Assignment #1

Instructions:

- For all questions, answer up to 4 decimal places.
- This assignment is due on **Tuesday, Mar 9, 2021 before class (11.00).**
- Write your answer in either digital or ordinary paper. For digital paper, export pages into a single PDF file. For ordinary paper, take photos of your writing and convert them into a single PDF file as well.
- There is no need to rewrite the question. Assign number item, ie. 1 a., clearly before your answer is sufficient.
- Submit your assignment into Moodle.
- Name your file as StudentID_Nickname (in Thai) such as 123456789_ไม้

1. Given this information

$n = 30$	$\sum_{i=1}^{n} X_i = 366$	$\sum_{i=1}^{n} Y_i = 631$	$\overline{X} = 12.20$	$\overline{Y} = 21.03$
$\sum_{i=1}^{n} (X_i)^2 = 5,564$	$\sum_{i=1}^{n} X_i Y_i = 7,524$	$\sum_{i=1}^{n} \left(X_i - \overline{X}\right)^2 = 1098.8$		$\sum_{i=1}^{n} (Y_i - \overline{Y})^2 = 882.97$
$\sum_{i=1}^{n} (X_i - \overline{X})(Y_i - \overline{Y}) = -174.20$		$\sum_{i=1}^{n} \hat{u}_i^2 = 873.14$		

Answer the following questions. Show your work.

a) From regression model: $Y_i = \beta_1 + \beta_2 X_i + u_i$, $u_i \sim NIID\left(0, \sigma^2\right)$ (normally, identically and independently distributed), find the estimators of β_1 and β_2 with OLS method and explain the meaning of the model.

b) Find r^2 and explain its meaning.

c) If $X_i = 5$, estimate the value of $\hat{Y}_i$ and explain its meaning.

d) Find the estimators of $\text{var}(u_i)$, $\text{var}\left(\hat{\beta}_1\right)$ and $\text{var}\left(\hat{\beta}_2\right)$

e) Test the hypothesis whether coefficients are different from zero at 0.05 level of significance.

f) Test the hypothesis whether coefficients are less than zero at 0.01 level of significance.

2. Given that Y is market price of a car (USD) while X is how long a car aged (years), results of the regression are as follows.

$$\hat{Y}_i = 7{,}836 - 502.4X_i$$
$$(52) \qquad (411.8)$$

Given that u_i is normally, identically and independently distributed with zero mean and σ^2 variance, total number of observations is 11,

$\bar{X} = 7.45,$

$\hat{\sigma}^2 = 212{,}877,$

$\sum(X_i - \bar{X})^2 = 78.73,$

Answer the following questions. Show your work.

a) Does the sign of $\hat{\beta}_2$ make economic sense? Provide your explanation.

b) If you are a car expert and someone asks you to estimate how much his car will be **averagely** priced at when his car is 5 years old, how much is the market price range that you would estimate that you can make sure that for 95% of the time, market price will be within the specific range?

c) If you multiply all the X with 10, report the new SRF with the standard error resulted from the multiplication.

d) Calculate the elasticity of market price when a car is 10 years old.

1. Given this information

$n = 30$	$\sum_{i=1}^{n} X_i = 366$	$\sum_{i=1}^{n} Y_i = 631$	$\overline{X} = 12.20$	$\overline{Y} = 21.03$
$\sum_{i=1}^{n}(X_i)^2 = 5{,}564$	$\sum_{i=1}^{n} X_iY_i = 7{,}524$	$\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2 = 1098.8$	$\sum_{i=1}^{n}(Y_i - \bar{Y})^2 = 882.97$	
$\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y}) = -174.20$		$\sum_{i=1}^{n}\hat{u}_i^2 = 873.14$		

Answer the following questions. Show your work.

a) From regression model: $Y_i = \beta_1 + \beta_2 X_i + u_i$, $u_i \sim NIID\left(0, \sigma^2\right)$ (normally, identically and independently distributed), find the estimators of β_1 and β_2 with OLS method and explain the meaning of the model.

$$\beta_2 = \frac{\Sigma(X_i - \bar{X})(Y_i - \bar{Y})}{\Sigma(X_i - \bar{X})^2} = \frac{-174.20}{1098.8} = -0.1585$$

$$\beta_1 = \bar{Y} - \beta_2\bar{X} = 21.03 - (-0.1585)(12.2)$$

$$= 22.9641$$

The model means that when X is 0, Y is 22.9641 and when X increases by 1 Y decrease by 0.1585.

b) Find r^2 and explain its meaning.

$$r^2 = \frac{ESS}{TSS} = \frac{\Sigma(\hat{Y}_i - \bar{Y})^2}{\Sigma(Y_i - \bar{Y})^2} \quad \text{or}$$

$$r^2 = 1 - \frac{RSS}{TSS} = 1 - \frac{\Sigma\hat{u}_i^2}{\Sigma(Y_i - \bar{Y})^2}$$

$$= 1 - \frac{873.14}{882.97} = 0.0111$$

1.11% of the variation in Y is explained by the variation in X. The remaining 98.89% is unexplained as residual.

c) If $X_i = 5$, estimate the value of Y_i and explain its meaning.

$$\hat{Y}_i = 22.9641 - 5(0.1585)$$

$$= 22.1716$$

suppose $X = 5$

Y_i is estimated to be 22.1716 by the SRF.

d) Find the estimators of $\text{var}(u_i)$, $\text{var}\left(\hat{\beta}_1\right)$ and $\text{var}\left(\hat{\beta}_2\right)$

$$\text{var}(u_i) = \sigma^2 \approx \hat{\sigma}^2 = \frac{\Sigma(Y_i - \hat{Y}_i)^2}{n-k} = \frac{873.14}{30-2} = 31.1836$$

$$\text{var}(\hat{\beta}_1) = \sigma^2_{\hat{\beta}_1} = \frac{\Sigma X_i^2}{n\Sigma x_i^2}\sigma^2 = \frac{(5564)}{30(1098.8)}(31.1836) = 5.2635$$

$$\sigma_{\beta_1} = 2.2942$$

$$\text{var}(\hat{\beta}_2) = \sigma^2_{\hat{\beta}_2} = \frac{\sigma^2}{\Sigma x_i^2} = \frac{31.1836}{1098.8} = 0.0284$$

$$\sigma_{\beta_2} = 0.1685$$

e) Test the hypothesis whether coefficients are different from zero at 0.05 level of significance.

