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**EE325 Section 1 HW 2 Due Thursday February 20th (23:00 hr.),2020**

**Use 4 decimal places for numerical answers**

1. In Table 1.a. $X_i$ is total microeconomics exam point (total points are 100) and $Y_i$ is GPA of each student.

**Table 1.a**

| Student | $Y_i$ | $X_i$ |
|---|---|---|
| 1 | 2.8 | 63 |
| 2 | 3.4 | 72 |
| 3 | 3 | 78 |
| 4 | 3.5 | 81 |
| 5 | 3.6 | 87 |
| 6 | 3.0 | 75 |
| 7 | 2.7 | 75 |
| 8 | 3.7 | 90 |

$\bar{Y} = 3.2125$

$\bar{X} = 77.625$

1.1 Now consider the two-variable $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Use OLS to find the estimator of $\beta_0$ and $\beta_1$. (Note: *NIID* = Normally, Identically, and Independently Distributed).

$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$$
$$\hat{\beta}_1 = \frac{\Sigma_{i=1}^n (Y_i - \bar{Y})(X_i - \bar{X})}{\Sigma_{i=1}^n (X_i - \bar{X})^2}$$

$$\beta_1 = \frac{(2.8-3.2125)(63-77.625)+(3.4-3.2125)(72-77.625)+(3-3.2125)(78-77.625)+(3.5-3.2125)(81-77.625)+(3.6-3.2125)(87-77.625)+(3-3.2125)(75-77.625)+(2.7-3.2125)(75-77.625)+(3.7-3.2125)(90-77.625)}{(2.8-3.2125)^2+(3.4-3.2125)^2+(3-3.2125)^2+(3.5-3.2125)^2+(3.6-3.2125)^2+(3-3.2125)^2+(2.7-3.2125)^2+(3.7-3.2125)^2}$$

$\beta_1 = 0.0340659$ #

$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$

$\hat{\beta}_0 = 3.2125 - 0.0340659(77.625)$

$\hat{\beta}_0 = 0.5681319$ #

1.2 For each observation i, find $\widehat{Y}_i$ and $\widehat{u}_i$ . Show that $\sum_{i=0}^{N} \hat{u}_i = 0$.

from $\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i$

$\hat{Y}_1 = 2.7143$
$\hat{u}_1 = 0.0857$

$\hat{Y}_2 = 3.0209$
$\hat{u}_2 = 0.3791$

$\hat{Y}_3 = 3.2252$
$\hat{u}_3 = -0.225$

$\hat{Y}_4 = 3.3275$
$\hat{u}_4 = 0.175$

$\hat{Y}_5 = 3.5319$
$\hat{u}_5 = 0.0681$

$\hat{Y}_6 = 3.1237$
$\hat{u}_6 = -0.1237$

$\hat{Y}_7 = 3.1237$
$\hat{u}_7 = -0.4237$

$\hat{Y}_8 = 3.6340$
$\hat{u}_8 = 0.066$

$\sum_{i=0}^{N} \hat{u}_i = 0.0027$

1.3 Find $var(\widehat{u_i})$, $var(\widehat{\beta_0})$, $var(\widehat{\beta_1})$

$$var(\hat{u}_i) = \sigma^2 = \frac{\Sigma \hat{u}_i^2}{n-2}$$

$$var(\hat{u}_i) = \sigma^2 = 0.0857^2 + 0.3791^2 + (-0.225^2) + 0.195^2 + 0.0681^2 + (-0.1231^2) + (-0.4231^2) + 0.066^2$$

$$var(\hat{u}_i) = \sigma^2 = \frac{0.4355}{6}$$

$$= 0.0726 \ \#$$

$$var(\hat{\beta}_1) = \frac{\sigma^2}{\Sigma x_i^2} = \frac{\sigma^2}{\Sigma_i (x_i - \bar{x})^2}$$

$$= \frac{0.0726}{511.875} = 0.0001 \ \#$$

$$var(\hat{\beta}_0) = \frac{\Sigma x_i^2}{n \Sigma x_i^2} \sigma^2$$

$$= \Sigma x_i^2 \cdot \frac{\sigma^2}{n \Sigma x_i^2}$$

$$= 48{,}717 \cdot 0.0001$$

$$= 48{,}717 \ \#$$

2. Data is listed in the table

| $X_i$ | $Y_i$ | $(x_i - \bar{x})$ | $(y_i - \bar{y})$ | |
|---|---|---|---|---|
| 10 | 0 | (−10) | (−9.1) | = 91 |
| 12 | 2 | (−8) | (−7.1) | = 56.8 |
| 14 | 5 | (−6) | (−4.1) | = 24.6 |
| 16 | 6 | (−4) | (−3.1) | = 12.4 |
| 18 | 7 | (−2) | (−2.1) | = 4.2 |
| 22 | 10 | 2 | (0.9) | = 1.8 |
| 24 | 10 | 4 | (0.9) | = 3.6 |
| 26 | 15 | 6 | (5.9) | = 35.4 |
| 28 | 16 | 8 | (6.9) | = 55.2 |
| 30 | 20 | 10 | (10.9) | = 109 |

$$\bar{x} = \frac{200}{10} = 20$$

$$\bar{y} = \frac{91}{10} = 9.1$$

2.1 From the simple regression model $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Find estimators of $\beta_0$ and $\beta_1$ from the OLS method and interpret the meaning.

