Consumer Behavior:
Advanced topics
EE311

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Agenda

- Hicks and Slutsky substitution effect
- Finding consumer surplus from ICs
- Revealed Preferences
- Index numbers
- Duality

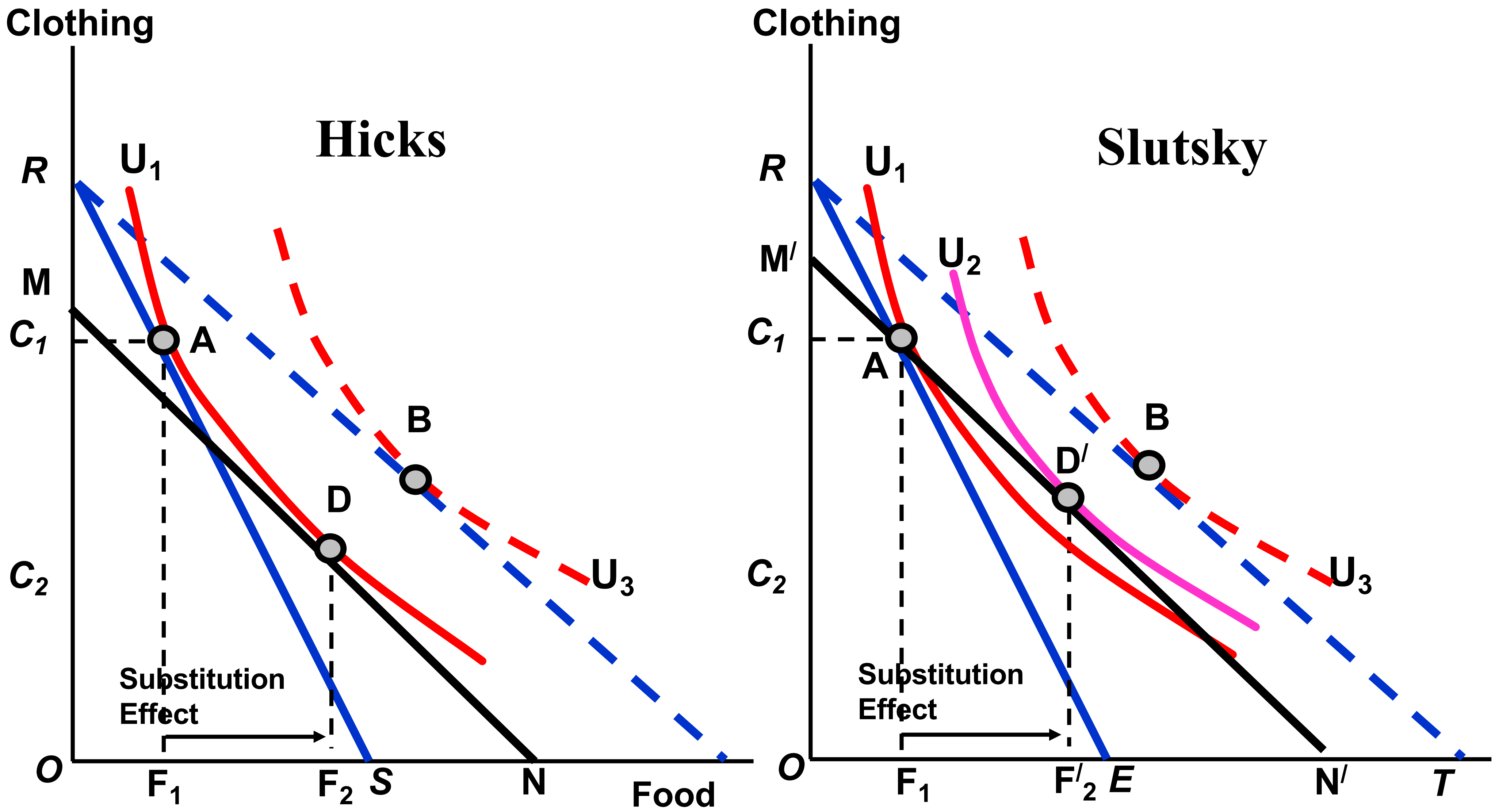
Hicks and Slutsky substitution effect

- To calculate the substitution effect, we need to compensate the consumer for the change in real income.
- The substitution effect defined earlier compensate the income change until the consumer receives the same level of utility.
  - This is called Hicks substitution effect.
- But we cannot measure utility, so Hicks substitution effect is not observable.

Hicks and Slutsky substitution effect

- The alternative way is to compensate the income change until the consumer can consume exactly the same bundle.
  - This is called Slutsky substitution effect.
- With this new budget, at the new relative prices, the consumer will choose a new optimal bundle which is observable.

Hicks v.s. Slutsky Substitution Effect



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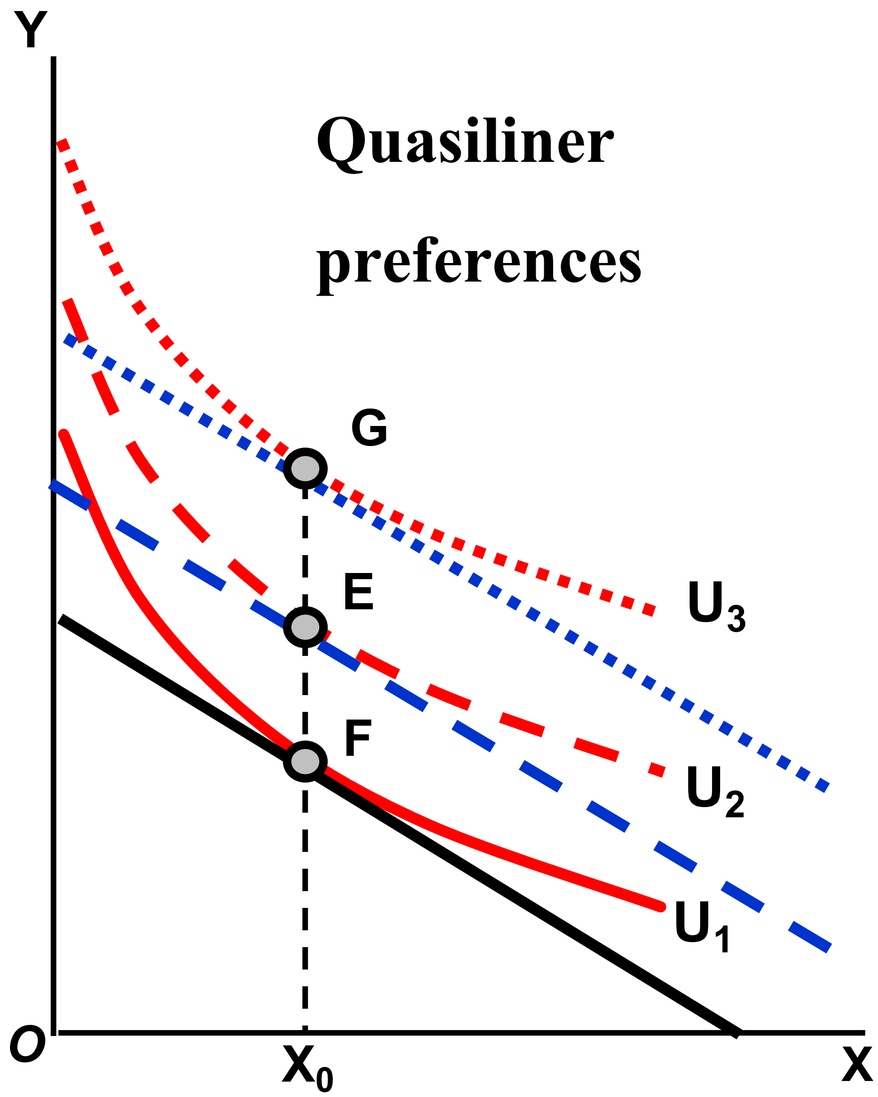
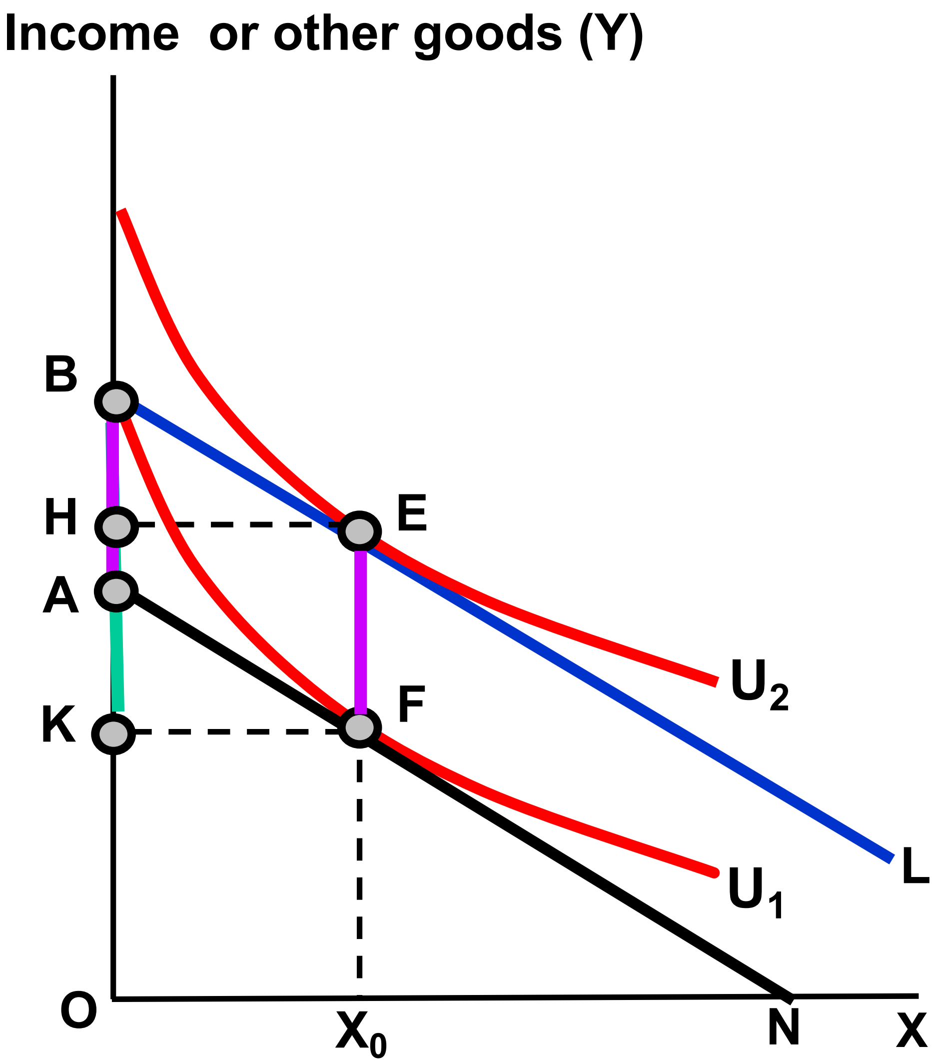
Finding consumer surplus from indifference curves



- Assume that the preferences are given by $U = Y + X^a$
- The utility function is called quasilinear where all indifference curves are "shift" versions of the lower one. Therefore the value of MRS at the same level of X is always the same.
- Income effect for this case is always zero.

