Fixed Income (FN351)

<u>**Structured securities**</u>

Bond prices and returns (Part 1)

Dr. Anchada (Aida) Charoenrook

Road map/Key ideas

- Spot rates
- The term structure of interest rates
- Forward rates

Spot rate: Compounding interest

- Bank I offers 10% interest rate compounded annually
- Bank II offers 10%/365 interest rate compounded daily
- Investment of $1000 today in 1 year will grow to:

Bank I:	After 1 year	$1000*(1+0.1) = $1100
Bank II:	After 1 day	$1000*(1+0.1/365) = $1000.274
	After 2 days	$1000*(1+0.1/365)^2 = $1000.548
	After 1 year	$1000*(1+0.1/365)^365 = $1105.2

- As an investor, which do you prefer?

Spot rate: Compounding interest

- Bank I must offer 10.52% compounded annually to attract investors.
- Investment of $1000 today in 1 year will grow to 1105.2
- In the market, interest rates must be comparable:

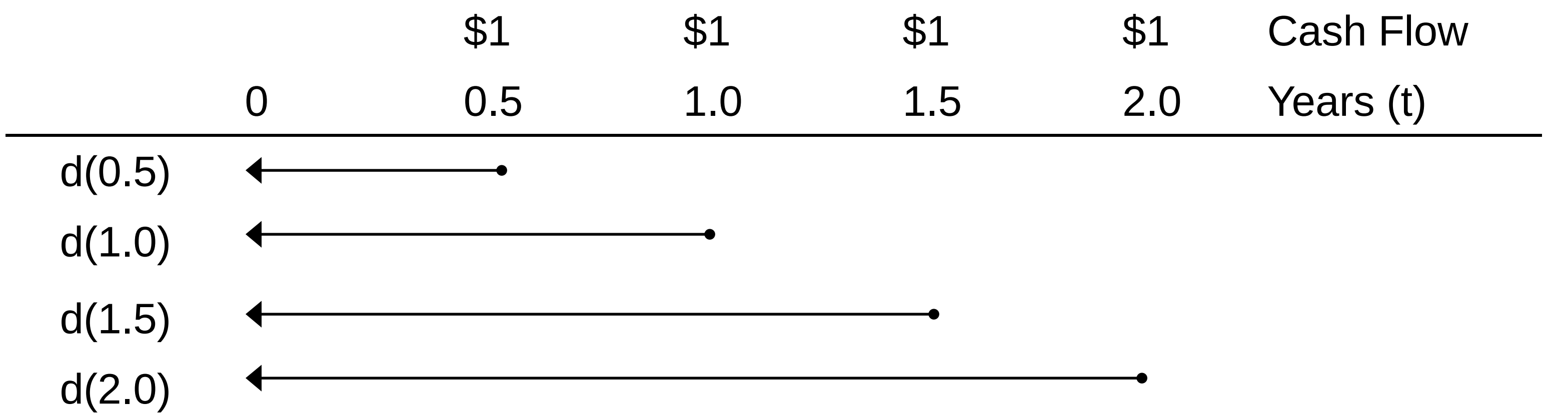
$$(1+r_{yearly})^{T} = (1+\frac{r_{daily}}{365})^{T*365} = (1+\frac{r_{semi-annual}}{2})^{T*2}$$

- It is customary to quote rates multiplied by its compounding period.
- Example: Bond equivalent yield or semi-annual rate=

$$r_{semi\text{-}annual} = \text{half-year rate} * 2$$

Spot Rate

- The discount factor, d(t)



- The ‘market’ discount factors from zeros

Mat.	Price	t	d(t) *
0.5	97.087	0.5	0.97087
1	93.897	1	0.93897
1.5	90.909	1.5	0.90909
2	87.336	2	0.87336

* d(t) = Price/Face Value

Spot rate

- The 'market' discount factors can also be extracted from coupon bonds (face value 100)

B_t	Bond	Price	Discount Factor
$B_{0.5}$	0.5-year zero coupon bond	97.087	100*d(0.5)
$B_{1.0}$	1-year 10% coupon bond	103.4462	5* d(0.5) +105*d(1.0)
$B_{1.5}$	1.5-year 6% coupon bond	99.36579	3* d(0.5) +3*d(1.0)+103*d(1.5)

- Solving the three equations for the three discount prices, we get

B_t	t	d(t)
$B_{0.5}$	0.5	0.97087
$B_{1.0}$	1.0	0.93897
$B_{1.5}$	1.5	0.90909

Spot rate

The discount factor d(2) = 0.87336 is equivalent to the following compounding rates

t=0	0.5	1.0	1.5	2.0
0.87336				1.0

Annual compounding at 7%

0.87336		0.935		1.0

Semi-annual compounding at 6.88%

0.87336	0.904	0.935	0.967	1.0

Rates of higher compounding intervals are lower because the interest payments get reinvested more often

Spot rate

- **The spot rate, y_t,is the <u>yield</u> that is equivalent to the discount rate at time t**.
- The relation between the semi-annual spot rate and the discount rate

