



EE 320 Introductory Mathematical Economics

Semester 1/2015

Homework 5

Due 3 December 2015

Question 1:

Given the production function

$$Q = f(K, L) = K^{0.5}L^{0.5}$$

Suppose that the per unit input prices for K and L are \$20 and \$5, respectively.

- If the producer needs to maintain the output level at $\bar{Q} = 20$ units, what are the values K^* and L^* that minimizes the total cost, and what is the corresponding minimum cost? Use the bordered Hessian to verify that the second-order sufficient condition is met.
- Suppose now that the output level is not fixed, but the producer has a budget constraint at \$400. Assume that this producer spends his entire budget on the production. Determine the values K^* and L^* that maximizes the total output. Also, use the bordered Hessian to verify that the second-order sufficient condition is met.

Question 2:

Pakorn's utility function depends on the consumption of two commodities, x and y , and it is given by

$$U(x, y) = 2xy$$

Suppose that his income is \$72, and the prices per unit of x and y are \$4 and \$6, respectively. Assume that Pakorn spends all of his income, and the values of x and y are both non-zero.

- Use the Lagrange method to determine the values of x^* and y^* that maximize Pakorn's utility given an income constraint. Verify that the second-order sufficient conditions are satisfied.
- Determine the maximum utility level and the Lagrange multiplier. Interpret the economic interpretation of the Lagrange multiplier.
- Suppose that the income is now \$73. Approximate the new maximum utility level.

Question 3: Integration

Suppose that a monopolist faces the demand function $Q = 16 - P$. Its total cost function is given by $TC(Q) = 4Q + Q^2$.

- Suppose that this monopolist cannot price-discriminate. Use integral to calculate the consumer and producer surplus at the profit-maximizing quantity and price
- If the monopolist can now practice price-discrimination; that is, he can perfectly identify its new profit-maximizing output level. Also, use integral to calculate the consumer and producer surplus at this new quantity, and discuss the change in total welfare.

Question 4: Cost minimization problem

Consider a cost minimization problem where firm chooses for optimal combination of capital (K) and labor (L). Suppose that r and w are the prices per unit of capital and labor, respectively. And assume further that the production technology of this firm is given by $\sqrt{K} + L = Q$. Consider the following problem

- Solve for the optimal combination of capital and labor.
- State the condition under which both types of factor inputs are used by firm.

- c. Derive the cost function.

Question 5 *Integration*

- a. Suppose the demand and supply curves are $P = \frac{6000}{Q+50}$ and $P = Q + 10$. Find the equilibrium price and quantity, and compute the consumer and producer surplus.
- b. Let $MR = 25 - 5x - 2x^2$ and $MC = 10 - 3x - x^2$, where x is the unit of output. Assume that fixed cost is \$7. Determine the level of production that contributes to maximum profit and determine the level of maximized profit.

Question 6

The production function for a company's product is $Q = 100L + 50K - L^2 - K^2$, where Q is the output that results from L units of labor and K units of capital. The unit costs of labor and capital are \$6 and \$3, respectively. Consider the following problem

- a. If the company wants the total cost of inputs to be 30, determine the greatest output possible subject to this budget constraint.
- b. Suppose market price of the product is \$12 per unit. Determine the optimal combination of labor and capital that yields this company the highest level of profit.