Finding consumer surplus from indifference curves



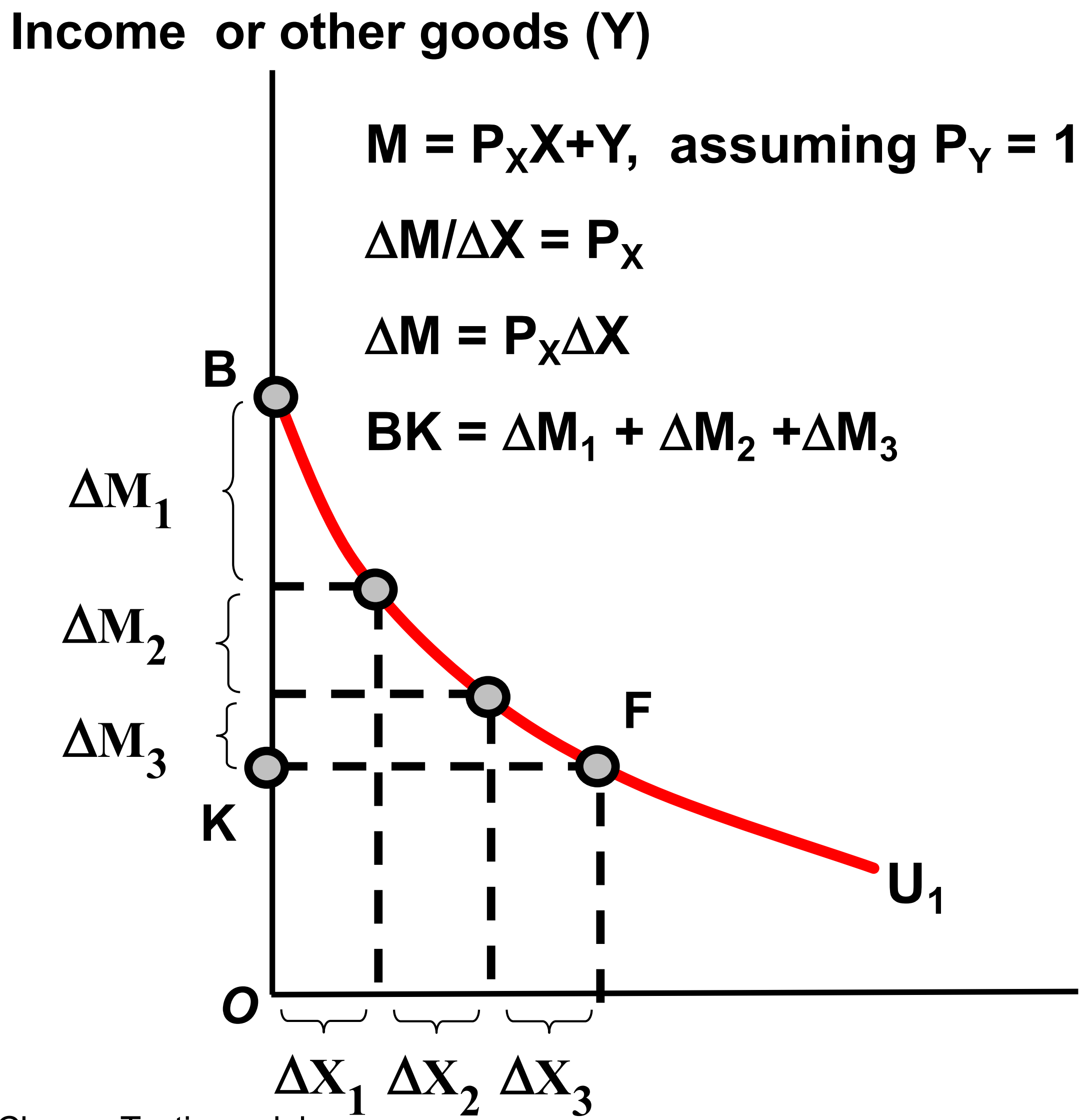
- Assume that price of Y is 1 and consumer income is OB.
- Hence the vertical axis measure both other goods and income. Further more the slope of the budget line BL will equal to the price of X (denote by P).

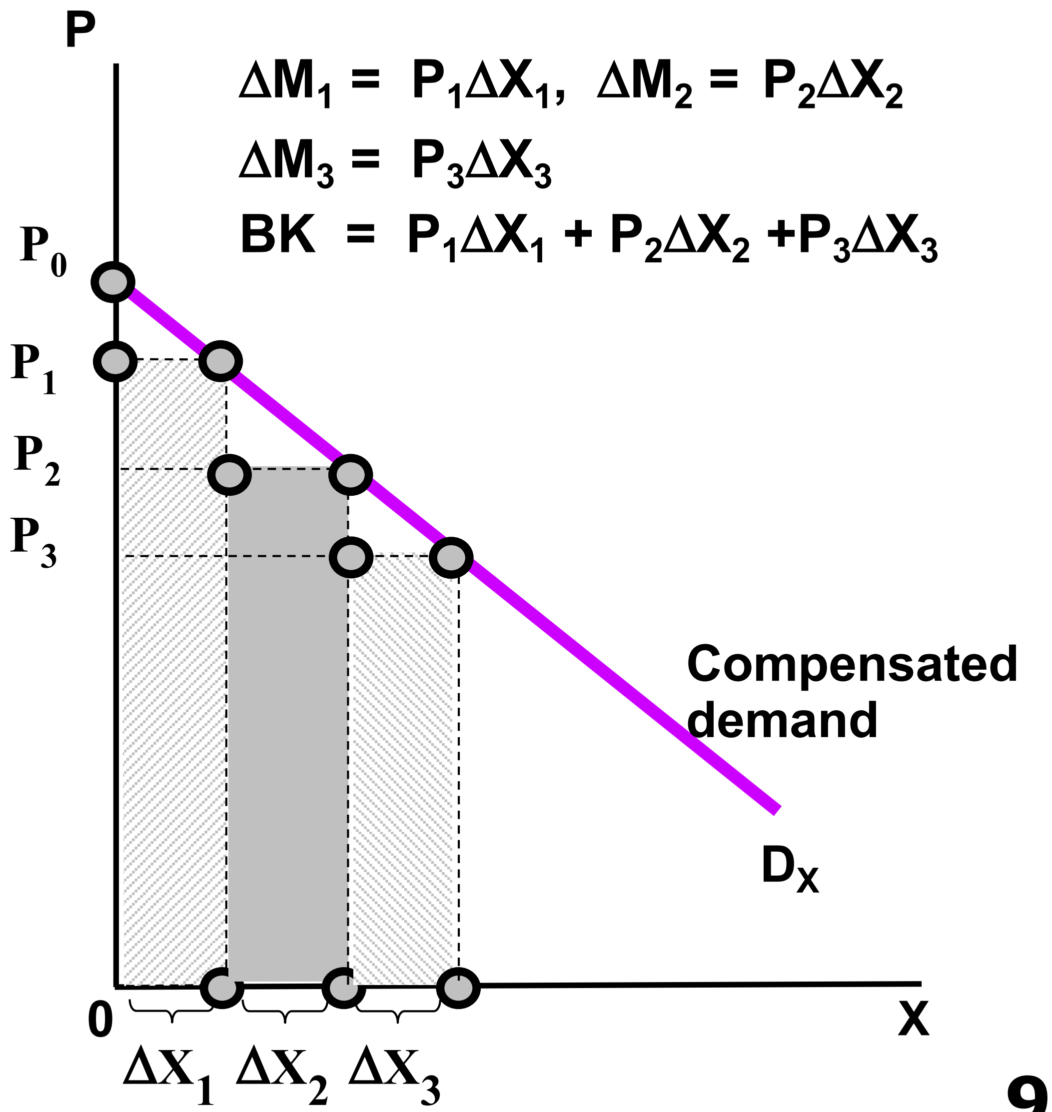
- Consumer maximizes utility, U_2, at point E, buying Y = OH and left with BH for buying X_0.