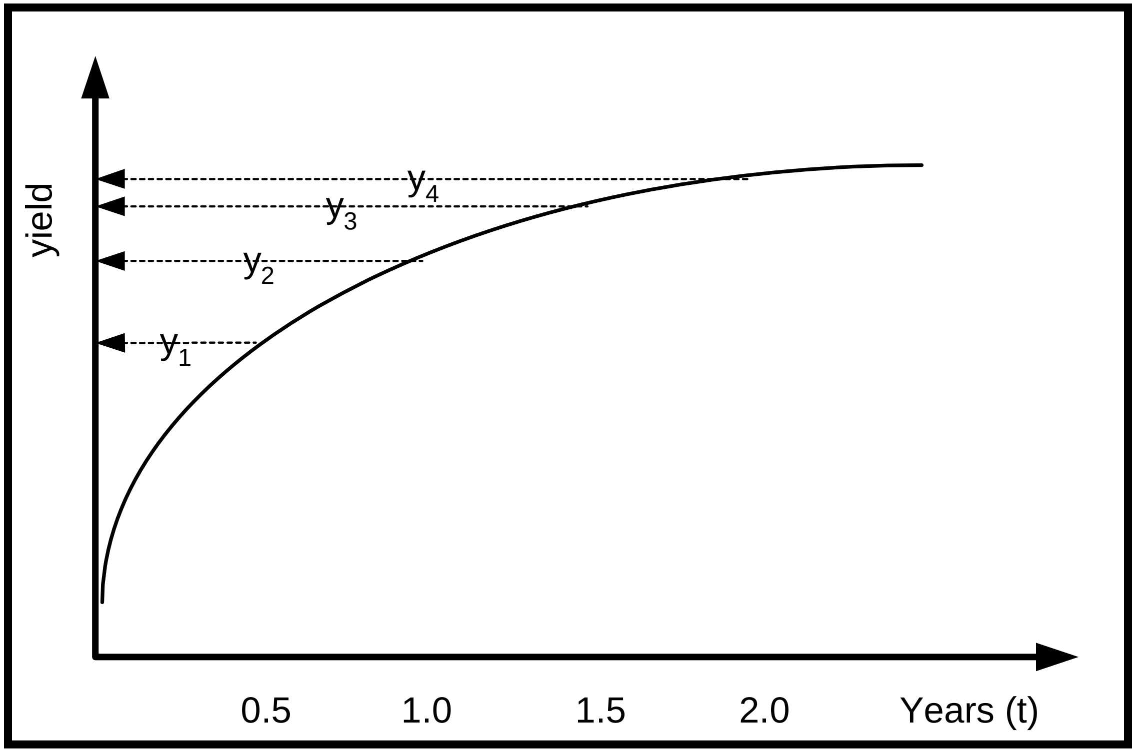
$$y_t = 2 * \left((1/d(t))^{1/year*2} - 1 \right)$$

- Getting spot rates directly from zero-coupon bonds

$$y_t = 2 * \left(\left(\frac{F}{B_t} \right)^{1/year*2} - 1 \right)$$

where B_t=price of a zero coupon bond that matures at time t and F=face value

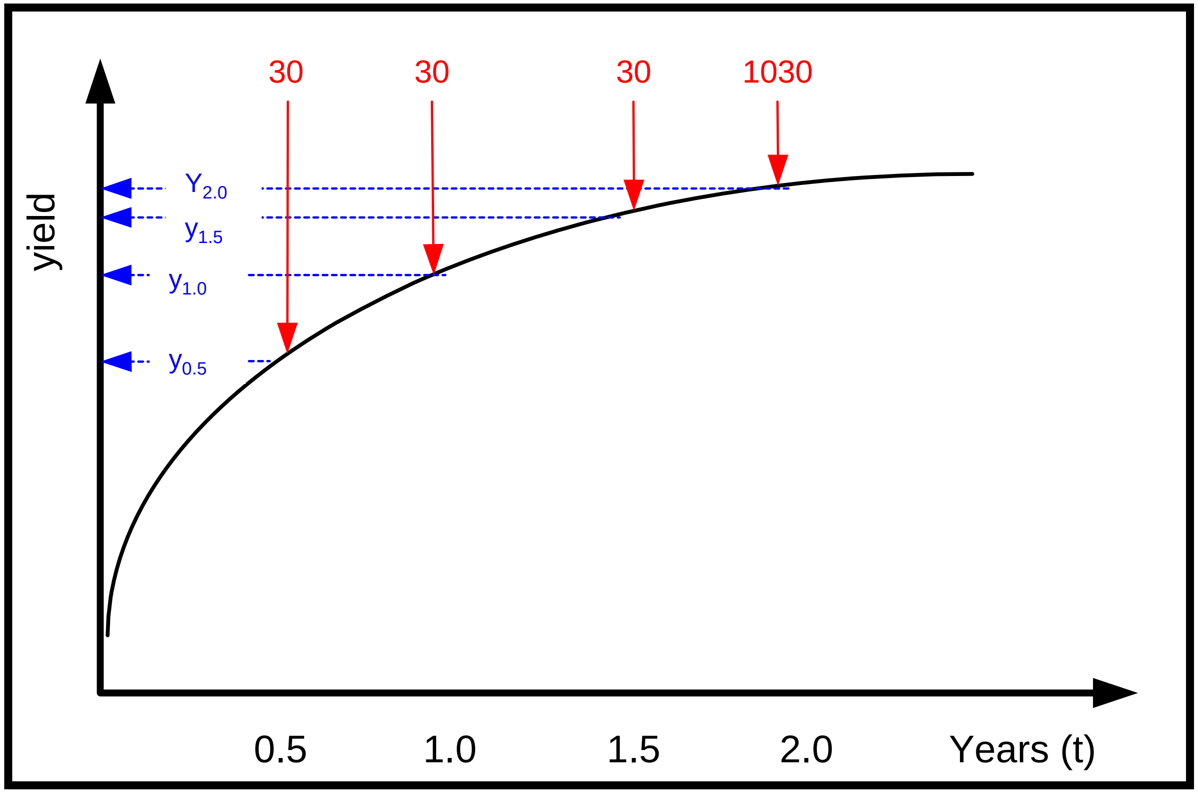
Term structure of interest rates (TIR)



t	d(t)	$y_t = 2 * \left(\left(1/d(t) \right)^{1/t*2} - 1 \right)$
0.5	0.97087	6.0%
1.0	0.93897	6.4%
1.5	0.90909	6.46%
2.0	0.87336	6.88%