$$\hat{\beta} = \frac{\Sigma (x_i - \bar{x})(y_i - \bar{y})}{\Sigma (x_i - \bar{x})^2}$$

$$= \frac{394}{440}$$

$$= 0.8955 \ \#$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

$$= 9.1 - 0.8955(20)$$

$$= -8.8091 \ \#$$

2.2 Find the value of $\widehat{Y}_i$ and $\widehat{u}_i$ . Show that $\sum_{i=0}^{N} \hat{u}_i = 0$.

from $\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$

$\hat{u}_i = Y_i - \hat{Y}_i$

$\hat{y}_1 = 0.1455$

$\hat{u}_1 = -0.1455$

$\hat{y}_2 = 1.9364$

$\hat{u}_2 = 0.0636$

$\hat{y}_3 = 3.7273$

$\hat{u}_3 = 1.2727$

$\hat{y}_4 = 5.5182$

$\hat{u}_4 = 0.4818$

$\hat{y}_5 = 7.3091$

$\hat{u}_5 = -0.3091$

$\hat{y}_6 = 10.8909$

$\hat{u}_6 = -0.8909$

$\hat{y}_7 = 12.6818$

$\hat{u}_7 = -2.6818$

$\hat{y}_8 = 14.4727$

$\hat{u}_8 = 0.5273$

$\hat{y}_9 = 16.2636$

$\hat{u}_9 = -0.2636$

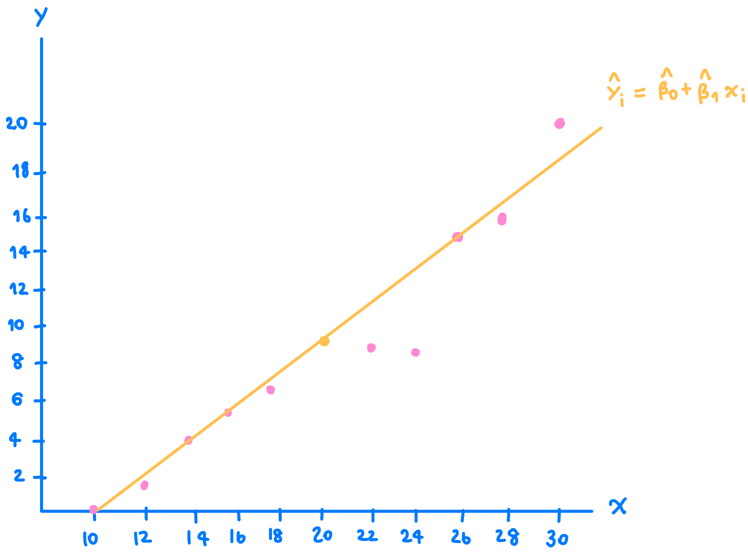
$\hat{y}_{10} = 18.0545$

$\hat{u}_{10} = 1.9455$

$\downarrow$

$\Sigma_i \hat{u}_i = 0$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?




from 2.1, $\bar{x} = 20$, $\bar{y} = 9.1$

$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$

$\hat{y}_i = -8.8091 + 0.8955(20)$

$\hat{y}_i = 9.1$

$\therefore$ the line passes through $\bar{x}, \bar{y}$

2.4 If $X_i$ =16, what is the predicted Y?

$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$

$= -8.8091 + 0.8955(16)$

$= 5.5189$

2.5 Find $var(\widehat{u}_i)$, $var(\widehat{\beta_0})$, $var(\widehat{\beta_1})$

$$var(\hat{u}_i) = \hat{\sigma}^2 = \frac{\Sigma \hat{u}_i^2}{n-2}$$

$$= (-0.1455)^2 + (0.0636)^2 + (1.2727)^2 + (0.4818)^2 + (-0.3091)^2 + (-0.8909)^2 + (-2.6818)^2 + (0.5273)^2 + (-0.2636)^2$$

$$= \frac{14.0909}{10-2}$$

$$= 1.7614 \ \#$$

$$var(\hat{\beta}_1) = \frac{\hat{\sigma}^2}{\Sigma x_i^2} = \frac{1.7614}{440}$$

$$= 0.004 \ \#$$

$$var(\hat{\beta}_0) = \frac{\Sigma X_i^2}{n \Sigma x_i^2} \hat{\sigma}^2$$

$$= \frac{4400 \cdot 0.004}{10}$$

$$= 1.7614 \ \#$$

3. Consider the below regression function:

$$Y_i = \beta_1 X_i + u_i \ ,$$

Where $u_i \sim NIID(0, \sigma^2)$. Find an OLS estimator of $\beta_1$. Then, provide a proof that this is an unbiased estimator. Please state the SLR assumption(s) when used.

$$\hat{\beta}_i = \frac{\Sigma_i (y_i - \bar{y})(x_i - \bar{x})}{\Sigma_i (x_i - \bar{x})^2}$$

let $A = x_i - \bar{x}$

$B = \frac{x_i - \bar{x}}{\Sigma_i (x_i - \bar{x})^2}$

$$\hat{\beta}_i = \Sigma_i (y_i - \bar{y}) B_i$$

$$= \Sigma_i (\beta_1 x_1 + u_i - \beta_1 \bar{x}) B_i + \Sigma_i u_i B_i$$

$$= \beta_1 \Sigma_i (x_1 - \bar{x}) B_i + \Sigma_i u_i B_i$$

$$= \beta_1 \Sigma_i A \cdot \frac{A}{\Sigma_i A^2} + \Sigma_i u_i B_i$$

$$= \beta_1 \cdot \frac{\Sigma_i A^2}{\Sigma_i A^2} + \Sigma_i u_i B_i$$

$$E(\hat{\beta}_1) = E(\beta_1 + \Sigma_i u_i B_i]$$

$$= \beta_i + E(\Sigma_i u_i B_i)$$

SLR4 make it to be 0

$$E(\hat{\beta}_1) = \Sigma_i B_i E(u_i)$$

, so $E(\hat{\beta}_1) = \beta_1$

$\therefore$ $\hat{\beta}_1$ is an unbiased estimator under assumption SLR 1-4