Finding consumer surplus from indifference curves

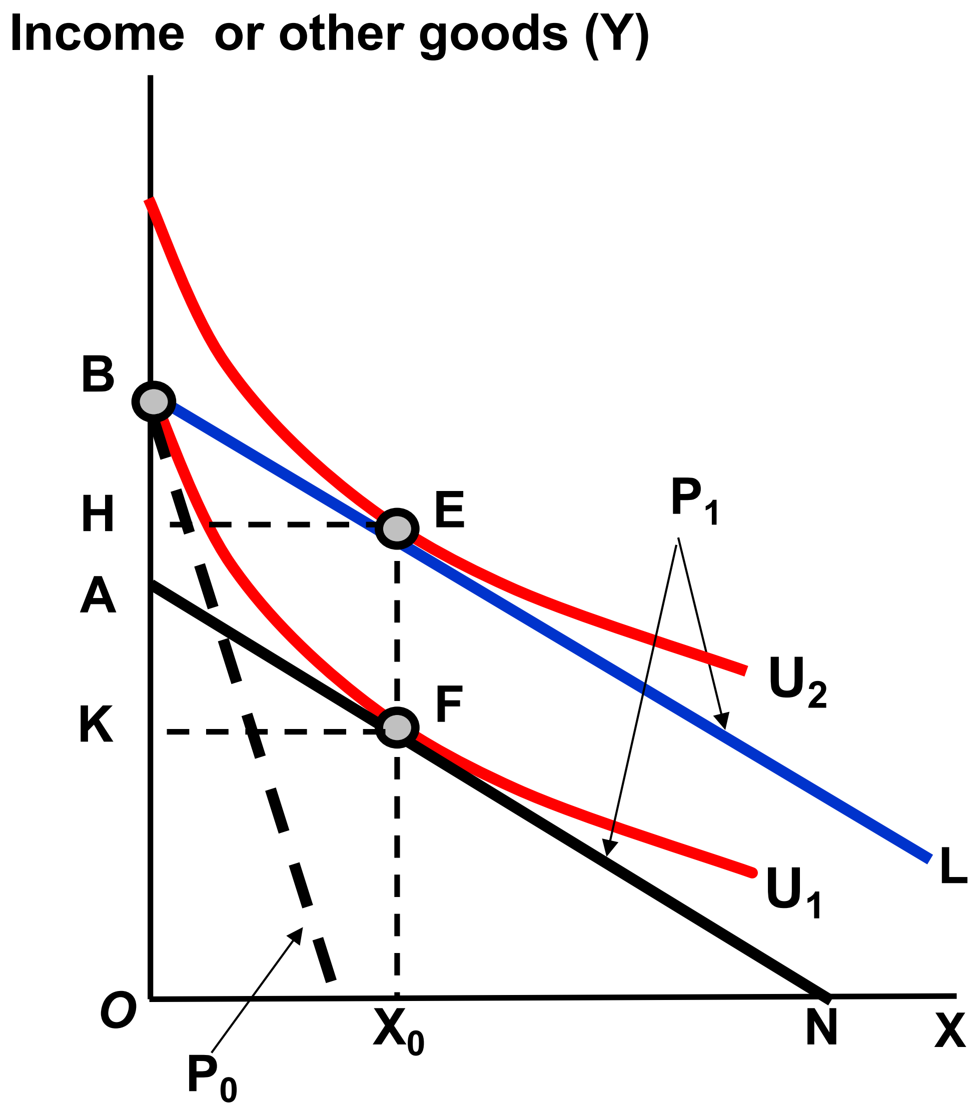
- If consumer cannot buy X at all, he will buy Y = OB. Utility will be reduced to U_1.
- With quasilinear preferences, the budget line AN tangent indifference curve U_1 at F. Note that consumer can consume the same amount of X = X_0 but actually pay only AK (OA-OK).
- Since point B and F are on the same IC, consumer is willing to give up BK units of Y to get X_0 units of X.
- Since consumer surplus is the difference between the value that consumer willing to pay and the the value that he actually paid, it is = BK - AK =BA = EF.

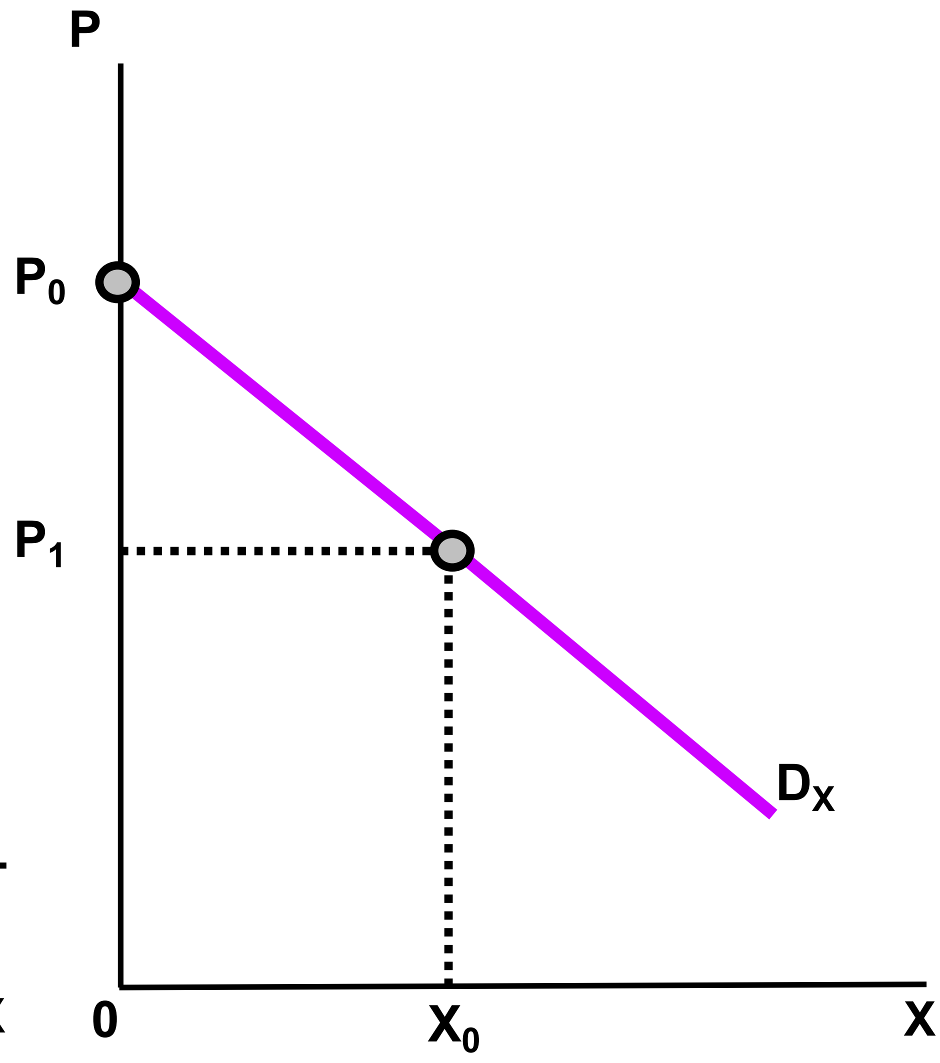
Finding consumer surplus from indifference curves





Finding consumer surplus from indifference curves

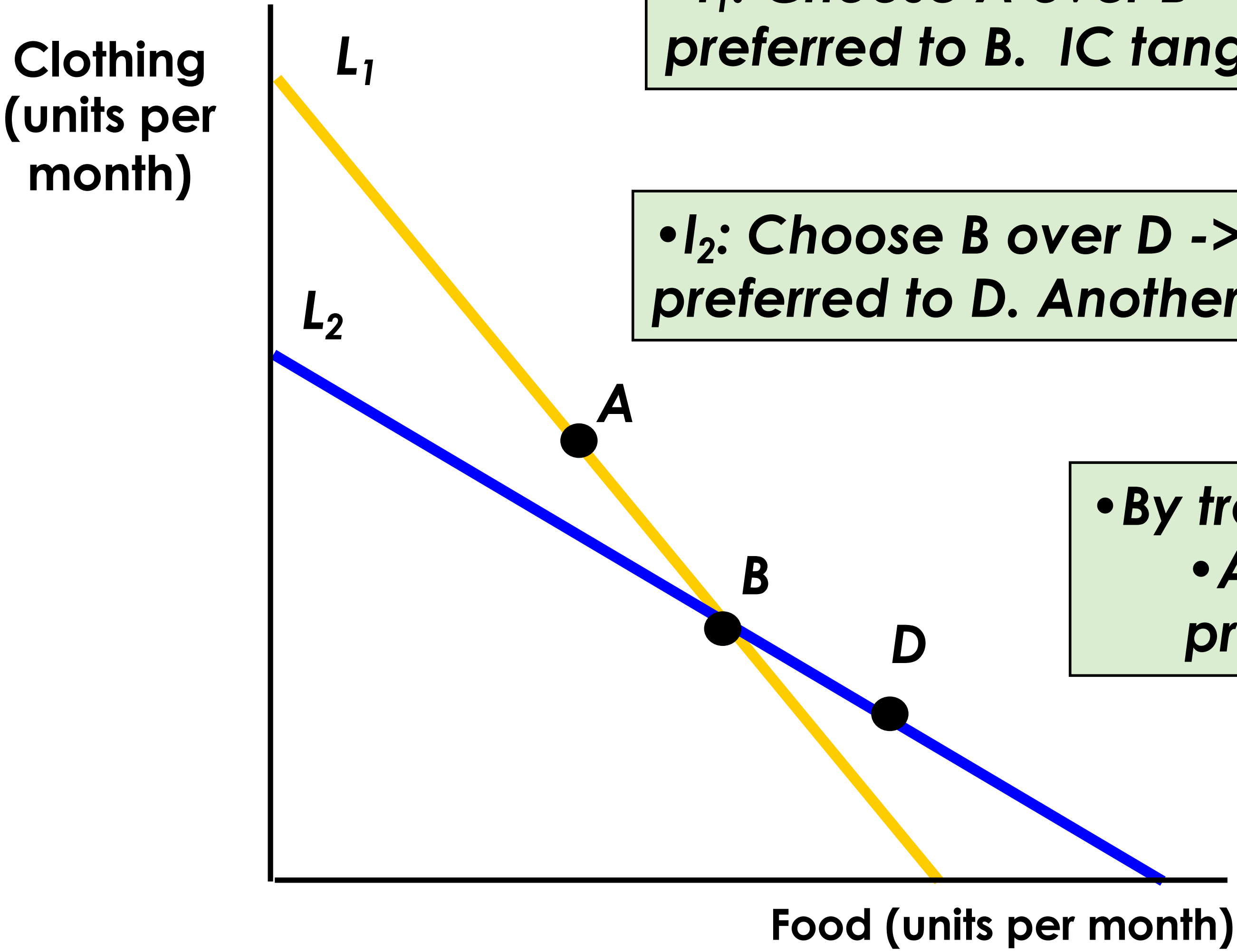




Revealed Preferences

- According to the theory, for given preferences, a consumer will choose bundle of goods subject to the budget constraint.
- In reality, preferences are unknown.
- But if we know the choices a consumer has made, we can determine what their preferences are if we have information about a sufficient number of choices that are made when prices and incomes vary.
- [Read Varian, Ch. 7 - 8]