TIR: Pricing bonds

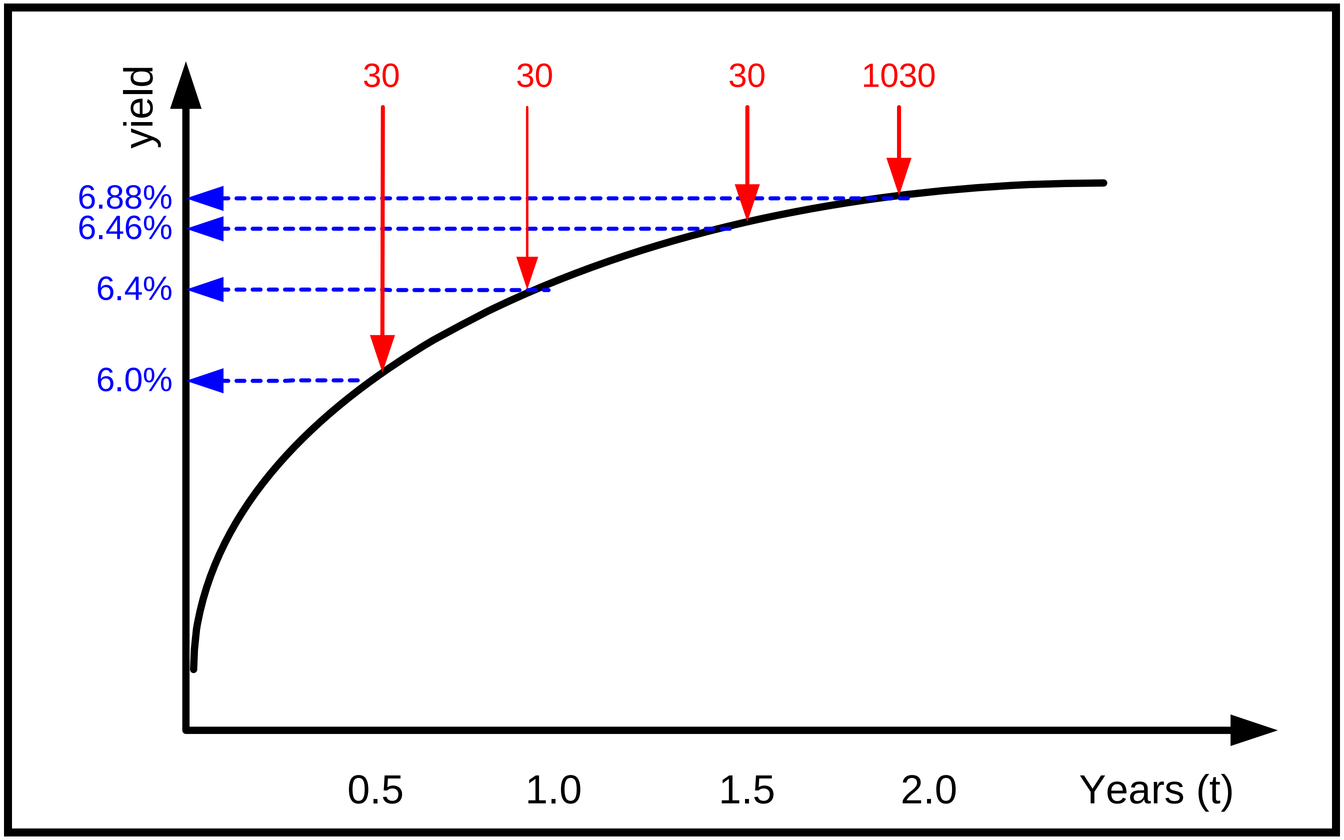
Bond price of a coupon bond = NPV of cash flows



$$\text{Price of a coupon bond} = \sum_t \frac{C_t}{(1+\frac{y_t}{2})^{t*2}}, \ t = \text{year}$$

