Student ID................................

EE431/438 Economics of Financial Markets and Institutions

Exercise 3: Capital Asset Pricing Model (CAPM)

Guidance and Solutions (sketch)

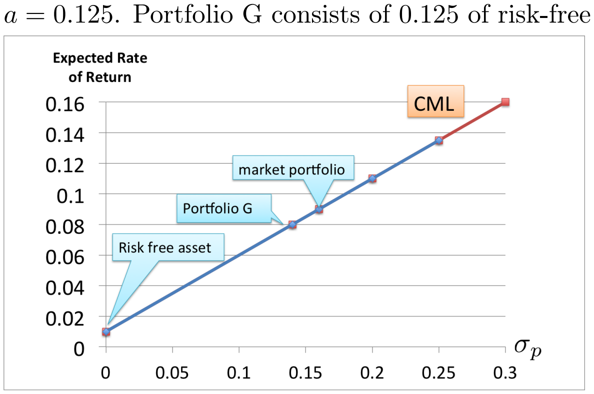
1. Let the expected rate of return on the market portfolio M ($E(R_m)$) be equal to 0.09. The standard deviation of the market portfolio M is equal to 0.16. The risk-free interest rate is equal to 0.01.

   (a) Find the equation for the CML and interpret its meaning.
   Solutions.

$$\begin{aligned} E(R_p) &= R_f + \left(\frac{E(R_M) - R_f}{\sigma_m}\right)\sigma_p \\ &= 0.01 + \left(\frac{0.09 - 0.01}{0.16}\right)\sigma_p \\ &= 0.01 + 0.5\sigma_p \end{aligned}$$

   CML shows the relationship between market required rate of return and risk (measured by standard deviation) for the efficient portfolios (portfolios consisting of the risk-free asset and the market portfolio). As the level of risk (σ_p) of the efficient portfolio increases, the market requires a higher rate of return in order to hold the portfolio.

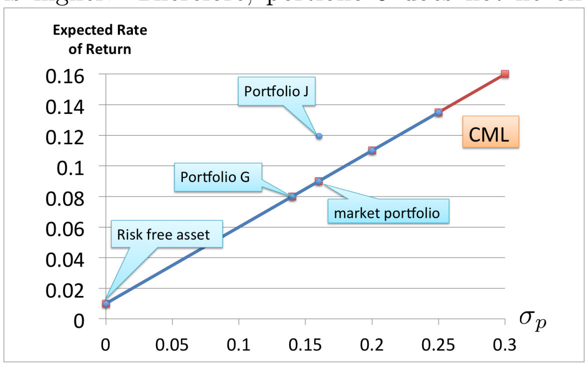
   (b) Let G be a portfolio. The expected rate of return on the portfolio G is equal to 0.08 and the variance of portfolio G is equal to 0.0196. Does the portfolio G lie on CML? Graphically illustrate and explain. If the portfolio G lie on the CML, derive the weight of the risk-free asset and the weight of the market portfolio in the portfolio G.
   Solutions.
   On the CML, a portfolio with $\sigma_p = \sqrt{0.0196} = 0.14$ must have the expected rate of return equal to $E(R_p) = 0.01 + (0.5 \times 0.14) = 0.08$.
   Portfolio G has 0.08 expected rate of return and 0.0196 variance. Therefore, portfolio G must lie on the CML.
   $R_G = aR_f + (1-a)R_m = 0.08$. $0.08 = 0.01a + 0.09 - 0.09a$. $0.08a = 0.01$. Therefore, $a = 0.125$. Portfolio G consists of 0.125 of risk-free asset and 0.875 of the market portfolio.



   (c) Let J be a portfolio. The expected rate of return on the portfolio J is equal to 0.12 and the variance of portfolio J is equal to 0.0256. Does the portfolio J lie on CML? Graphically illustrate and explain. If the portfolio J lie on the CML, derive the weight of the risk-free asset and the weight of the market portfolio in the portfolio J.

Solutions.

On the CML, a portfolio with $\sigma_p = \sqrt{0.0256} = 0.16$ must have the expected rate of return equal to $E(R_p) = 0.01 + (0.5 \times 0.16) = 0.09$. On the CML, the portfolio with $\sigma_p = \sigma_m$ must have the expected rate of return equal to the market portfolio. Portfolio J has the same variance as the market portfolio but the expected rate of return is higher. Therefore, portfolio J does not lie on the CML. Portfolio J lies above CML.



