Chapter 3

Consumer Preferences and the Concept of Utility

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Ch 3

BL

Ch 4

Choice

Ch 5

Chapter Three Overview

1. Motivation
2. Consumer Preferences and the Concept of Utility
3. The Utility Function
    - *Marginal Utility and Diminishing Marginal Utility*
4. Indifference Curves
5. The Marginal Rate of Substitution
6. Some Special Functional Forms

Motivation

- Why study consumer choice?
    - Study of how consumers with limited resources choose goods and services
    - Helps derive the demand curve for any good or service
    - Businesses care about consumer demand curves
    - Government can use this to determine how to help and whom to help buy certain goods and services

Consumer Preferences

what a consumer likes ?

Consumer Preferences tell us how the consumer would rank (that is, compare the desirability of) any two combinations or allotments of goods, assuming these allotments were available to the consumer at no cost.

These allotments of goods are referred to as **baskets** or **bundles**. These baskets are assumed to be available for consumption at a particular time, place and under particular physical circumstances.

Consumer Preferences

Assumptions

Complete and Transitive

Preferences are **complete** if the consumer can rank any two baskets of goods (A preferred to B; B preferred to A; or indifferent between A and B)

Preferences are **transitive** if a consumer who prefers basket A to basket B, and basket B to basket C also prefers basket A to basket C

→ ~~$A \succ B$, $B \succ C \Rightarrow A \succ C$~~

ORDINAL RANKING

CARDINAL RANKING (QUANTITATIVE)

$\succ$

IF $A \succ B$ and $B \succ C$, then $A \succ C$

is preferred to

$A \rightarrow B \rightarrow C$ ✓

$A \rightarrow B \rightarrow C \rightarrow A$ ✗

Consumer Preferences

Assumptions

Monotonic / Free Disposal

Preferences are **monotonic** if a basket with more of *at least one* good and no less of any good is preferred to the original basket.

(= More is preferred to less)

= "More is better"

Ex: $A(\overset{x}{5}, \overset{y}{5}) \prec B(\overset{x}{5}, \overset{y}{7})$

Types of Ranking

Example:

Students take an exam. After the exam, the students are ranked according to their performance. An ordinal ranking lists the students in order of their performance (i.e., Harry did best, Joe did second best, Betty did third best, and so on). A cardinal ranking gives the mark of the exam, based on an absolute marking standard (i.e., Harry got 80, Joe got 75, Betty got 74 and so on). Alternatively, if the exam were graded on a curve, the marks would be an ordinal ranking.

The Utility Function

The three assumptions about preferences allow us to represent preferences with a utility function.

Utility function

– a function that measures the level of satisfaction a consumer receives from any basket of goods and services.

– assigns a number to each basket so that more preferred baskets get a higher number than less preferred baskets.

– U = u(y) ✓ amount of good y per time period

The Utility Function

Implications:

- An ordinal concept: the precise magnitude of the number that the function assigns has no significance.
- Utility not comparable across individuals.
- Any transformation of a utility function that preserves the original ranking of bundles is an equally good representation of preferences. e.g. $U = \sqrt{y}$ vs. $U = \sqrt{y} + 2$ represent the same preferences.

Marginal Utility

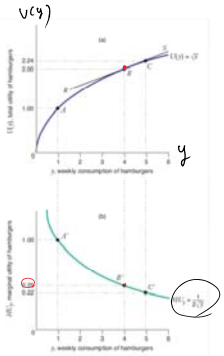
Marginal Utility of a good y

- additional utility that the consumer gets from consuming a little more of y
- i.e. the rate at which total utility changes as the level of consumption of good y rises
- $\boxed{MU_y = \Delta U/\Delta y}$
- slope of the utility function with respect to y

Diminishing Marginal Utility

The principle of diminishing marginal utility states that the marginal utility falls as the consumer consumes more of a good.

U(y)



y

$U(y) = \sqrt{y}$

Diminishing Marginal Utility

$y = 4.00 \rightarrow U(y) = \sqrt{4} = 2$

$y' = 4.01 \rightarrow U(y') = \sqrt{4.01} = 2.0025$

$\Delta y = 0.01$

$\Delta U = .0025$

$MU = \frac{\Delta U}{\Delta y} = \frac{0.0025}{0.01} = 0.25$

Marginal Utility

The marginal utility of a good, x, is the additional utility that the consumer gets from consuming a little more of x when the consumption of all the other goods in the consumer's basket remain constant.