Revealed Preferences – Two Budget Lines

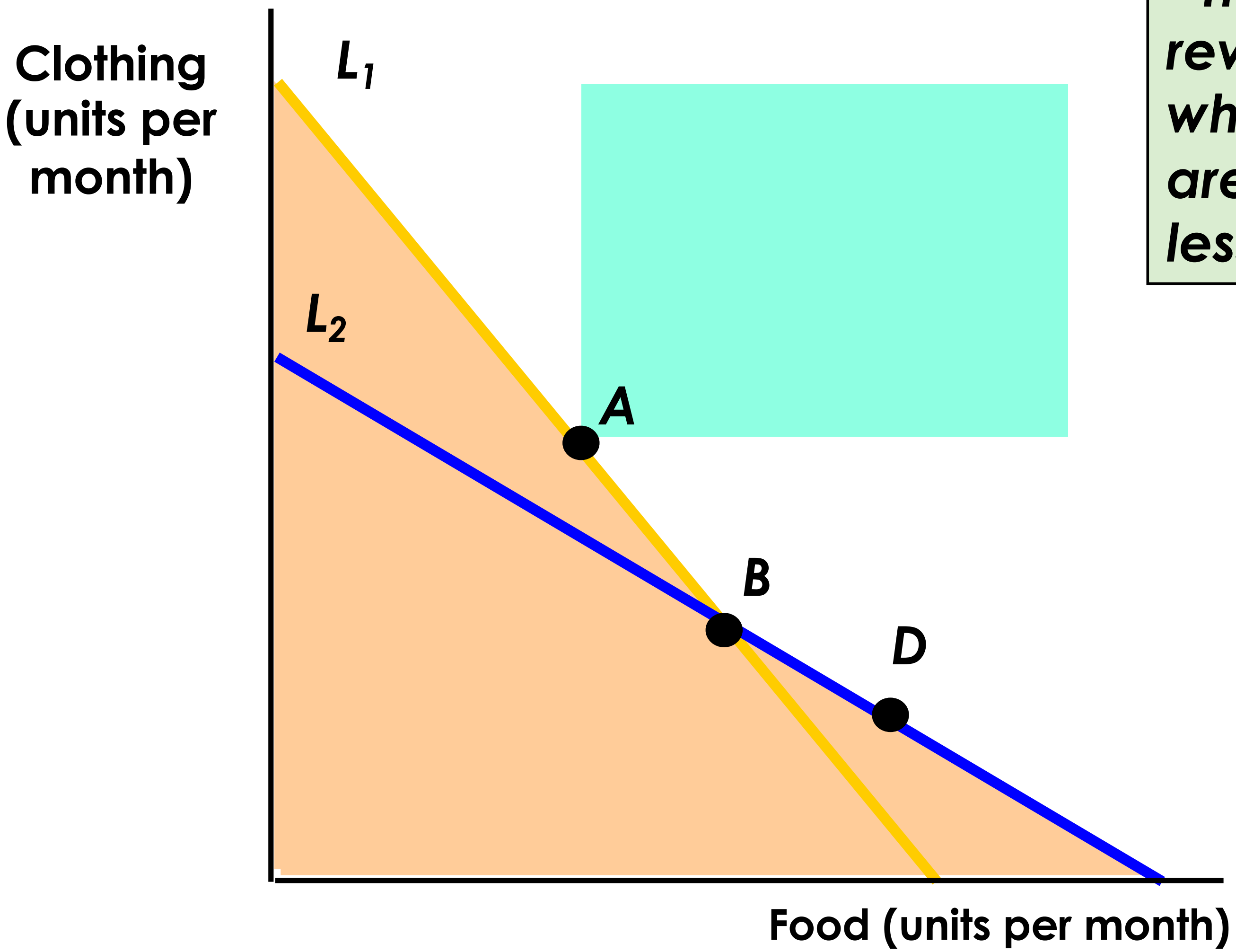


- *I_1: Choose A over B ->A is revealed preferred to B. IC tangents at point A*

- *I_2: Choose B over D ->B is revealed preferred to D. Another IC tangents at B*

- *By transitive:*
  - *A is revealed preferred to D*

Revealed Preferences – Two Budget Lines

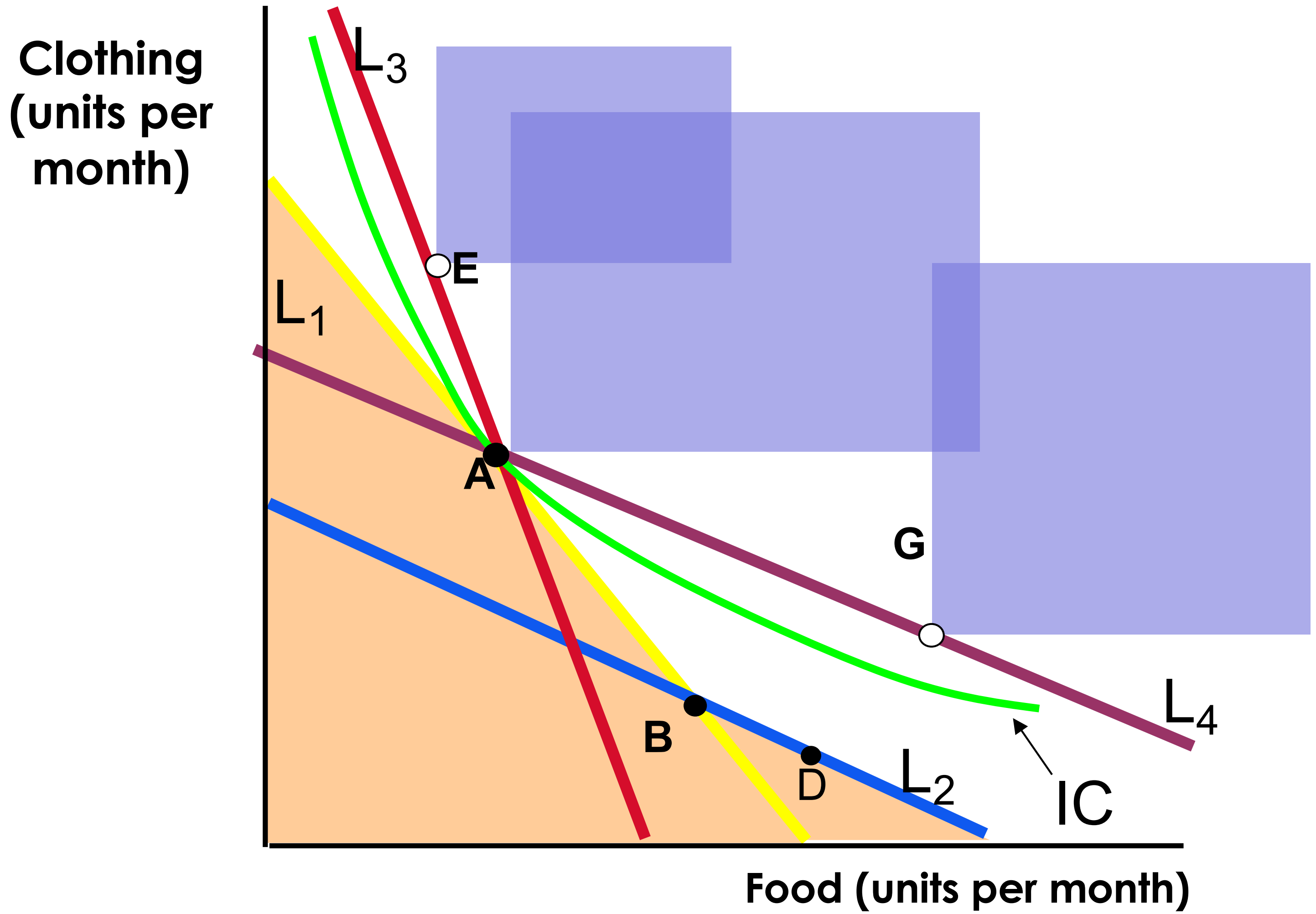


•The pink areas are revealed prefer to A while the yellow areas are revealed less prefer to A.

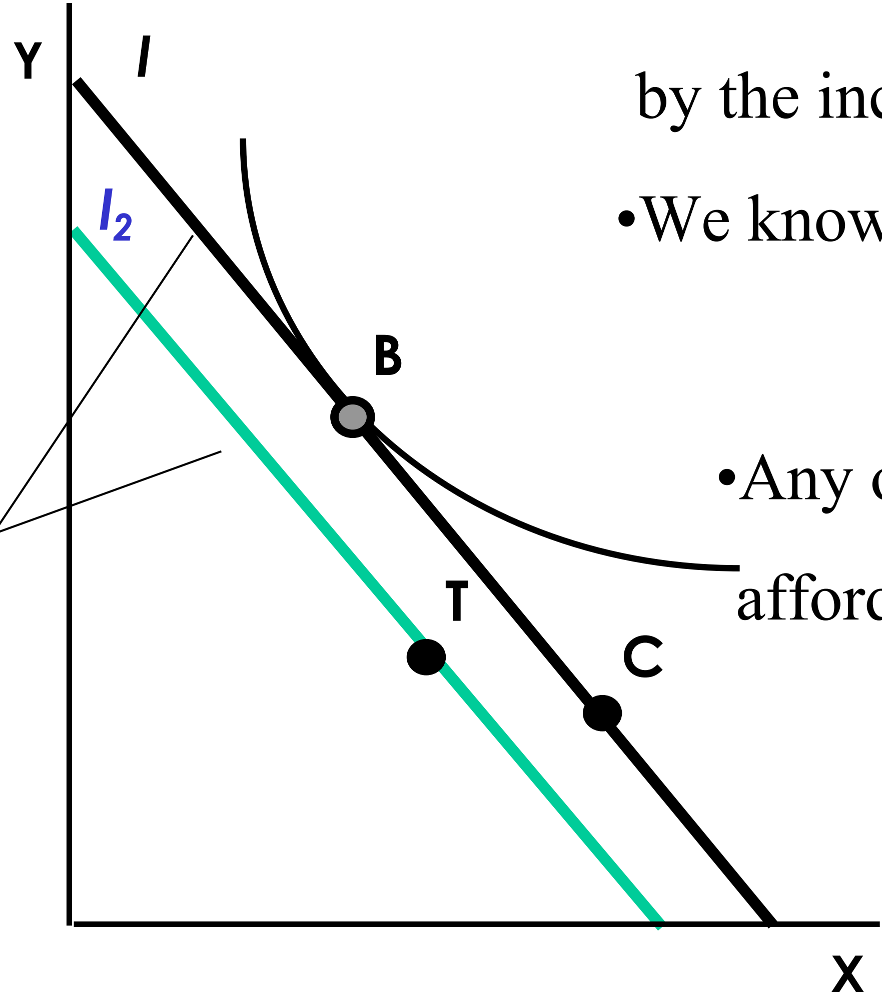
Revealed Preference

- As you continue to change the budget line, individuals can tell you which basket they prefer to others.
- The more the individual reveals, the more you can discern about their preferences
- Eventually you can map out an indifference curve