(d) According to James Tobin's investment decision process, which portfolio(s) investors might choose to hold? Describe James Tobin's investment decision process. Indentify the portfolio(s) investors might choose to hold. Explain the reason.

Solutions.

According to James Tobin's investment decision process, each investor will have a utility-maximising portfolio that is a combination of the risk-free asset and a portfolio (or fund) of risky assets that is determined by the line drawn from the risk-free rate of return tangent to the investor's effiecient set of risky assets. If investors have homogenous beliefs, the they all have the same linear efficient set called the Capital Market Line (CML). Therefore, they will try to hold some combination of the risk-free aseet, R_f and the tangency porfolio M. At equilibrium, all assets must be held. All prices will be adjusted so that the demand is equal to supply. The tangency portfolio must be the market portfolio, M. [If V_i is the market value of the ith asset, then the percentage of wealth in each asset is equal to the ratio of the market value of the asset to the market value of all assets. $w_i = \frac{V_i}{\sum_{i=1}^{N} V_i}$.]

Investors will hold portfolios along the CML. Therefore, investors might hold portfolio M and G. No investor will choose to hold portfolio J.

(e) According to James Tobin's investment decision process and the CML in (a), what are the market required rates of return on the portfolio M, G and J? Is the given information sufficient to determine the market required rate of returns of the three portfolios?

Solutions.

The given information is sufficient to determine the market required rate of return for portfolio M and G. Portfolio M and G lie on the CML. They are efficient. Their market required rate of returns are determined by the relationship between the market required rate of return and standard deviation along the CML. Therefore, the market required rate of return on portfolio M and G are equal to 0.10 and 0.08, respectively. The given information is insufficient to determine the market requied rate of return for portfolio J. This is because portfolio J does not lie on the CML. Portfolio J is inefficient. Eventhough we know the variance and the rate of return on portfolio J, we cannot be sure at what rate of return that the market will require from asset J because it is not on the CML. Therefore, we cannot use our knowledge of the mean and the variance of asset J to determine the rate of return that the market will require from asset J in order to hold it in equilibrium. The variance may not be the correct measure of riskiness for an individual asset. Only the portion of total variance that is correlated with the economy(covariance risk) is relevant. Any portion of total risk that is not correlated with the economy is irrelevant and can be avoided at zero cost through diversification. We need the information on how the rate of return on asset J is correlated with the economy(covariance risk, systematic risk) to determine the market required rate of return on asset J.

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2. The following information is provided for a stock market.

	ER_i	σ_i	r_{im}
Asset 1	0.04	0.02	0.8
Asset 2	0.06	0.04	0.4
Asset 3	0.08	0.08	0.8
Market Portfolio (M)	0.09	0.10	1

ER_i is the expected rate of return on asset i. σ_i is the standard deviation of the rate of return on asset i. r_{im} is **the correlation coefficient** between the rate of return on asset i and the rate of return on the market portfolio M. At equilibrium, the expected rate of return on the market portfolio is equal to 0.09 and the standard deviation of the market portfolio is equal to 0.10. The risk free interest rate is 4%. (Hint: $\sigma_{xy} = r_{xy} \times \sigma_x \times \sigma_y$, $r_{xy} = \dfrac{COV(X,Y)}{\sigma_x \sigma_y}$)

(a) In the context of CAPM, find the beta-coefficient for each asset.

Solutions.

	σ_i	r_{im}	$\beta_i = \dfrac{r_{im} \times \sigma_i \times \sigma_m}{\sigma_m^2}$
Asset 1	0.02	0.8	0.16
Asset 2	0.04	0.4	0.16
Asset 3	0.08	0.8	0.64
Market Portfolio (M)	0.10	1	1.0

