Assignment 2

1. Let $P(x)$ be the predicate "$\frac{x}{(x+2)^3} < 0$." Find the *truth set* of $P(x)$ if the domain of predicate variable x is the set of:
   (a) real numbers $\mathbb{R}$, (b) positive real numbers $\mathbb{R}^+$, (c) integers $\mathbb{Z}$.

2. Let $Q(x, y)$ be the predicate "If $\frac{x}{y} < 1$ then $x^2 < y^2$" with domain for both x and y being the set $\mathbb{R}$ of real numbers. Determine the truth value of $Q(x, y)$ when
   (a) $x = -3, y = 1$, (b) $x = 3, y = 1$.

3. Let the set $\mathbb{Z}^+$ of all positive integers be the domain of the predicate variable x.

   (a) Find the truth set for each predicate: " $\sqrt{x} > 2$ " and "$x^2 > 23$. "

   (b) Determine if the following statements (i) and (ii) are true or false. Give counterexamples for the statements that are false.
   (i) $\sqrt{x} > 1 \Rightarrow x^2 > 23$, (ii) $\sqrt{x} > 2 \Leftrightarrow x^2 > 23$.

4. Let D be the set of all students who are in a math class, and for $s \in D$,
   let $M(s)$ be "s is a math major,"
   let $C(s)$ be "s is a computer science student," and
   let $E(s)$ be "s is an engineering student."
   Express each of the following statements using quantifiers, variables, and the predicates $M(s)$, $C(s)$, and $E(s)$.

   (a) There is an engineering student who is a math major.

   (b) Every computer science student is an engineering student.

   (c) No computer science students are engineering students.

   (d) Some computer science students are also math majors.

   (e) Some computer science students are engineering students and some are not.

5. Write a negation for each statement.

   (a) $\exists x \in \mathbb{R}$, if $(x-1)(x+1) > 0$, then $x > 1$ or $x < -1$.

   (b) $\forall$ integers a, b and c, if $a - b$ is even and $b - c$ is even, then $a - c$ is even.

   (c) The limit of a real-valued function $f(x)$ at point a is L, $\lim\limits_{x \to a} f(x) = L$, which is defined as

   $$\forall \varepsilon \exists \delta \forall x \left(|x - a| < \delta \to |f(x) - L| < \varepsilon \right),$$

   where the domain for ε, δ, x is the set of real numbers. I.e. Find the negation of $\lim\limits_{x \to a} f(x) = L$ (which is "$\lim\limits_{x \to a} f(x) \neq L$") in terms of multiple quantified statement.

6. The notation $\exists!$ stands for the words "there exists a unique." Thus, for instance, "$\exists! x$ such that x is prime and x is even" means that there is **one and only one** even prime number. Which of the following statements are true and which are false? Explain.

   (a) $\exists!$ real number x such that $\forall$ real numbers y, $xy = y$.

(b) $\exists!$ integer x such that $1/x$ is an integer.

(c) $\forall$ real numbers x, $\exists!$ real number y such that $x + y < 0$.

7. Show that each of the following arguments is valid by **universal modus ponens** or **universal modus tollens**, or show that it is invalid from the **converse error** or the **inverse error**. In addition, use also the **diagram** to confirm that each argument is valid or invalid.

(a) All honest people pay their taxes.
Darth is not honest.
$\therefore$ Darth does not pay his taxes.

(b) No vegetarians eat meat.
All vegans are vegetarian.
$\therefore$ No vegans eat meat.