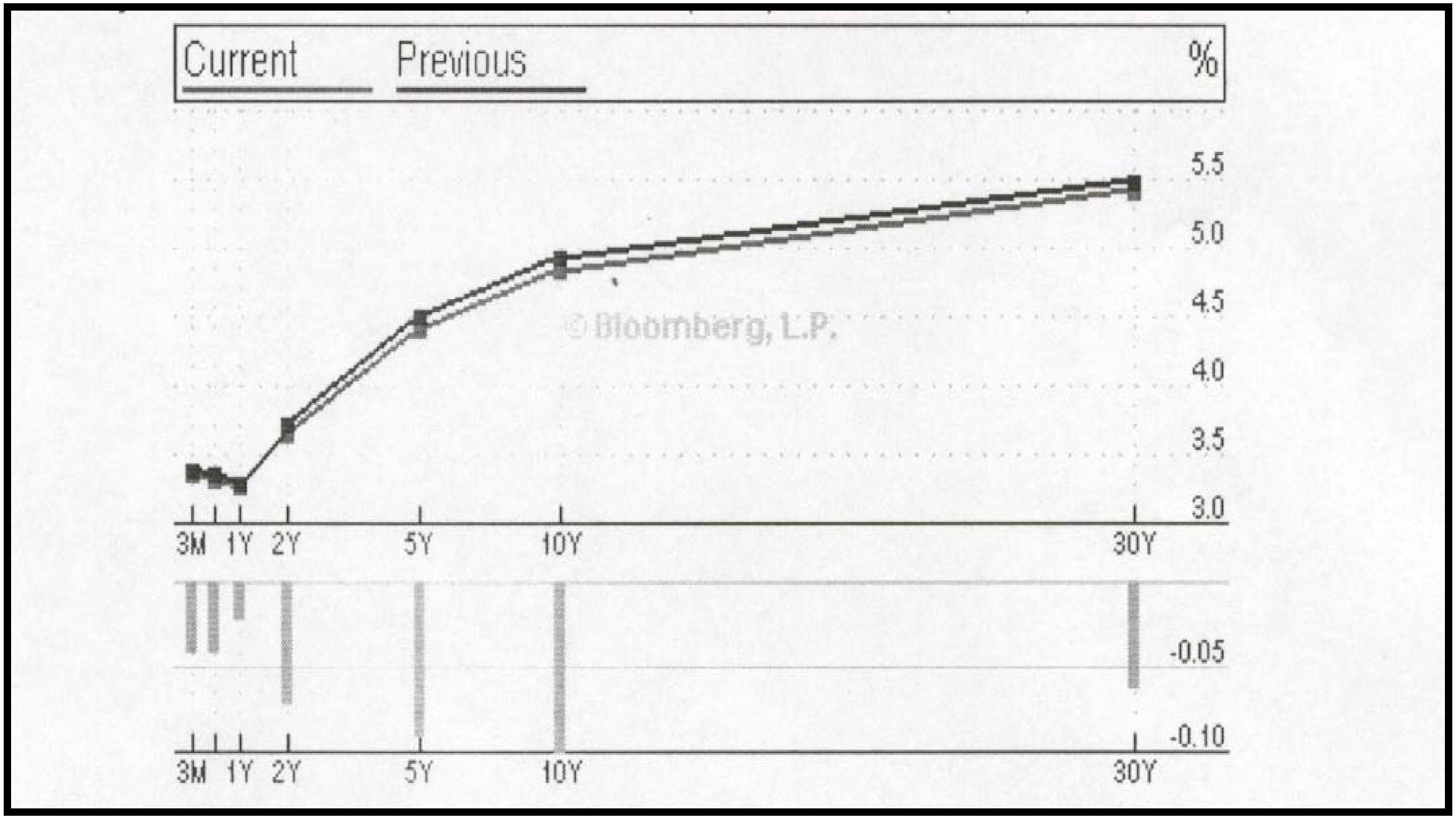
TIR: Pricing bonds

A 2 year 6% coupon bond



$$Price = \frac{30}{(1.03)} + \frac{30}{(1.032)^2} + \frac{30}{(1.0323)^3} + \frac{1030}{(1.0344)^4} = 984.24$$

TIR



Current
Previous
%
5.5
5.0
4.5
4.0
3.5
3.0
© Bloomberg, L.P.
3M
1Y
2Y
5Y
10Y
30Y
-0.05
-0.10
3M
1Y
2Y
5Y
10Y
30Y

TIR: U.K. government bonds

	Coupon	Date	Price	Yield
3 Months	0.00000	11/29/02	3.900	3.900
1 Years	6.50000	12/07/03	103.190	3.901
2 Years	5.00000	06/07/04	101.790	3.941
3 Years	8.50000	12/07/05	112.410	4.385
4 Years	7.50000	12/07/06	111.640	4.474
5 Years	7.25000	12/07/07	112.570	4.541
6 Years	5.00000	03/07/08	102.200	4.545
7 Years	5.75000	12/07/09	107.040	4.599
8 Years	6.25000	11/25/10	110.980	4.630
9 Years	9.00000	07/12/11	131.280	4.654
10 Years	5.00000	03/07/12	102.860	4.625
15 Years	8.00000	12/07/15	132.880	4.651
20 Years	8.00000	06/07/21	142.390	4.602
30 Years	4.25000	06/07/32	96.750	4.448

Concept check

Your company is bidding for a contract from the US government. This project will provide cash flows of 1M a year from today and 2M 2 years from today. You want to calculate the present value of this stream of cash flows to determine what price to bid. What is the PV? (certain cash flow and investment cost)