- $U(x, y) = x + y$
- $\Delta U/\Delta x$ (y held constant) = MU_x
- $\Delta U/\Delta y$ (x held constant) = MU_y

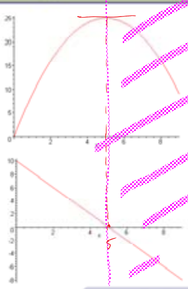
Marginal Utility

Example of U(H) and MU_H

$U(H) = 10H - H^2$
$MU_H = 10 - 2H$

H	H^2	U(H)	MU_H
2	4	16	6
4	16	24	2
6	36	24	-2
8	64	16	-6
10	100	0	-10

Marginal Utility



$U(H) = 10H - H^2$

$MU_H = 10 - 2H$

Marginal Utility

Example of U(H) and MU_H

- The point at which he should stop consuming hotdogs is the point at which $MU_H = 0$
- This gives $H = 5$.
- That is the point where Total Utility is flat.
- You can see that the utility is diminishing.

Marginal Utility – multiple goods

$$U = xy^2$$
$$MU_x = y^2$$
$$MU_y = 2xy$$

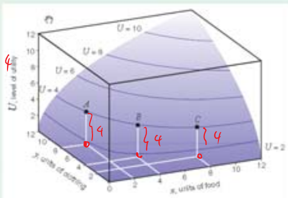
- More is better? More *y* more and more *x* indicates more *U* so yes it is monotonic
- Diminishing marginal utility?
  - MU of x is not dependent of x. So the marginal utility of x (movies) does not decrease as the number of movies increases.
  - MU of y increases with increase in number of operas (y) so neither exhibits diminishing returns.

MATLAB / STISTICA to plot

$U = \sqrt{xy}$

$U = \sqrt{2 \cdot 8} = \sqrt{16} = 4$

FIGURE 3.4 Graph of the Utility Function $U = \sqrt{xy}$
The level of utility is shown on the vertical axis, and the amounts of food (x) and clothing (y) are shown, respectively, on the right and left axes. Contours representing lines of constant utility are also shown. For example, the consumer is indifferent between baskets A, B, and C because they all yield the same level of utility (U = 4).

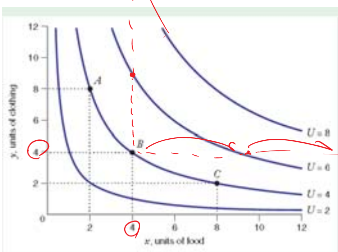


$U = \sqrt{4 \cdot 4} = \sqrt{16} = 4$

Indifference Curves

An **Indifference Curve** or **Indifference Set:** is the set of all baskets for which the consumer is indifferent

An **Indifference Map** : Illustrates a set of indifference curves for a consumer



$U = \sqrt{xy}$

- monotonic ✓

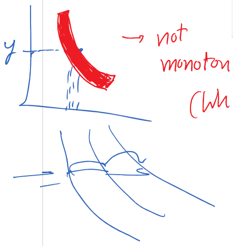
Indifference Curves

Key Properties

1) **Monotonicity** => indifference curves have negative slope – and indifference curves are not "thick"

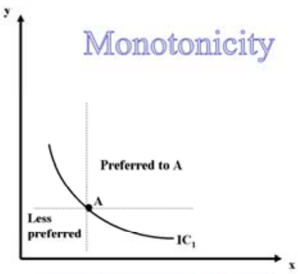
2) **Transitivity** => indifference curves do not cross

3) **Completeness** => each basket lies on only one indifference curve



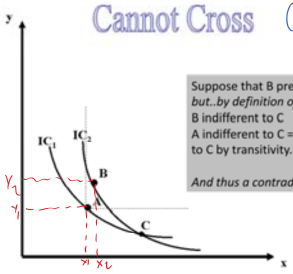
→ not monotonic ! (why?)

Indifference Curves



Indifference Curves

Cannot Cross (TRANSITIVITY)



Suppose that B preferred to A.
but..by definition of IC_i
B indifferent to C
A indifferent to C => B indifferent to C by transitivity.

And thus a contradiction.

