Exercise 3

Derivatives of trigonometric/inverse/exponential/logarithmic/hyperbolic functions

1. Find the derivative for each of the given function.
   (a) $f(x) = \sin^{-1}\left(\frac{x}{x+1}\right)$
   (b) $f(x) = 3\cot^{-1}(2x^3) + x\cot(x)$
   (c) $f(x) = \frac{\cos(x)}{\cos^{-1}(x)}$
   (d) $f(x) = \arctan(\sin(\frac{x}{2}))$
   (e) $f(x) = \log_2(\ln(\ln(x)))$
   (f) $f(x) = \sinh^{-1}(\sin(x)) - \cos^{-1}(\cosh(x))$
2. Find dy/dx in terms of x and y.
   (a) $x^2 + y^2 = 2^{x/y}$
   (b) $y = x\sin(e^{xy})$
   (c) $y = e^{e^{x^3}}$
   (d) $y = e^x + x^e$
   (e) $y = 3^{x+y} + x^3 - \ln(|\cos(x)|)$
   (f) $y = x\log_3(x) + \ln(x^2) + \ln(3)$
   (g) $\sin^{-1}(y) - \cos^{-1}(x) = e$
3. Find the slope of the tangent line to the graph of $y = \sin^{-1}\left(\frac{x}{2}\right)$ at $x = 1$.
4. Find an equation of the tangent line to the graph $f(x) = x\arctan(x)$ at $x = 1$.
5. Find $\frac{d^3y}{dx^3}$ for $y = e^{x^2}$.
6. Find all points on the graph $f(x) = e^{\cos(x)}$ where the tangent lines are horizontal.
7. Find the slope of the tangent to the graph $y = \ln(e^{3x} + x)$ at $x = 0$.
8. Use *logarithmic differentiation* to find dy/dx.
   (a) $y = x^{\sin(x)}$
   (b) $y = [\sin(x)]^x$
   (c) $y = x(x-e)^x$
   (d) $y = \frac{(x+1)^{0.1}(x+2)^{0.2}(x+3)^{0.3}}{(x^2+4)^{0.4}(x^2+5)^{0.5}}$
9. Find $\frac{dy}{dx}$ for
   (a) $y = x^{\sin(x)} + |\sin(x)|^x + \sin(x^x)$
   (b) $y = x^x e^{x^x}$.
   (c) $y = x\cosh(x^2) + \coth(x)$
   (d) $y = \ln\left(\tanh^{-1}(e^{2x})\right)$
10. Find $\frac{d^2y}{dx^2}$ for $y = \sqrt[3]{x^x}$.