Revealed preferences and the construction of an IC



Reveal preference condition



$$\frac{P_X^B}{P_Y^B}$$

- If B is revealed prefer to any bundle affordable by the income I.
- We know that

$$P_X^B X^B + P_Y^B Y^B = I$$

- Any other bundle, such as T or C, is affordable by income I, hence

$$P_X^B X^T + P_Y^B Y^T \leq I$$

Reveal preference condition

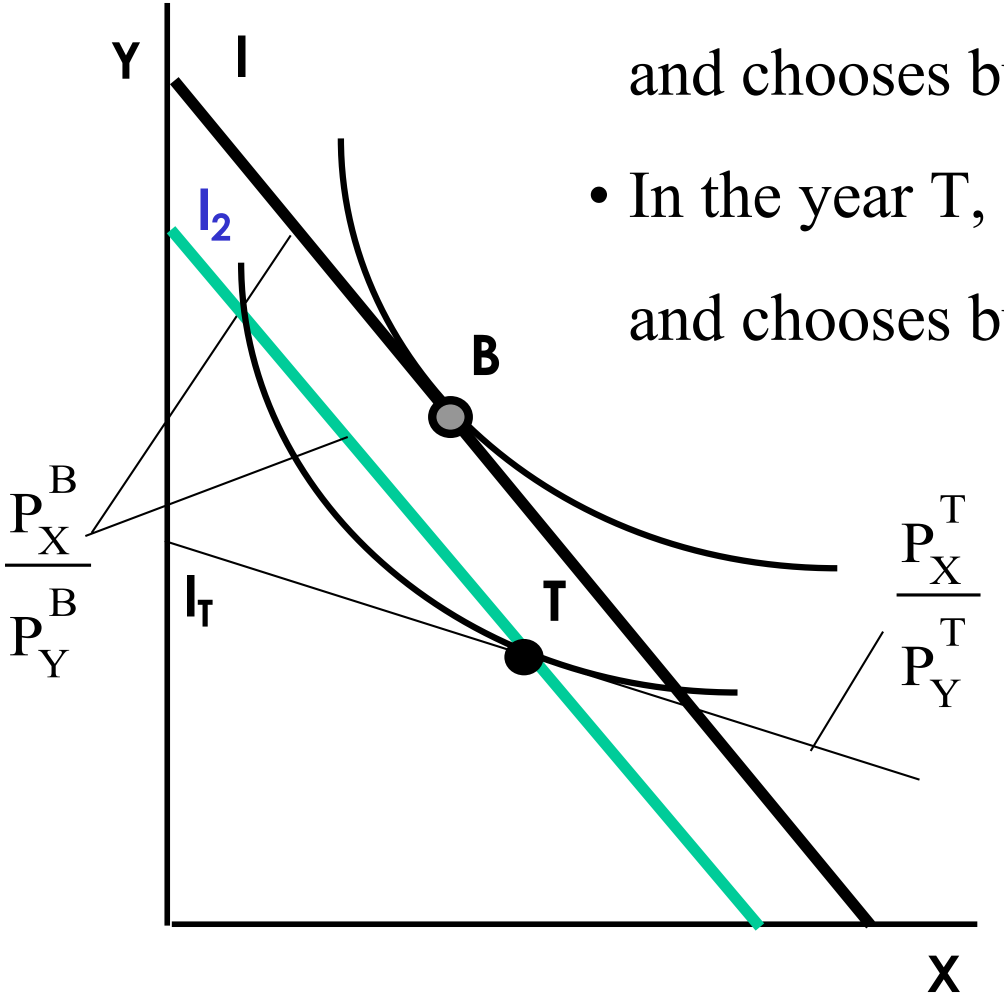
$$P_X^B X^B + P_Y^B Y^B = I$$

$$P_X^B X^T + P_Y^B Y^T \leq I$$

$$\boxed{P_X^B X^B + P_Y^B Y^B \geq P_X^B X^T + P_Y^B Y^T}$$

- For a given income I, if a consumer chooses bundle B, the expenditure for B must be greater than or equal to the expenditure for any other bundle.
- If the expenditure for bundle B is greater than or equal to the expenditure for bundle T and the consumer chooses B, B must be preferred to T.

Reveal preference condition



- In the base year, consumer is facing budget line I and chooses bundle B.
- In the year T, consumer is facing budget line I_T and chooses bundle T.

- If we know that

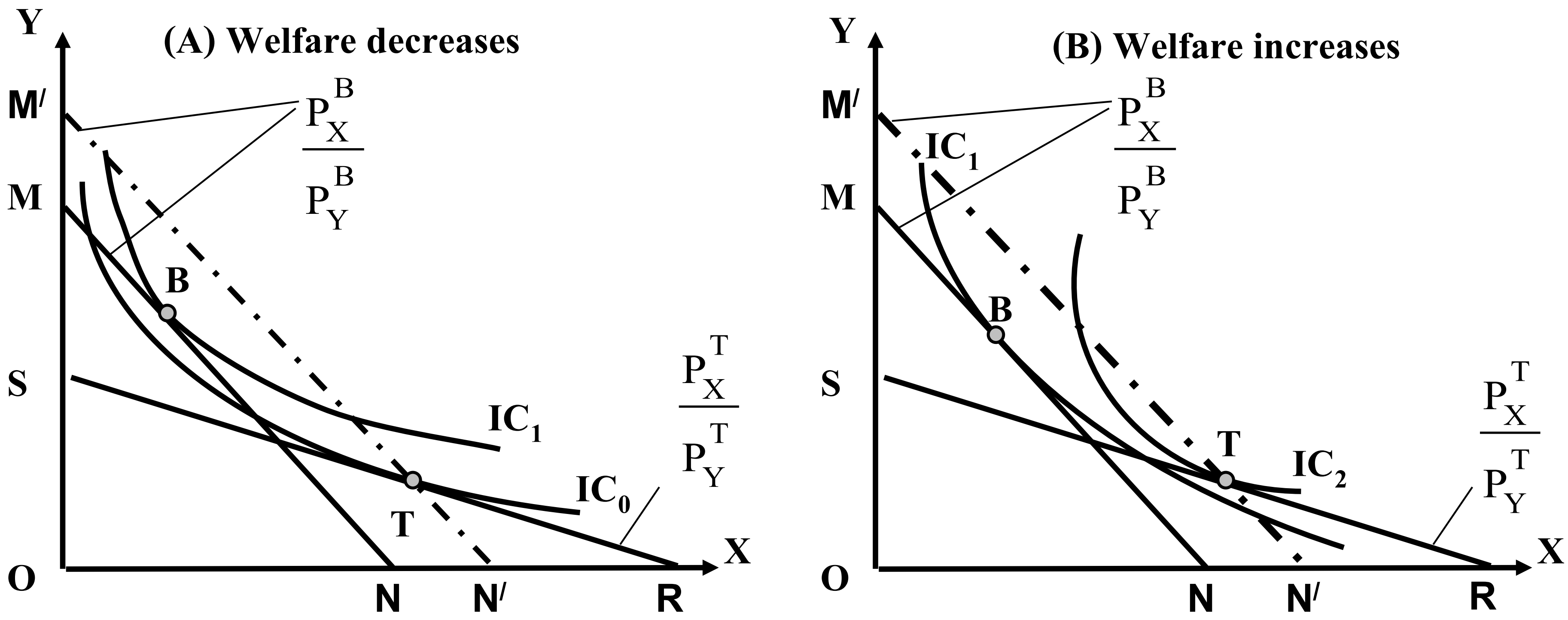
$$P_X^B X^B + P_Y^B Y^B \geq P_X^B X^T + P_Y^B Y^T$$

B must be preferred to T or the consumer is worse off in period T compared to B.

Reveal preference condition

- However, we cannot conclude anything if

$$P_X^B X^B + P_Y^B Y^B < P_X^B X^T + P_Y^B Y^T$$



- This is because bundle T is not affordable in period B

Index numbers

- We may construct quantity indexes of price indexes to estimate welfare change from period B to period T.

Period	Prices	Quantities
Base (B)	P_X^B , P_Y^B	X^B , Y^B
Present time (T)	P_X^T , P_Y^T	X^T , Y^T

Quantity Indexes

$$I_Q = \frac{w_X X^T + w_Y Y^T}{w_X X^B + w_Y Y^B}$$

- where I_Q is a general form of quantity index; w_i is the weight for each good i.
- One possible weight is the price.
- **Laspeyres** uses the base period price in the past, **Paasche** uses the current period price.

Quantity Indexes

- **Laspeyres** uses the base period prices P^B as weights to construct

Laspeyres Quantity Index

$$L_Q = \frac{P_X^B X^T + P_Y^B Y^T}{P_X^B X^B + P_Y^B Y^B} = \frac{\sum P_i^B Q_i^T}{\sum P_i^B Q_i^B}$$

- **Paasche** uses the period T prices P^T as weights to construct

Paasche Quantity Index

$$P_Q = \frac{P_X^T X^T + P_Y^T Y^T}{P_X^T X^B + P_Y^T Y^B} = \frac{\sum P_i^T Q_i^T}{\sum P_i^T Q_i^B}$$