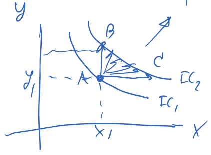
ON IC_1 : $A \sim C$
ON IC_2 : $B \sim C$
Then by transitivity
IF $A \sim C$ and $B \sim C$,
then it must be that $A \sim B$

1^{st} statement

However, B has more of X and Y.

2^{nd} statement.

By more-is-better assumption, $B \succ A$!



Indifference Curves

Example

$$U = xy^2$$

Check that underlying preferences are complete, transitive, and monotonic.

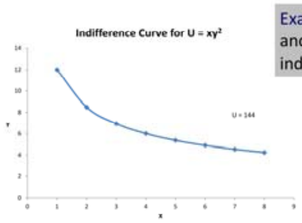
$$MU_x = y^2$$

$$MU_y = 2xy$$

$$for \quad U = 144$$

x	y	xy^2
8	4.24	143.8
4	6	144
3	6.93	144.07
1	12	144

Indifference Curves



Example: Utility and the single indifference curve.

Marginal Rate of Substitution (MRS)

The marginal rate of substitution: is the maximum rate at which the consumer would be willing to substitute a little more of good x for a little less of good y;

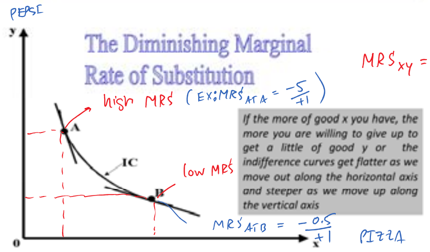
It is the increase in good x that the consumer would require in exchange for a small decrease in good y in order to leave the consumer just indifferent between consuming the old basket or the new basket;

It is the rate of exchange between goods x and y that does not affect the consumer's welfare;

It is the negative of the slope of the indifference curve:

$MRS_{x,y} = -\Delta y/\Delta x$
(for a constant level of preference)

Marginal Rate of Substitution



$MRS_{xy} = \frac{\Delta y}{\Delta x}$ = slope of IC

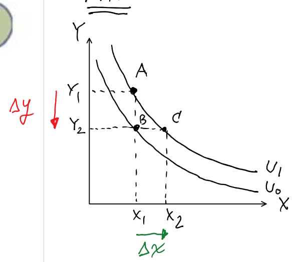
Marginal Rate of Substitution

$MU_x(\Delta x) + MU_y(\Delta y) = 0$ ...along an IC...

$MU_x/MU_y = -\Delta y/\Delta x = MRS_{x,y}$

- Positive marginal utility implies the indifference curve has a negative slope *(implies monotonicity)*
- Diminishing marginal utility implies the indifference curves are convex to the origin *(implies averages preferred to extremes)*

MRS



Point of departure : Point A

- Moving from $A(x_1, y_1)$ to $B(x_1, y_2)$ involves w/ " Utility loss " :
  $\overset{2}{MU_y} \cdot \overset{-10}{\Delta y}$
- Moving from $B(x_1, y_2)$ to $C(x_2, y_2)$ involves w/ " Utility gain " :
  $\overset{4}{MU_x} \cdot \overset{+5}{\Delta x}$
- $A \to B$ & $B \to C$ = $A \to C$

$MU_y \cdot \Delta y + MU_x \cdot \Delta x = \Delta U = 0$ (NO CHANGE IN UTILITY AS $A \to C$ IS ON THE SAME IC)

$MU_y \cdot \Delta y = - MU_x \cdot \Delta x$

$\frac{\Delta y}{\Delta x} = -\frac{MU_x}{MU_y}$

$-\frac{\Delta y}{\Delta x} = \frac{MU_x}{MU_y} = MRS_{xy}$

MRS_{xy} = " the negative " of the slope of indifference curve

Marginal Rate of Substitution

The Marginal Rate of Substitution

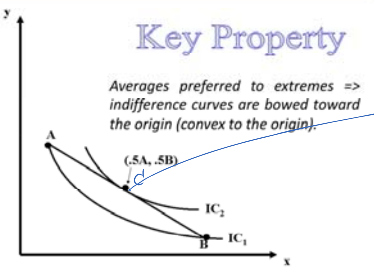
Implications of this substitution:

- Indifference curves are negatively-sloped, bowed out from the origin, preference direction is up and right
- Indifference curves do not intersect the axes