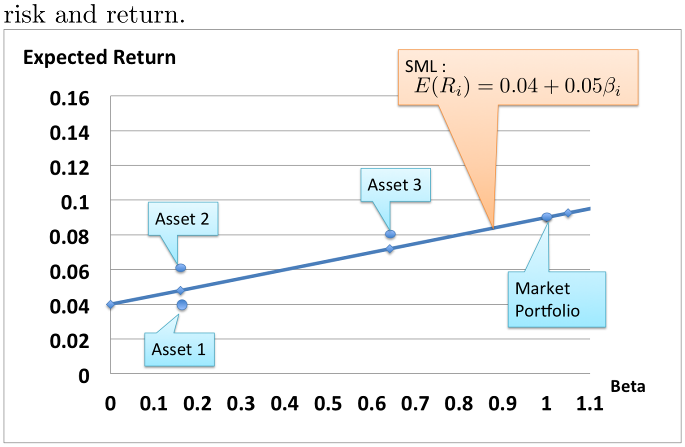
(b) Construct SML from the given information and interpret its meaning. Graphically illustrate SML and show the points where each asset lie in the graph.

Solutions.

$E(R_i) = R_f + \beta_i(E(R_M) - R_f))$.

$E(R_i) = 0.04 + 0.05\beta_i$.

Expected return on asset i is equal to risk free rate plus risk premium. Asset i's risk is measured by β_i. The unit price of risk is equal to the market risk premium. A high beta asset must pay high rate of return. Otherwise, it contradicts CAPM relationship between risk and return.



(c) In the context of CAPM, detemine whether each asset is overpriced, underpriced.or correctly priced. Explain the price adjustment process that might happen (if any).

Solutions.

	β_i (from (a))	SML : $E(R_i) = 0.04 + 0.05\beta_i$.	$E(R_i)$ (from the question)
Asset 1	0.1	0.048	0.04
Asset 2	0.1	0.048	0.06
Asset 3	0.4	0.072	0.08
Market Portfolio (M)	1.0	0.09	0.09

- Asset 1 lies below the SML. Asset 1 is overpriced. Investors will sell asset 1. The price of asset 1 will go down and the rate of return on asset 1 will go up. At equilibrium, asset 1 must lie on the SML. (Another implication is that we have found an empirical evidence against CAPM.)
- Asset 2 lies above the SML. Asset 2 is underpriced. Investors will buy asset 2. The price of asset 2 will go up and the rate of return on asset 2 will go down. At equilibrium, asset 2 must lie on the SML. (Another implication is that we have found an empirical evidence against CAPM.)
- Asset 3 lies above the SML. Asset 3 is underpriced. Investors will buy asset 3. The price of asset 3 will go up and the rate of return on asset 3 will go down. At equilibrium, asset 3 must lie on the SML. (Another implication is that we have found an empirical evidence against CAPM.)
- In the context of CAPM, $E(R_i) = 0.04 + 0.05\beta_i$. The beta of the market portfolio is equal to one. From question (b), SML equation is constructed by using the information on the return of the market portfolio and the risk free rate. Therefore, market portfolio lies on SML.

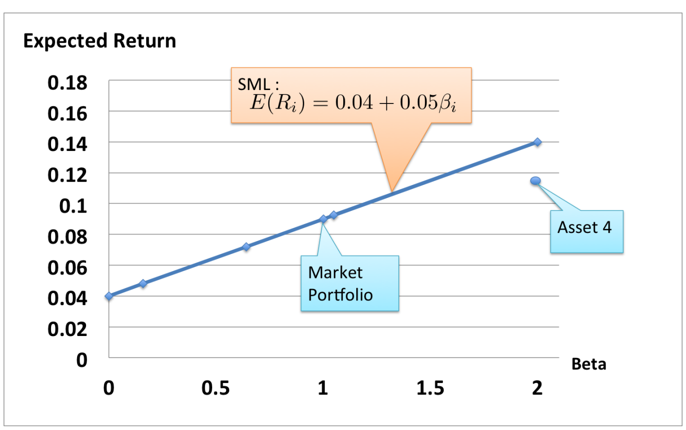
(d) You are informed that the fourth asset, with the beta-coefficient is equal to 2 is available. Empirical evidence reveals that its expected rate of return is 11%. Determine whether this asset is overpriced, underpriced or correctly priced. In the context of CAPM, explain the price adjustment process that might happen(if any).

Solutions.

	β_i	SML : $E(R_i) = 0.04 + 0.05\beta_i$.	$E(R_i)$ (from the question)
Asset 4	2	0.14	0.11

Asset 4 lies below the SML. Asset 4 is overpriced. Investors will buy asset 4. The price of asset 4 will decrease and the rate of return on asset 4 will increase. At equilibrium asset 4 must lie on the SML.