TIR

t	0.5	1.0	1.5	2.0
yield	4.0%	4.3%	4.6%	4.8%

G) 1M/(1.043) + 2M/(1.048) = 2.87M

Y) 1M/(1.043) + 2M/(1.048)^2 = 2.779M

R) 1M/(1.0215)^2+2M/(1.024)^4 = 2.777M

The forward rate

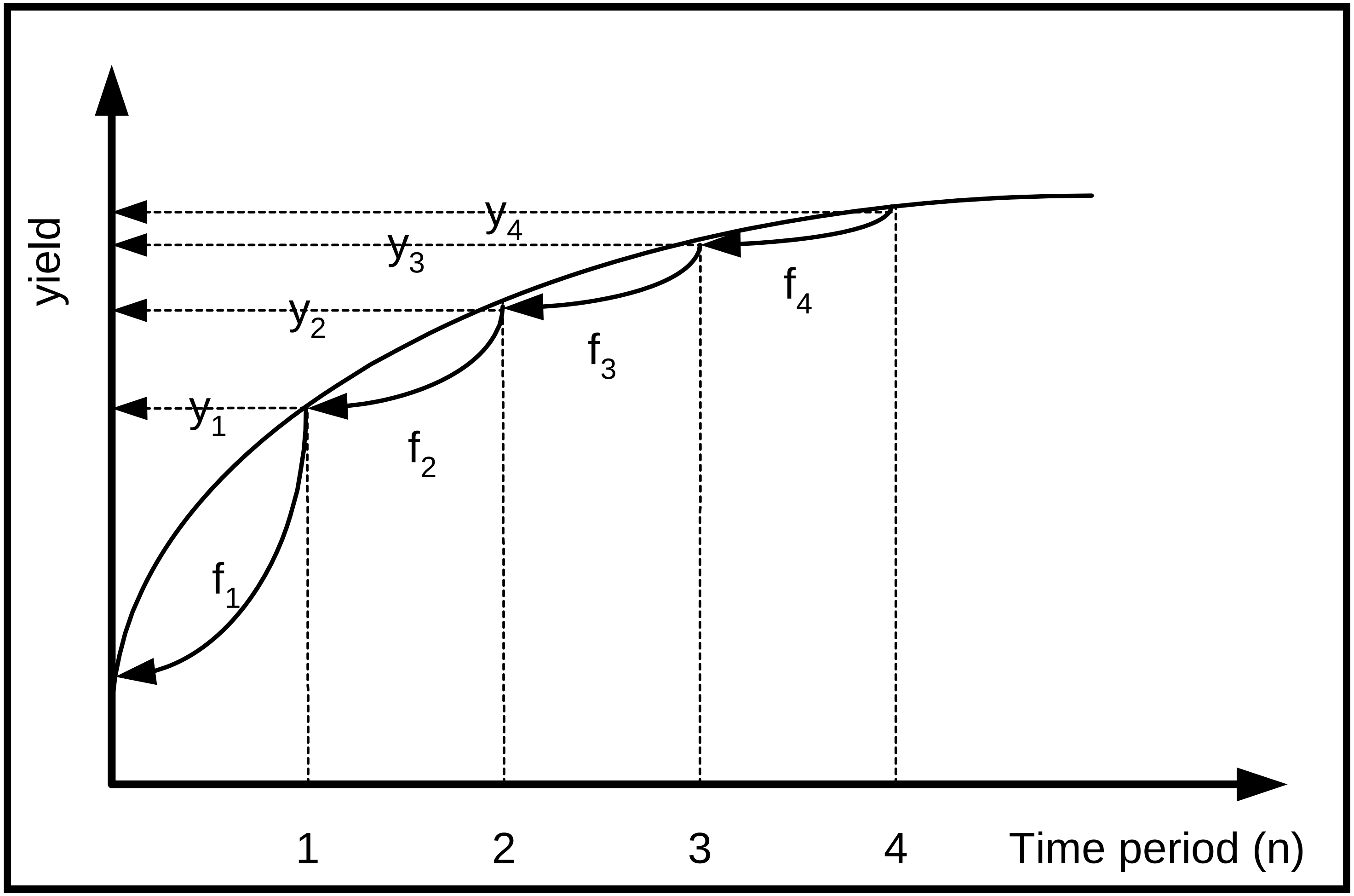
Is the interest rate in the n^{th} period that equates returns from:

a) Holding an n-period zero-coupon bond to maturity

b) The strategy of holding an (n-1) zero for n-1 periods and rolling it over through the n^{th} period

$$(1+f_n)(1+y_{n-1})^{n-1} = (1+y_n)^n$$

The forward rate



$$(1+f_n)(1+y_{n-1})^{n-1} = (1+y_n)^n$$

The forward rate

For semi-annual rates:

$$(1+\frac{f_t}{2})(1+\frac{y_{t-0.5}}{2})^{2t-1}=(1+\frac{y_t}{2})^{2t}$$

Solving for the forward rate:

$$f_t = 2*\left(\frac{(1+ y_t/2)^{2t}}{(1+ y_{t-0.5}/2)^{2t-1}}-1\right)$$

The spot rate yt can be written in terms of the forward rate as:

$$(\frac{y_t}{2}+1)^{2t}=(1+\frac{f_{0.5}}{2})(1+\frac{f_{1.0}}{2})\cdots(1+\frac{f_t}{2})$$

The forward rate

TIR:	Year	Yield	Forward?
	0.5	4.5%	
	1.0	4.7%	
	1.5	4.8%	
	2.0	4.9%	

The forward rate

TIR:	Year	Yield	Forward
	0.5	4.5%	4.5%
	1.0	4.7%	4.9% (2.45%/half year)
	1.5	4.8%	5.0%
	2.0	4.9%	5.2%

Forward rate between 0.5 and 1:

$$f_{1.0} = 2*[\frac{(1+.047/2)^2}{(1+.045/2)^1} - 1] = 0.049$$

Forward rate between 1 and 1.5:

$$f_{1.5} = 2*[\frac{(1+.048/2)^3}{(1+.047/2)^2} - 1] = 0.05$$

The forward rate

For semi-annual rates:

$$(1+\frac{f_t}{2})(1+\frac{y_{t-0.5}}{2})^{2t-1} = (1+\frac{y_t}{2})^{2t}$$

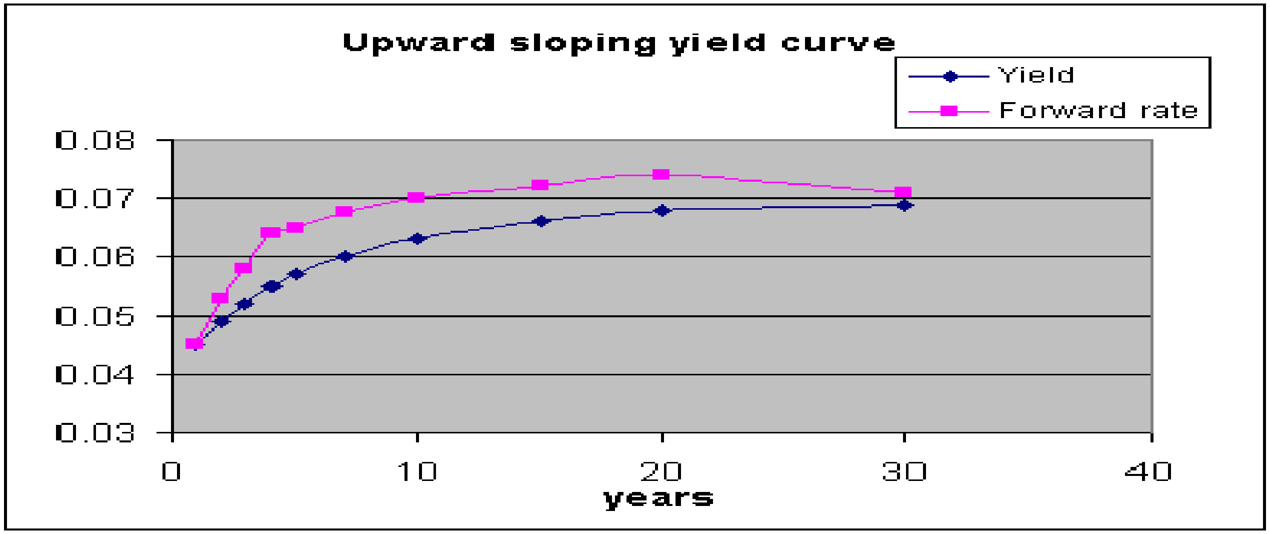
Solving for the forward rate:

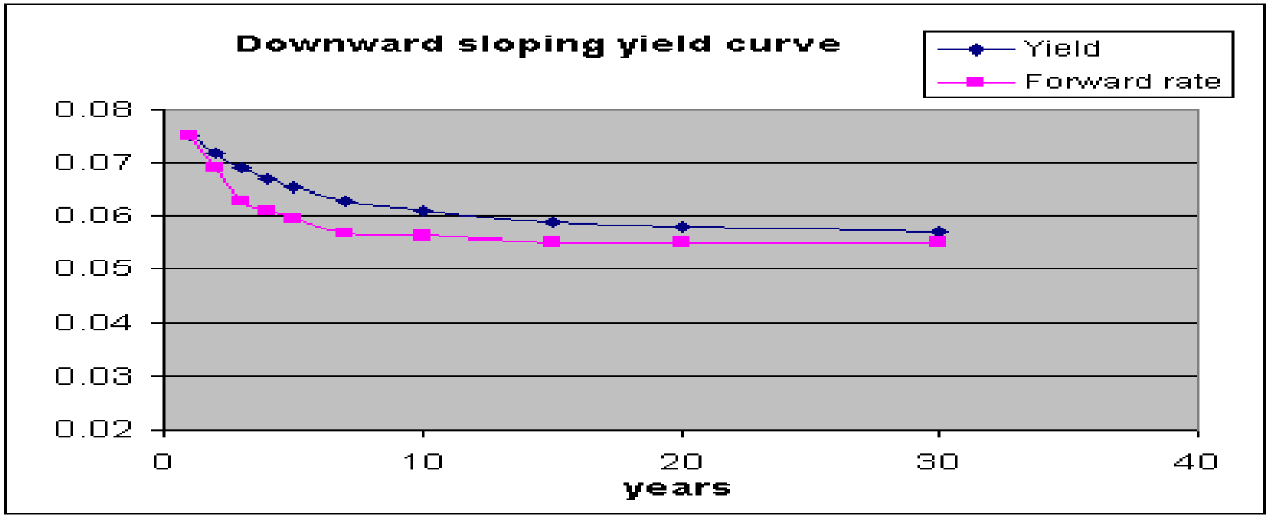
$$f_t = 2 * \left(\frac{(1+{y_t}/{2})^{2t}}{(1+{y_{t-0.5}}/{2})^{2t-1}} - 1 \right)$$

The spot rate yt can be written in terms of the forward rate as:

$$(\frac{y_t}{2}+1)^{2t} = (1+\frac{f_{0.5}}{2})(1+\frac{f_{1.0}}{2})\cdots(1+\frac{f_t}{2})$$

The forward rate





The forward rate

TIR:

Year	Yield	Forward
0.5	4.5%	4.5%
1.0	4.7%	4.9% (2.45%/half year)
1.5	4.8%	5.0%
2.0	4.9%	5.2%

- Price of a 10% 2-year bond?

The forward rate

TIR:	Year	Yield	Forward
	0.5	4.5%	4.5%
	1.0	4.7%	4.9% (2.45%/half year)
	1.5	4.8%	5.0%
	2.0	4.9%	5.2%

$$\text{Price of a } 10\% \text{ 2-year bond?}$$

$$\frac{50}{(1.0225)} + \frac{50}{(1.0225)(1.0245)} + \frac{50}{(1.0225)(1.0245)(1.025)} + \frac{1050}{\cdots} = 1096.3$$

$$\text{or using the yields}$$

$$\frac{50}{(1.0225)} + \frac{50}{(1.0235)^2} + \frac{50}{(1.024)^3} + \frac{1050}{(1.0245)^4} = 1096.3$$

Concept check

TIR:	year	yield
	0.5	4.4%
	1.0	4.6%

- Use the forward rates to price a 8% 1-year bond ?