$\boxed{MRS_{xy} = \frac{MU_x}{MU_y}}$

$\boxed{MRS_{xy} = -\frac{\Delta y}{\Delta x}}$

Indifference Curves



x , y
A (2 , 10)
C (6 , 6)
B (10 , 2)

C ≻ A
C ≻ B

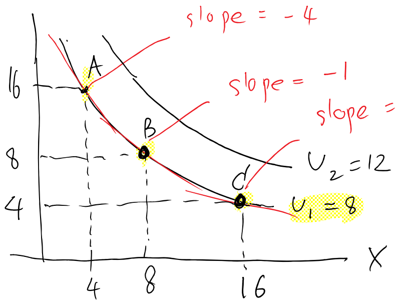
Ex : $\boxed{U = \sqrt{xy}}$ or $U = x^{\frac{1}{2}}$...

Find MU_x , MU_y , MRS_{xy}

$MU_x = \frac{y}{2\sqrt{xy}}$

$MU_y = \frac{x}{2\sqrt{xy}}$

$MRS_{xy} = \frac{y}{x}$



A (4, 16) → $U = \sqrt{4 \cdot 16}$ ...

B (8, 8) → $U = \sqrt{8 \cdot 8}$ ...

C (16, 4) → $U = \sqrt{16 \cdot 4}$ ...

$\boxed{MRS_{xy} = \frac{y}{x}}$

Indifference Curves

Do the indifference curves intersect the axes?

A value of x = 0 or y = 0 is inconsistent with any positive level of utility.

Since MRS is diminishing ,

$y^{\frac{1}{2}}$

$= \sqrt{4} \cdot \sqrt{16} = 2 \cdot 4 = 8$

$= \sqrt{64} = 8$

$= \sqrt{16} \cdot \sqrt{4} = 4 \cdot 2 = 8$

$RS'_{AT\,A} = \frac{16}{4} = 4$

$RS'_{AT\,B} = \frac{8}{8} = 1$

1   x

Since MRS is diminishing, it implies that IC's are convex to the origin.

Marginal Rate of Substitution

The Marginal Rate of Substitution

Example: $\boxed{U = Ax^2+By^2}$; $MU_x=2Ax$; $MU_y=2By$

(where: A and B positive)

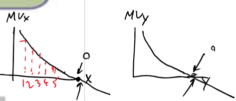
$MRS_{x,y} = MU_x/MU_y = 2Ax/2By = Ax/By$

Marginal utilities are positive (for positive x and y)
Marginal utility of x increases in x;
Marginal utility of y increases in y

Indifference Curves

$U = \sqrt{xy}$

Example: $U=(xy)^{.5}$; $MU_x=y^{.5}/2x^{.5}$; $MU_y=x^{.5}/2y^{.5}$



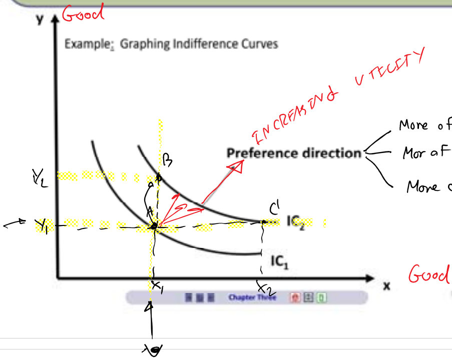
A. *Is more better for both goods?* Yes, since marginal utilities are positive for both.

B. *Are the marginal utility for x and y diminishing?* Yes. (For example, as x increases, for y constant, MU_x falls.)

C. *What is the marginal rate of substitution of x for y?*
$MRS_{x,y} = MU_x/MU_y = y/x$

Indifference Curves

Example: Graphing Indifference Curves



More of x → U↑
More of y → U↑
More of x & y → U↑

- A good is "Good" if more is preferred to less.
- A good is "Bad" if less is preferred to more.
- A good is "Neuter" if having more or having less does not affect utility.