Quantity Indexes

1) **If L_Q is less than or equal to 1**

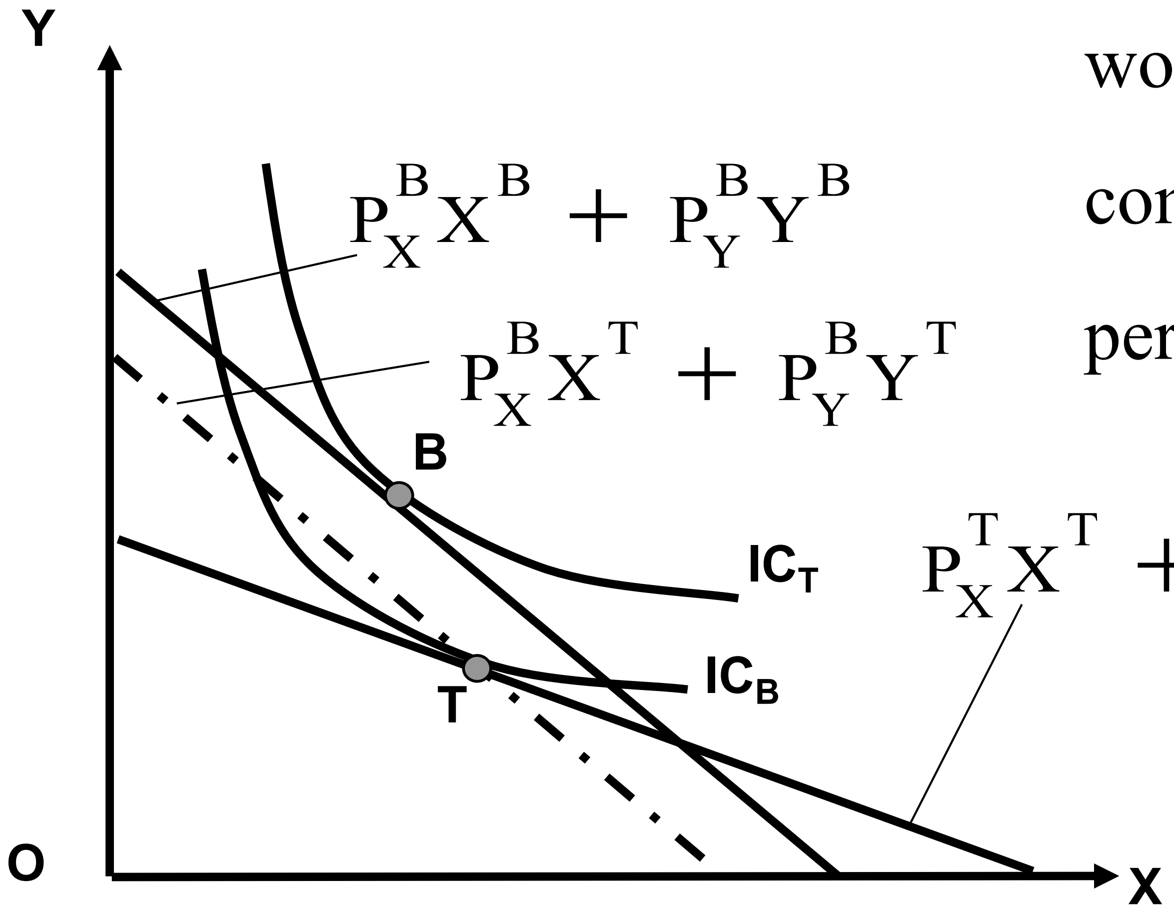
$$L_Q = \frac{P_X^B X^T + P_Y^B Y^T}{P_X^B X^B + P_Y^B Y^B} \leq 1$$

$$P_X^B X^T + P_Y^B Y^T \leq P_X^B X^B + P_Y^B Y^B$$

- This is a reveal preference condition.
- Consumers choose B over T when both are affordable

  -> **worse off in period T compared to the base period B**.

L_Q is less than or equal to 1



worse off in period T compared to the base period B.

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Quantity Indexes

2) **If L_Q is greater than 1**

$$L_Q = \frac{P_X^B X^T + P_Y^B Y^T}{P_X^B X^B + P_Y^B Y^B} > 1$$

$$P_X^B X^T + P_Y^B Y^T > P_X^B X^B + P_Y^B Y^B$$

- This is **not** a revealed preference condition.
- It only tells that bundle T is not affordable in period B.
- **Inconclusive** about the welfare change.

Quantity Indexes

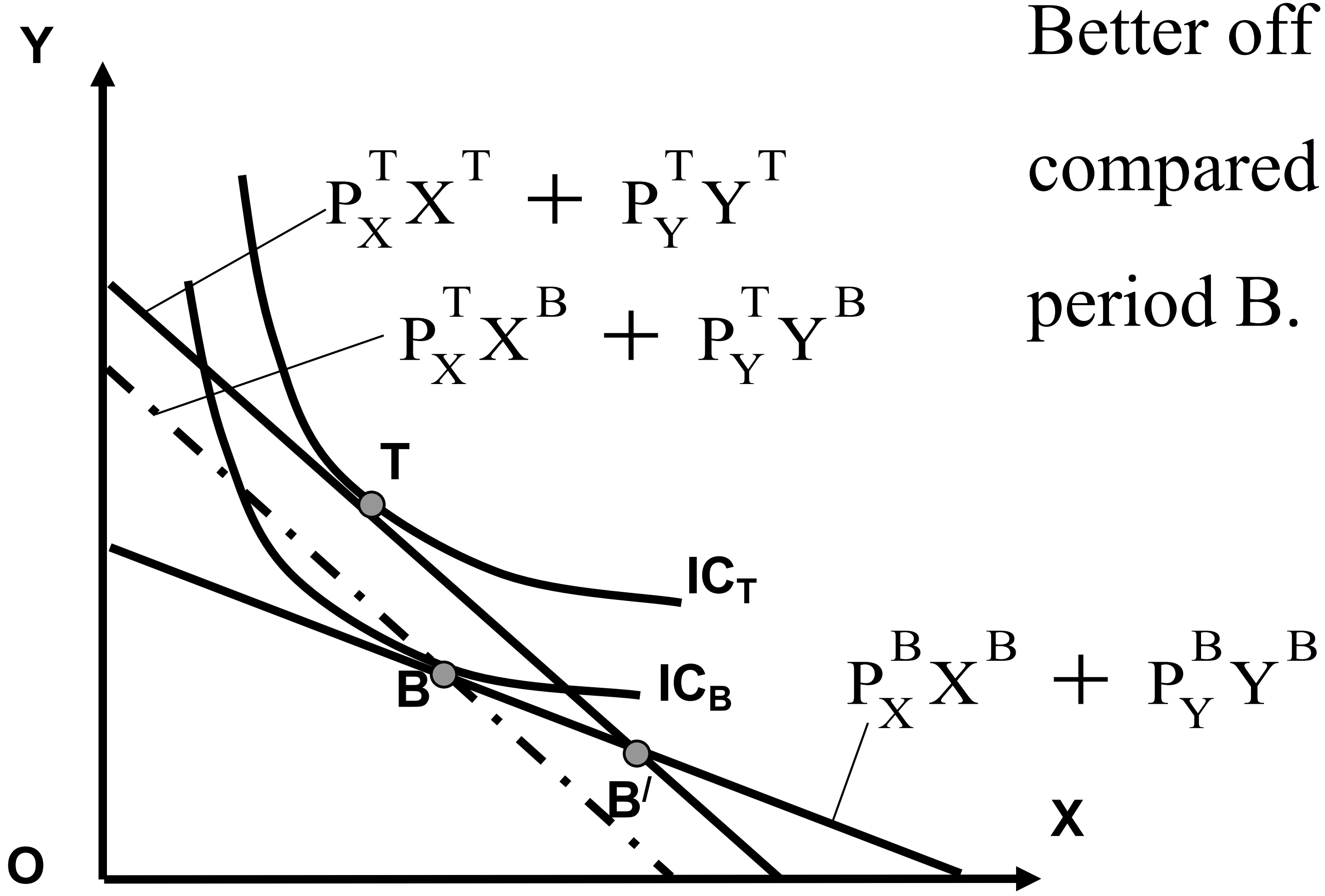
3) **If P_Q is greater than or equal to 1**

$$P_Q = \frac{P_X^T X^T + P_Y^T Y^T}{P_X^T X^B + P_Y^T Y^B} \geq 1$$

$$P_X^T X^T + P_Y^T Y^T \geq P_X^T X^B + P_Y^T Y^B$$

- This is a reveal preference condition.
- Consumers choose T over B when both are affordable -> **Better off in period T compared to the base period B**.

P_Q is greater than or equal to 1



Better off in period T compared to the base period B.

Quantity Indexes

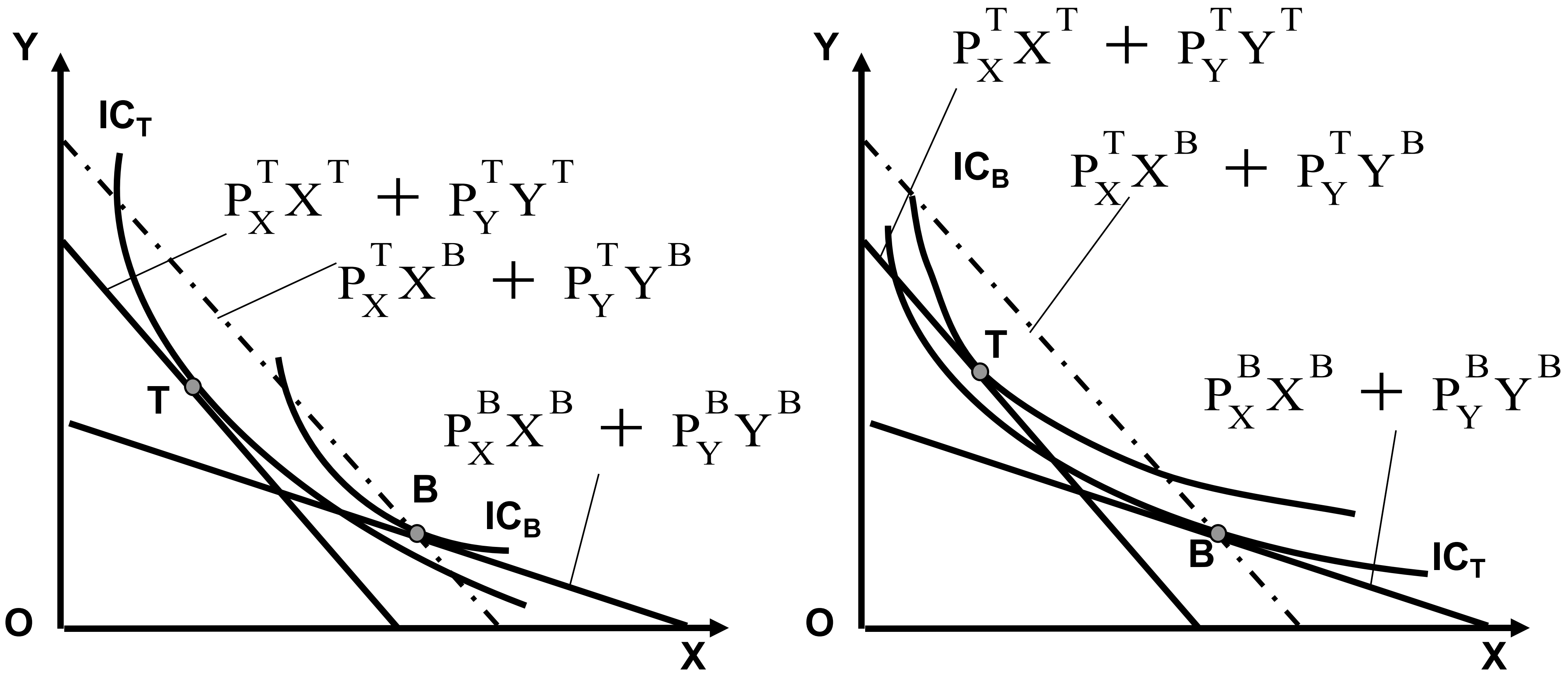
4) If P_Q is less than 1

$$P_Q = \frac{P_X^T X^T + P_Y^T Y^T}{P_X^T X^B + P_Y^T Y^B} < 1$$

$$P_X^T X^T + P_Y^T Y^T < P_X^T X^B + P_Y^T Y^B$$

- This is **not** a revealed preference condition.
- Inconclusive about the welfare change.