Another implication is that we have found an empirical evidence against CAPM.



3. Given that

$$CML : E(R_p) = R_f + \left(\frac{E(R_m) - R_f}{\sigma_m}\right)\sigma_p,$$

show how to derive CAPM. (Imagine you hold the market portfolio. You add a risky asset, i.)

Solution

$$\begin{aligned}
E(R_p) &= aE(R_i) + (1-a)E(R_m) \\
\sigma_p^2 &= a^2\sigma_i^2 + 2a(1-a)\sigma_{im} + (1-a)^2\sigma_m^2 \\
&= \left(a^2\sigma_i^2 + 2a(1-a)\sigma_{im} + (1-a)^2\sigma_m^2\right)^{1/2}
\end{aligned}$$

Slope of the efficient frontier at the market portfolio is equal to $\lim\limits_{a\to 0}\dfrac{dE(R_p)}{d\sigma_p}$.

$$\begin{aligned}
\frac{dE(R_p)}{d\sigma_p} &= \frac{\partial E(R_p)/\partial a}{\partial\sigma_p/\partial a} \\
\frac{\partial E(R_p)}{\partial a} &= E(R_i) - E(R_m) \\
\frac{\partial\sigma_p}{\partial a} &= \frac{1}{2}\left(a^2\sigma_i^2 + 2a(1-a)\sigma_{im} + (1-a)^2\sigma_m^2\right)^{-1/2}\left(2a\sigma_i^2 + 2(1-a)\sigma_{im} - 2a\sigma_{im} + (-2+2a)\sigma_m^2\right) \\
\left.\frac{\partial\sigma_p}{\partial a}\right|_{a=0} &= \frac{1}{2}\left(\sigma_m^2\right)^{-1/2}\left(2\sigma_{im} - 2\sigma_m^2\right) \\
&= \left(\sigma_m\right)^{-1}\left(\sigma_{im} - \sigma_m^2\right) \\
&= \frac{\left(\sigma_{im} - \sigma_m^2\right)}{\sigma_m} \\
\frac{dE(R_p)}{d\sigma_p} &= \left(E(R_i) - E(R_m)\right) \times \frac{\sigma_m}{\left(\sigma_{im} - \sigma_m^2\right)}
\end{aligned}$$

At equilibrium : Slope CML = Slope of efficient frontier

$$\begin{aligned}
\left(\frac{E(R_m) - R_f}{\sigma_m}\right) &= \left(E(R_i) - E(R_m)\right) \times \frac{\sigma_m}{\left(\sigma_{im} - \sigma_m^2\right)} \\
E(R_m) - R_f &= \left(E(R_i) - E(R_m)\right) \times \frac{\sigma_m^2}{\left(\sigma_{im} - \sigma_m^2\right)} \\
E(R_m) - R_f &= \frac{\sigma_m^2}{\left(\sigma_{im} - \sigma_m^2\right)}E(R_i) - \frac{\sigma_m^2}{\left(\sigma_{im} - \sigma_m^2\right)}E(R_m) \\
E(R_i) &= \frac{\left(\sigma_{im} - \sigma_m^2\right)}{\sigma_m^2} \times \left[E(R_m) + \frac{\sigma_m^2}{\left(\sigma_{im} - \sigma_m^2\right)}E(R_m) - R_f\right] \\
&= \frac{\left(\sigma_{im} - \sigma_m^2\right)}{\sigma_m^2}E(R_m) + E(R_m) - \frac{\left(\sigma_{im} - \sigma_m^2\right)}{\sigma_m^2}R_f \\
&= \left(\frac{\left(\sigma_{im} - \sigma_m^2\right) + \sigma_m^2}{\sigma_m^2}\right)E(R_m) - \frac{\sigma_{im}}{\sigma_m^2}R_f + R_f \\
&= \frac{\sigma_{im}}{\sigma_m^2}E(R_m) - \frac{\sigma_{im}}{\sigma_m^2}R_f + R_f \\
&= R_f + \left(E(R_m) - R_f\right)\frac{\sigma_{im}}{\sigma_m^2} \\
\beta_i &= \frac{\sigma_{im}}{\sigma_m^2} \\
E(R_i) &= R_f + \left(E(R_m) - R_f\right)\beta_i
\end{aligned}$$