Special Functional Forms

Cobb-Douglas: $U = Ax^{\alpha}y^{\beta}$

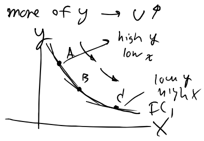
where: $\alpha + \beta = 1$; A, α, β positive constants

$MU_x = \alpha Ax^{\alpha-1}y^{\beta}$ → positive MU
$MU_y = \beta Ax^{\alpha}y^{\beta-1}$ → positive MU

Meaning?
Answer: more of x → U↑
more of y → U↑

$MRS_{x,y} = (\alpha y)/(\beta x)$
"Standard" case

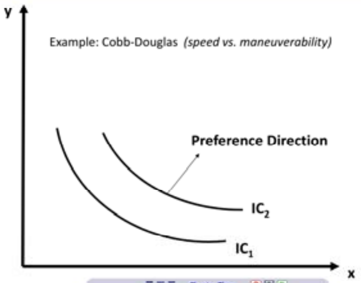
$\boxed{MRS = \frac{\alpha y}{\beta x}}$
is diminishing (Why?)



$es'_{AT\ B} = \frac{8}{8} = 1$

$es'_{AT\ C'} = \frac{4}{16} = \frac{1}{4}$

Special Functional Forms



Special Functional Forms

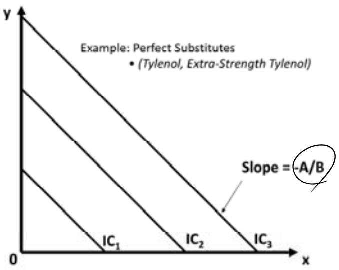
$\boxed{\text{Perfect Substitutes: } U = Ax + By}$

Where: A, B positive constants

$MU_x = A$
$MU_y = B$

$MRS_{x,y} = A/B$ so that 1 unit of x is equal to B/A units of y *everywhere* (constant MRS).

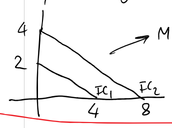
Special Functional Forms



$MRS = -\frac{\Delta Y}{\Delta X}$

$MRS = -(-A/B) = A/B$

Y (orange juice)



$MRS = -\left(-\frac{1}{2}\right) = \frac{1}{2} = 0.5$

$\to$

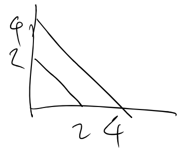
$Ax + By$

$\boxed{U = 1 \cdot x + 2y}$

$MU_x = 1$.

$MU_y = 2$.

$MRS_{xy} = \frac{MU_x}{MU_y} = \frac{1}{2}$.



Special Functional Forms

Perfect Complements: U = Amin(x,y)

where: A is a positive constant.

$MU_x = 0$ or A
$MU_y = 0$ or A

$MRS_{x,y}$ is 0 or infinite or undefined (corner)

$U = A \cdot \min(x, y)$

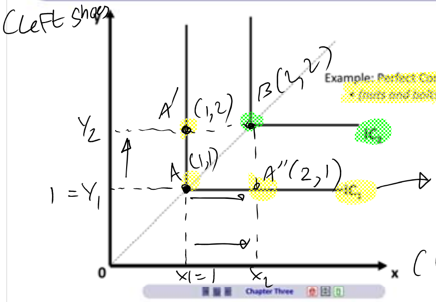
Ex: suppose $A = 1$

$U = \min(x, y)$

Ex: $U = \min(\overset{x}{5}, \overset{y}{2}) = 2$

$U = \min(10, 2) = 2$

Special Functional Forms



$U = \min(R, L)$

$A(1,1) \to U = \min(1,1) = 1$

$A'(1,2) \to U = \min(1,2) = 1$

$A''(2,1) \to U = \min(2,1) = 1$

$B(2,2) \to U = \min(2,2) = 2$

Special Functional Forms

Quasi-Linear Preferences:

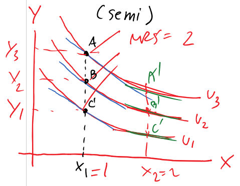
$U = v(x) + Ay$ → Utility function is linear in y, Utility function is non-linear in x. That's why we call "quasi-linear" (semi)

Where: A is a positive constant.

$MU_x = v'(x) = \Delta V(x)/\Delta x$, where Δ small $MU_y = A$

"The only thing that determines your personal trade-off between x and y is how much x you already have."

can be used to "add up" utilities across individuals



IC's are "parallel."

Meaning ???

Ans: At a given amount of x, let's say x_1 units/month, MRS_{xy} at basket A, B, C are all the same!!! (same slope)

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Special Functional Forms