Inconclusive when $P_Q < 1$



Price Indexes

$$I_P = \frac{w_X P^T + w_Y P^T}{w_X P^B + w_Y P^B}$$

- Where I_P is a general form of quantity index, w_i is the weight for each good i.
- One possible weight is the quantity.
- **Laspeyres** uses the base period quantity in the past, **Paasche** uses the current period quantity.

Price Indexes

- **Laspeyres** uses the base period quantity Q^B as weight to construct

Laspeyres Price Index

$$L_P = \frac{P_X^T X^B + P_Y^T Y^B}{P_X^B X^B + P_Y^B Y^B} = \frac{\Sigma P_i^T Q_i^B}{\Sigma P_i^B Q_i^B}$$

- **Paasche** uses the period T quantitiy Q^T as weight to construct

Paasche Price Index

$$P_P = \frac{P_X^T X^T + P_Y^T Y^T}{P_X^B X^T + P_Y^B Y^T} = \frac{\Sigma P_i^T Q_i^T}{\Sigma P_i^B Q_i^T}$$

Price Indexes

- We cannot simply compare the price index with 1 as in the case of the quantity index.
- For instance if $P_P > 1$, this means

$$P_X^T X^T + P_Y^T Y^T > P_X^B X^T + P_Y^B Y^T$$

- This is not a reveal preference condition because the price on both sides are not the same.

Price Indexes

- To solve the problem we need to construct another index to help, called expenditure index.

$$M = \frac{P_X^T X^T + P_Y^T Y^T}{P_X^B X^B + P_Y^B Y^B} = \frac{\sum P_i^T Q_i^T}{\sum P_i^B Q_i^B}$$

- It is the ratio between the actual expenditure in period T and the actual expenditure in the base period.

Price Indexes

1) **If L_P is less than or equal to M**

$$L_P = \frac{P_X^T X^B + P_Y^T Y^B}{P_X^B X^B + P_Y^B Y^B} \leq \frac{P_X^T X^T + P_Y^T Y^T}{P_X^B X^B + P_Y^B Y^B} = M$$

$$P_X^T X^B + P_Y^T Y^B \leq P_X^T X^T + P_Y^T Y^T$$

- This is the same as $P_Q >$ or $= 1$.
- Consumers choose T over B when both are affordable -> **Better off in period T compared to the base period B**.

Price Indexes

2) **If L_P is greater than M**

$$L_P = \frac{P_X^T X^B + P_Y^T Y^B}{P_X^B X^B + P_Y^B Y^B} > \frac{P_X^T X^T + P_Y^T Y^T}{P_X^B X^B + P_Y^B Y^B} = M$$

$$P_X^T X^B + P_Y^T Y^B > P_X^T X^T + P_Y^T Y^T$$

- This is the same as $P_Q < 1$.
- **Inconclusive** about the welfare change.

Price Indexes

3) **If P_P is greater than or equal to M**

$$P_P = \frac{P_X^T X^T + P_Y^T Y^T}{P_X^B X^T + P_Y^B Y^T} \geq \frac{P_X^T X^T + P_Y^T Y^T}{P_X^B X^B + P_Y^B Y^B} = M$$

$$P_X^B X^B + P_Y^B Y^B \geq P_X^B X^T + P_Y^B Y^T$$

- This is the same as L_Q < OR = 1
- Consumers choose B over T when both are affordable

-> **worse off in period T compared to the base period B**.

Price Indexes

4) If P_P is less than M

$$P_P = \frac{P_X^T X^T + P_Y^T Y^T}{P_X^B X^T + P_Y^B Y^T} < \frac{P_X^T X^T + P_Y^T Y^T}{P_X^B X^B + P_Y^B Y^B} = M$$

$$P_X^B X^B + P_Y^B Y^B < P_X^B X^T + P_Y^B Y^T$$

- This is the same as $L_Q > 1$
- Inconclusive about the welfare change.

Cost-of-Living Indexes

- Social security payments can be adjusted to compensate for inflation.
- The inflation rate is calculated from the Consumer Price Index (CPI) based on Laspeyres price index.
- Should we increase the payments equal to the rate of increase of the CPI?

Cost-of-Living Indexes

- Laspeyres price index
  - Amount of money at current year prices that an individual requires to purchase a bundle of goods/services chosen in a base year divided by the cost of purchasing the same bundle at base-year prices

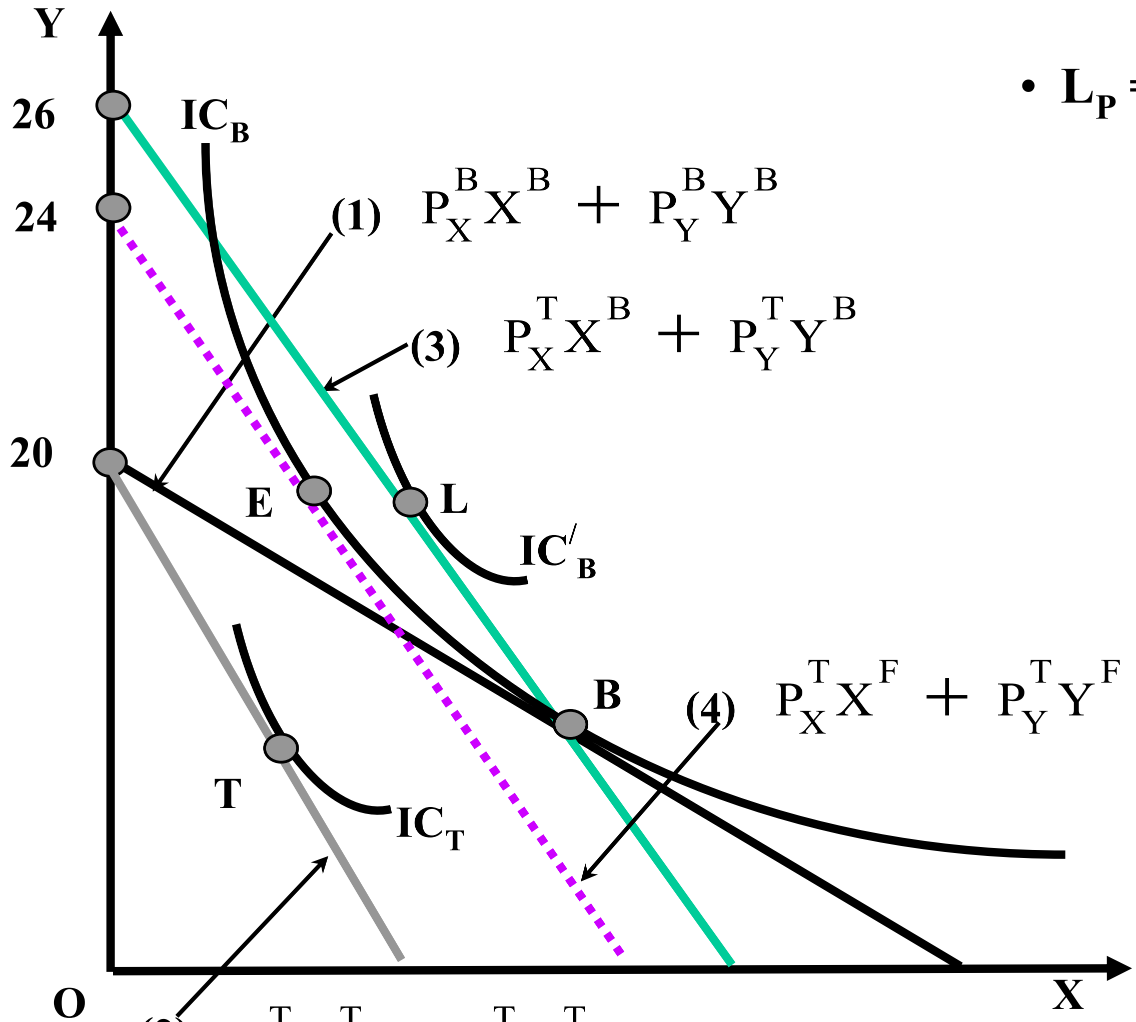
$$L_P = \frac{P_X^T X^B + P_Y^T Y^B}{P_X^B X^B + P_Y^B Y^B} = \frac{\sum P_i^T Q_i^B}{\sum P_i^B Q_i^B}$$

Cost-of-Living Indexes

- The Laspeyres price index assumes that consumers do not alter their consumption patterns as prices change
- Tend to overstate the true cost of living index
- Using the CPI to adjust retirement benefits will tend to overcompensate most recipients requiring greater government expenditure

Laspeyres price index overstates the cost of living

Assuming that $P_Y = 1$, Y axis also measure income.