G) $$\frac{40}{(1.022)} + \frac{1040}{(1.022)(1.023)} = 1033.87$$

Y) $$\frac{40}{(1.022)} + \frac{1040}{(1.022)(1.024)} = 1032.9$$

R) $$\frac{1040}{(1.023)} = 1016.62$$

Concept check

Which of the following statements is true?

G) A series of spot rates (TIR) contains more information about the market discount rates than a series of forward rates

Y) A series of spot rates (TIR) contains less information about the market discount rates than a series of forward rates

R) A series of spot rates (TIR) contains the same information about the market discount rates as a series of forward rates

The forward rate as future borrowing rate

Your company will work on a short project that starts one year from today and ends 1.5 years from today. It will need a loan at the start of the project and will get all the revenues to pay back the loan at the end of the project. The analysis of the project is based on current interest rates so you want to fix the future borrowing rate **today**. How would you use positions in the 1-year and 1.5-year bonds to fix your borrowing rate between year 1 and 1.5 (disregard your default risk for now)?

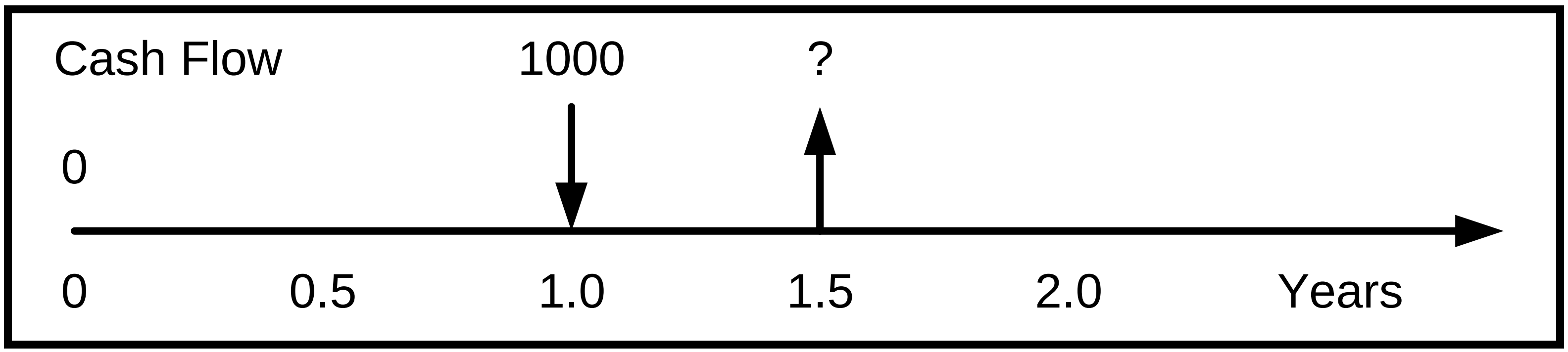
G) Long 1-year and short 1.5-year zeros

Y) Short 1-year and long 1.5-year zeros

R) Long 1-year and long 1.5-year zeros

The forward rate as future borrowing rate

You want to fix your borrowing rate today for borrowing $1000 between years 1 and 1.5. What to do?



Buy a 1 year zero

Sell a 1.5 year zero

The forward rate as a future borrowing rate

t	price	yield	forward
0.5	970.87	6.0%	
1.0	938.97	6.4%	0.0680
1.5	909.09	6.46%	0.0658

Cash Flow (year)	0	1.0	1.5
Buy one 1-year zero	-938.97	1000	
Sell 1.0329* of 1.5 yr zero	938.97	0	-1032.9
Net	0	1000	-1032.9

Interest rate = (1032.9-1000)/1000 = 3.29% for half year

= 6.58% per year

*1.0329 = (938.97/909.09)

Theory of the term structure

When the future is uncertain what should be our best estimate of the future short rate?

1. Market segmentation

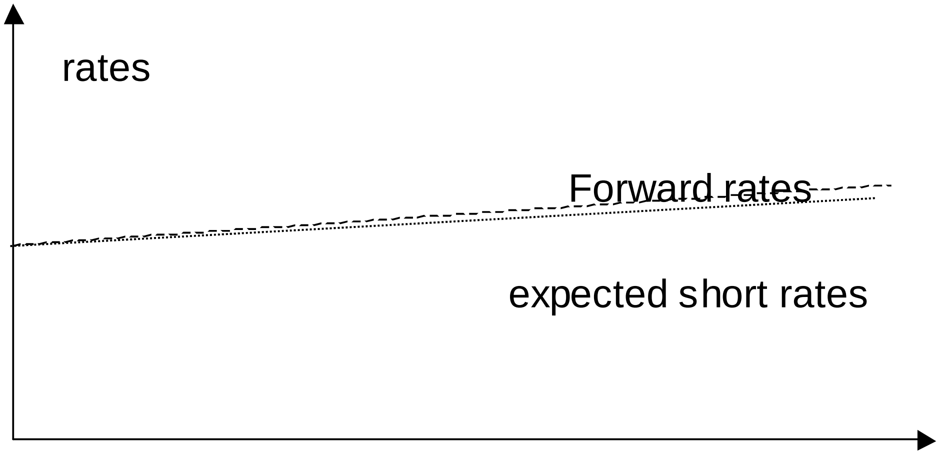
2. Market Expectations Theory
   – Term structure determined solely by expectations of future interest rates

