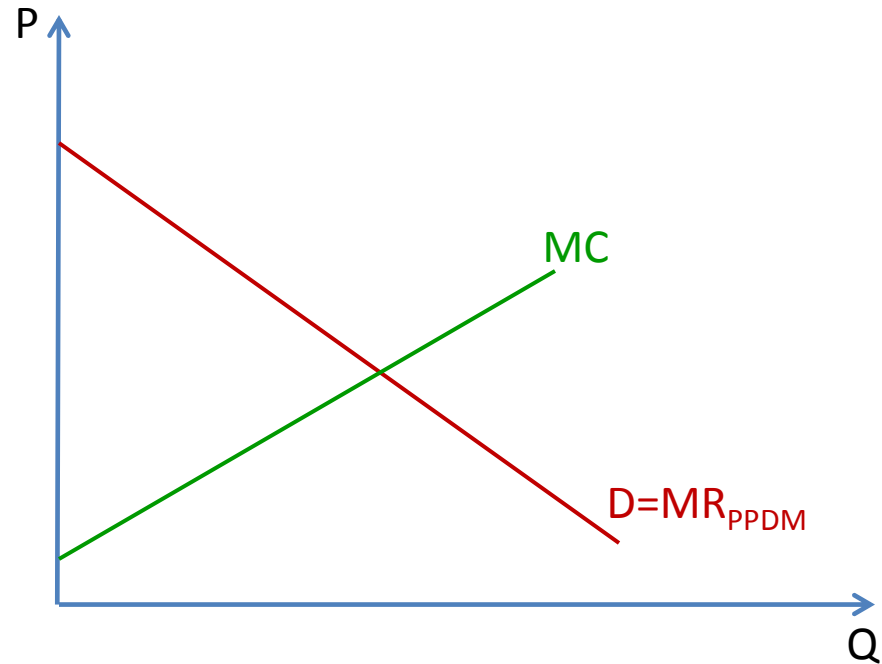
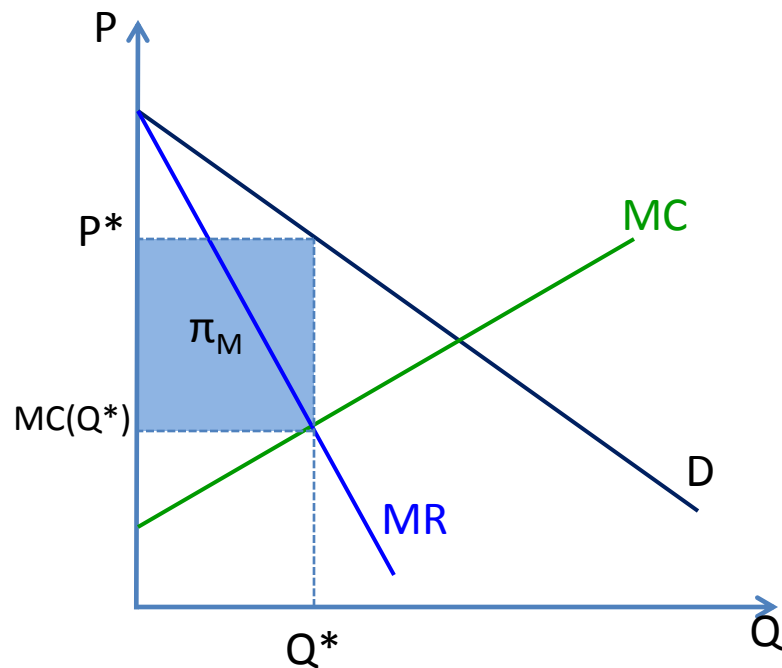
- From the previous example, if the demand changes so that $D(P) = \frac{30}{2} - \frac{1}{2}P$, what happens to consumer and producer surplus?

Welfare Effects of Tax Distortion

- From the previous example, if the government imposes a \$4 per unit tax on the producer, calculate the total welfare loss.

First-Degree (or Perfect) Price Discrimination

- **Perfect price discrimination** occurs when the monopolist can charge the *maximum price for each unit of output sold*.
- **(Regular) Monopolist**
- **Perfect price discriminating monopolist**



Example: Perfect Price Discrimination (1)

- Suppose that a monopolist faces a demand function $P = 24 - Q$ and $MC = 4 + 3Q$. Find the consumer surplus at the profit-maximizing quantity. (No price discrimination)

Example: Perfect Price Discrimination (2)

- If the monopolist can practice perfect price discrimination, find the total revenue that maximizes the profit.