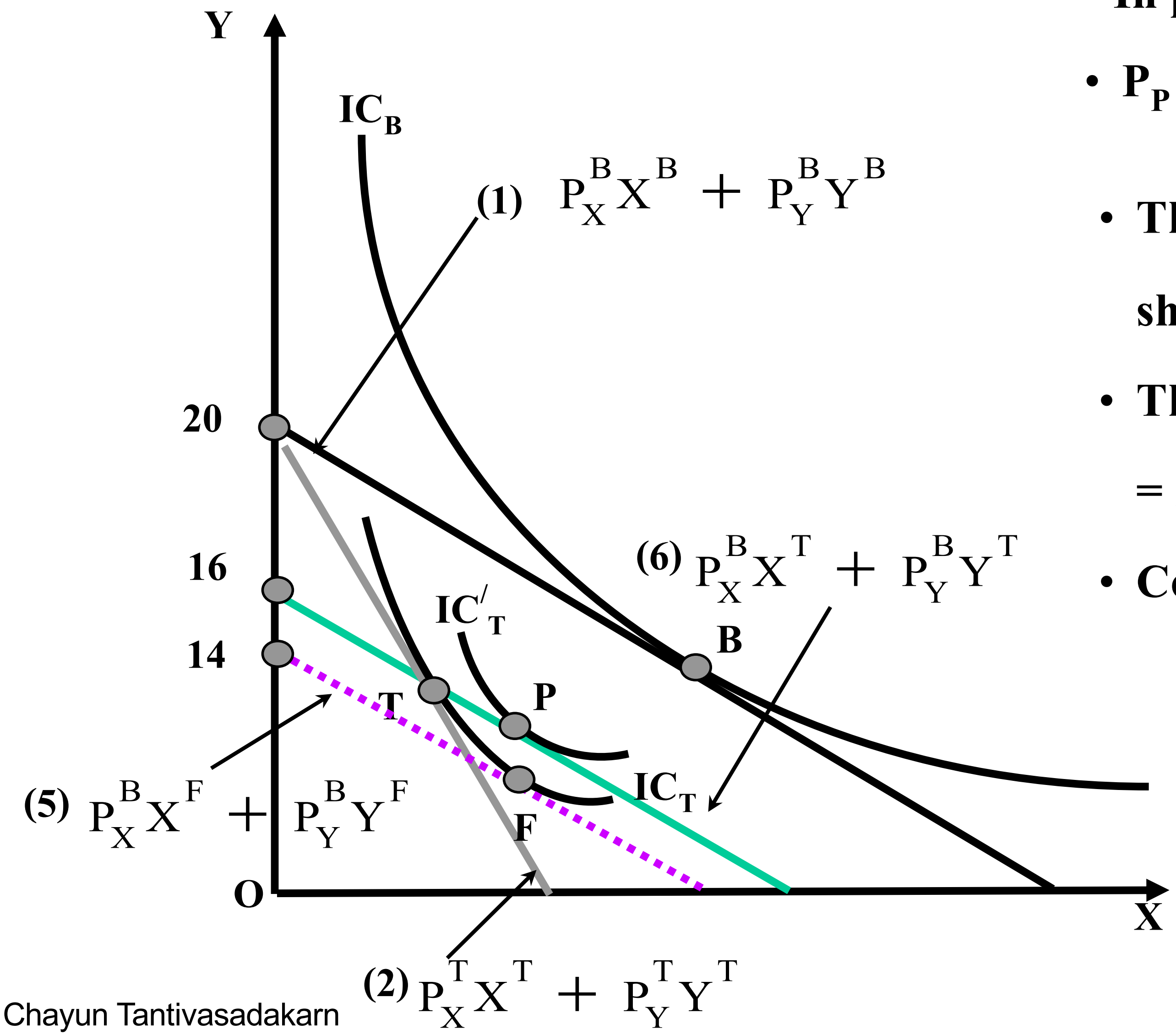


- The expenditure in period B is (1) = 20.
- In period T price of X increases.
- $L_P = \frac{\text{old bundle at new prices}}{\text{old bundle at old prices}}$
  - The income needed to compensate is only (4) - (2) = 24 - 20 = 4.
  - The income implies by L_P is (3) - (2) = 26 - 20 = 6.
  - Consume at L instead of E.
    - L_P = (3)/(1) = 26/20 = 1.3
    - True cost-of-living index = (4)/(1) = 24/20 = 1.2

    --> over estimate

Paasche price index understates the cost of living

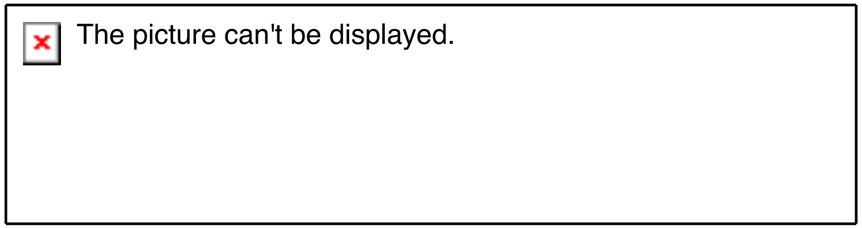
Assuming that $P_Y = 1$, Y axis also measure income.



- The expenditure in period B is (1) = 20.
- In period T price of X increases.
- $P_P = \dfrac{\text{new bundle at new prices}}{\text{new bundle at old prices}}$
  - The income needed to compensate should be (1) - (5) = 20 - 14 = 6.
  - The income implies by P_P is (1) - (6) = 20 - 16 = 4.
  - Consume at P instead of F.
    - P_P = (2)/(6) = 20/16 = 1.25
    - True cost-of-living index = (2)/(5) = 20/14 = 1.43 --> under estimate

Fisher Ideal Price Index

- To solve the over and under estimate problems of both indices, Irving Fisher has proposed the Fisher Ideal Price Index



Duality

- There exists a dual relationship between two optimization problems.

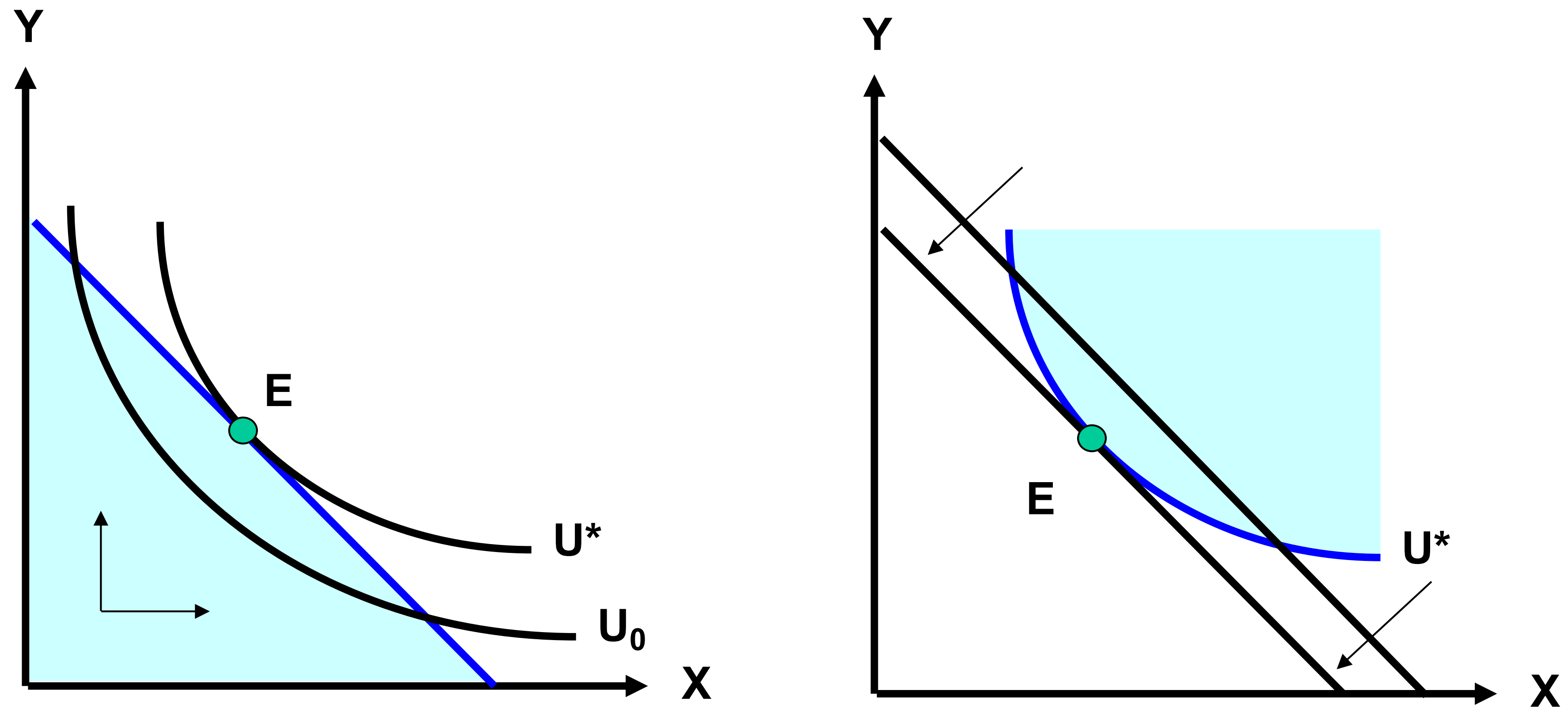
  Max $U(X,Y)$ subject to $I = P_X X + P_Y Y$ (1)

  Min $P_X X + P_Y Y$ subject to $U(X,Y) = U^*$ (2)

where U^* is the utility level attained from (1)

- Solving either problem will give the same optimum value of X^* and Y^*.

Duality



Maximize utility subject to budget constraint ⟺ **Minimize expenditure subject to utility constraint**

Duality: example

- Max $U = XY$ subject to $I = 20$, $P_X = P_Y = 1$
- Since $I = P_X X + P_Y Y$,

  $Y = (I - P_X X)/P_Y = 20 - X$
- Substitute this into U to get $U = X(20-X) = 20X - X^2$
- $dU/dX = 20 - 2X = 0$

  --> $X^* = 10$, $Y^* = (20 - 10) = 10$

  $U^* = (10)(10) = 100$

Duality: example

- Min $E = P_X X + P_Y Y$ subject to $U^* = 100 = XY$, $P_X = P_Y = 1$
- Since $Y = 100/X$
- Substitute this into E to get $E = P_X X + 100P_Y/X = X + 100/X$
- $dE/dX = 1 - 100/X^2 = 0$

--> $X^* = 10$, $Y^* = 100/10 = 10$

$E^* = 10 + 10 = 20 = I$ in the first problem

Duality

- The optimal expenditure function from the second problem is useful because it is observable.
- We can collect expenditure data to estimate expenditure function and get the Hicksian demand function.
- It is possible to prove that the derivative of the expenditure function is the Hicksian demand function which gives an accurate measurement for consumer surplus.