$$E[\tilde{r}_n] = f_n$$

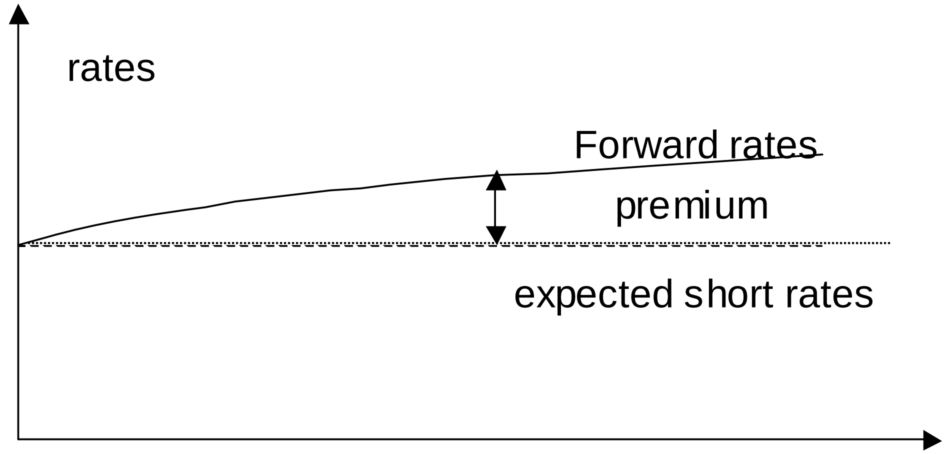
3. Liquidity Preference Theory
   – Term structure determined also by different levels of risk associated with different maturities

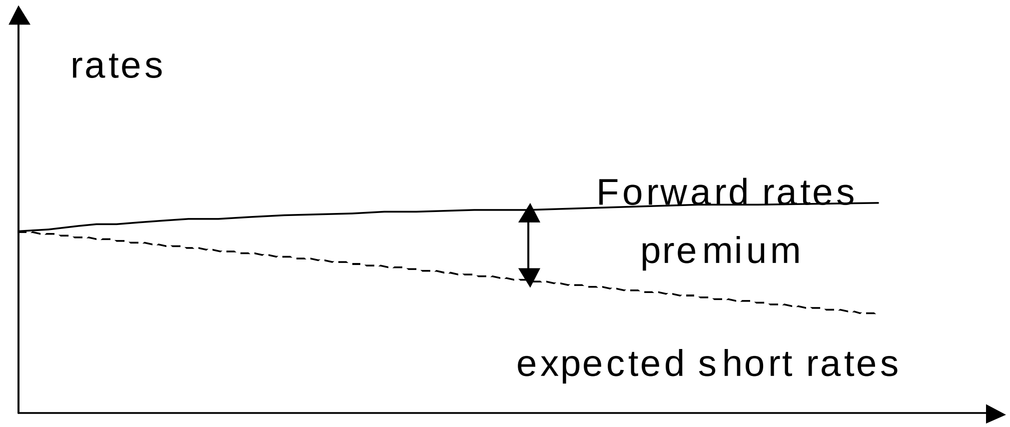
$$f_n = E[\tilde{r}_n] + premium$$

Expectations theory

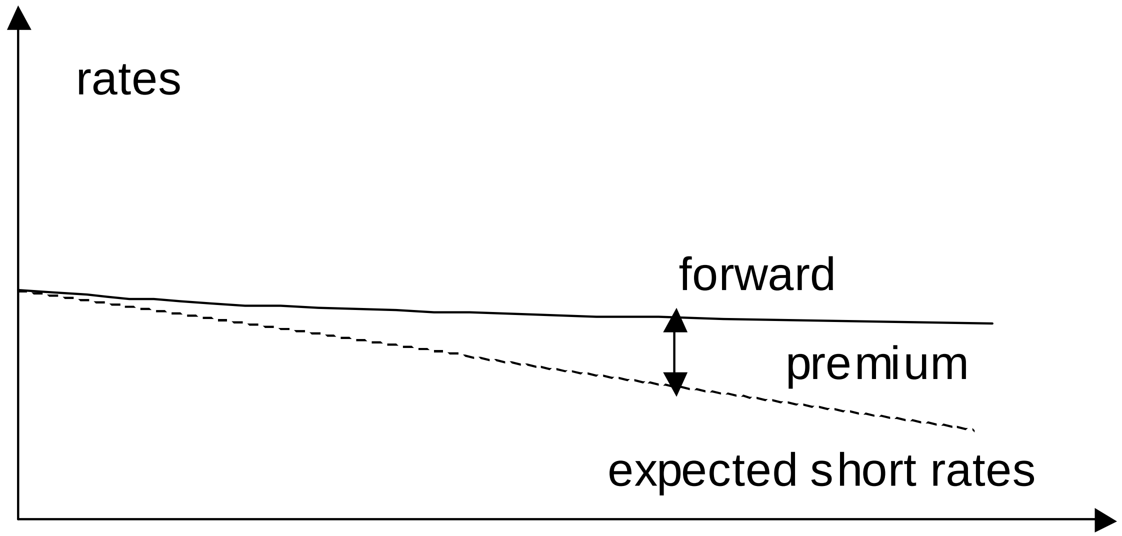


If most investors are short-term investors





If most investors are short-term investors



Summary

- Spot rates
- The term structure of interest rates
- Forward rates