β_1

1. $H_0: \beta_1 = 0$

$H_a: \beta_1 \neq 0$

2. $t_{cal} = \frac{\hat{\beta}_1 - \beta_1}{\sigma_{\beta_1}} = \frac{22.9641 - 0}{2.2942} = 10.0096$

3. upper bound $t_{\frac{\alpha}{2}} = 2.048$

lower " $t_{\frac{\alpha}{2}} = -2.048$

$\alpha = 0.05$

$n - k = 28$

4. $-2.048 < 10 \nless 2.048$

we can reject the null hypothesis

β_2

$H_0: \beta_2 = 0$

$H_a: \beta_2 \neq 0$

$t_{cal} = \frac{\hat{\beta}_2 - \beta_2}{\sigma_{\beta_2}} = \frac{0.1585 - 0}{0.1685} = .9407$

upper bound $t_{\frac{\alpha}{2}} = 2.048$

lower " $t_{\frac{\alpha}{2}} = -2.048$

$-2.048 < 0.9407 < 2.048$

we can't reject the null hypothesis.

f) Test the hypothesis whether coefficients are less than zero at 0.01 level of significance.

β_1

1. $H_0: \beta_1 < 0$

$H_a: \beta_1 \geq 0$

2. $t_{cal} = \frac{\hat{\beta}_1 - \beta_1}{\sigma_{\beta_1}} = \frac{22.9641 - 0}{2.2942} = 10.0096$

3. More than : AR on right of lower bound

Less than : AR on left of upper "

$\alpha = 0.01$

$n - k = 28$

Upper bound : $t_{0.01}$

$= 2.467$

$10.0096 \nless 2.467$

we can reject the null hypothesis

β_2

$H_0: \beta_2 < 0$

$H_a: \beta_2 \geq 0$

$t_{cal} = \frac{\hat{\beta}_2 - \beta_2}{\sigma_{\beta_2}} = \frac{0.1585 - 0}{0.1685} = .9407$

upper bound : $t_{0.01}$

$= 2.467$

$0.9407 < 2.467$

we cannot reject the null hypothesis

2. Given that Y is market price of a car (USD) while X is how long a car aged (years), results of the regression are as follows.

$$\hat{Y}_i = 7{,}836 - 502.4X_i$$
$$(52) \quad (411.8)$$

Given that u_i is normally, identically and independently distributed with zero mean and σ^2 variance, total number of observations is 11,

$\bar{X} = 7.45$,

$\hat{\sigma}^2 = 212{,}877$,

$\sum(X_i - \bar{X})^2 = 78.73$,

Answer the following questions. Show your work.

a) Does the sign of $\hat{\beta}_2$ make economic sense? Provide your explanation.

Yes, because cars' condition worsen over time, so it's logical that their market price decrease as time increases.

b) If you are a car expert and someone asks you to estimate how much his car will be **averagely** priced at when his car is 5 years old, how much is the market price range that you would estimate that you can make sure that for 95% of the time, market price will be within the specific range?

$$E(Y|X_0=5) = 7{,}836 - 502.4(5)$$
$$\hat{Y}_0 = 7836 - 2512 = 5{,}324$$

$$\text{var}(\hat{Y}_0) = \sigma^2\left[\frac{1}{n} + \frac{(X_0-\bar{X})^2}{\sum(X_i-\bar{X})^2}\right]$$

$$\sigma^2(\hat{Y}_0) = (212{,}877)\left[\frac{1}{11} + \frac{(5-7.45)^2}{78.73}\right]$$
$$= 35{,}582.5355$$

$$\sigma(\hat{Y}_0) = 188.6333$$

$$\Pr\left[\hat{Y}_0 - t_{\frac{\alpha}{2}} \cdot \sigma_{\hat{Y}_0} \le Y_0 \le \hat{Y}_0 + t_{\frac{\alpha}{2}} \cdot \sigma_{\hat{Y}_0}\right] = 1-\alpha$$

$\alpha = 0.05$

$n-k = 11-2$

$$\Pr[5{,}324 - 2.262(188.6333) \le Y_0 \le 5{,}324 + 2.262(188.6333)]$$

$$= \Pr[4{,}897.3115\$ \le Y_0 \le 5{,}750.6885\$] = 0.95 \text{ or } 95\%$$

5 yr. old car market price

c) If you multiply all the X with 10, report the new SRF with the standard error resulted from the multiplication.

$Z = 10X$

$$\hat{Y}_i = 7{,}836 - 50.24(Z)$$
$$se \quad (52) \quad (41.18)$$

d) Calculate the elasticity of market price when a car is 10 years old.

$x = 10$, $y = 2812 = 7836 - 502.4(10)$

$$\varepsilon = \frac{dy}{dx} \cdot \frac{x}{y} = 502.4 \cdot \frac{x}{y}$$

$$= 502.4 \cdot \frac{10}{2812} = 1.7866$$