CHAPTER 4

Basic Matrix Algebra and Applications

Topics: Application of Matrix Algebra

Outline:

- How to use matrix to solve system of simultaneous equations
- Partial market equilibrium
- The effect of specific tax on market equilibrium
- Simple Macroeconomic Model
- IS – LM

How to use matrix to solve system of simultaneous equations

When we have the system of simultaneous equations:

1.) Identify endogenous variables, exogenous variables, parameters

2.) Reduce the system of equations to have number of equations equal to number of endogenous variables and to have only necessary information

3.) Rearrange every equations in the system so that all endogenous variables are on one side.

From:

$$ax = k - by$$
$$dy = h - cx$$

To:

$$ax + by = k$$
$$cx + dy = h$$

4.) Rewrite the system into matrix form:

$$\begin{bmatrix} ax+by \\ cx+dy \end{bmatrix} = \begin{bmatrix} k \\ h \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} k \\ h \end{bmatrix}$$

$$\boxed{A_{n\times n}x_{n\times 1} = d_{n\times 1}}$$

A is matrix of parameters that are in front of endogenous variables.

x is column vector of endogenous variables to solve for its final values as a function of exogenous variables and parameters

d is column vector of parameters that don't multiply with endogenous variables and exogenous variables

5.) Check det A:

<table>
<tr><td colspan="4">To find solution to linear system of equations: $A_{n\times n}x_{n\times 1} = d_{n\times 1}$</td></tr>
<tr><td colspan="2">Vector d
Det A</td><td>$d_{n\times 1} \neq 0$</td><td>$d_{n\times 1} = 0$
(homogenous equation system)</td></tr>
<tr><td colspan="2">$|A| \neq 0$
(Linear independent eq)
Matrix A is non-singular
A^{-1} exist</td><td>Unique solution
$x = A^{-1}d \neq \underline{0}$
e.g.
$\begin{bmatrix} 5 & 3 \\ 6 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 30 \\ 8 \end{bmatrix}$</td><td>Unique solution
$x = A^{-1}\underline{0} = \underline{0}$
e.g.
$\begin{bmatrix} 5 & 3 \\ 6 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$</td></tr>
<tr><td rowspan="2">$|A| = 0$
Matrix A singular
A^{-1} does not exist</td><td>Linear dependent eq.
$|A_k| = 0, |A| = 0$</td><td>Infinite solutions
(not including $x = \underline{0}$)
e.g.
$\begin{bmatrix} 5 & 3 \\ 10 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 30 \\ 60 \end{bmatrix}$</td><td>Infinite solutions
(including $x = \underline{0}$)
e.g.
$\begin{bmatrix} 5 & 3 \\ 10 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$</td></tr>
<tr><td>Inconsistent eq.
$|A_k| \neq 0, |A| = 0$</td><td>No solution
$\begin{bmatrix} 5 & 3 \\ 10 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 30 \\ 65 \end{bmatrix}$</td><td>n/a</td></tr>
</table>

6.) Use inverse matrix or Cramer's rule to find the solution of the system of simultaneous equations:

$$x = A^{-1}d \quad or \quad x_k = \frac{|A_k|}{|A|}$$

Note: If determinant is found by using Cofactor for elements from the WRONG row or column, the determinant will be zero!!!!!

$$\begin{vmatrix} 4 & 1 & 2 \\ 5 & 2 & 1 \\ 1 & 0 & 3 \end{vmatrix} \qquad |C_{21}| = -\begin{vmatrix} 1 & 2 \\ 0 & 3 \end{vmatrix} = -3 \qquad |C_{22}| = \begin{vmatrix} 4 & 2 \\ 1 & 3 \end{vmatrix} = 10 \qquad |C_{23}| = -\begin{vmatrix} 4 & 1 \\ 0 & 1 \end{vmatrix} = 1$$

$$|A| = 4(-3) + 1(10) + 2(1) = 0$$

Exercise: Consider the following system of equations and examine whether there is(are) solution(s) to the system or not.

1.) $$\begin{aligned} 4x + y + 2z &= 5 \\ 5x + 2y + z &= 7 \\ x + z &= 1 \end{aligned}$$

2.) $$\begin{aligned} 3x &= 12 - 5y + 4z \\ 6x &= 9 + 10y \\ 8x &= 10 + 7y - 3z \end{aligned}$$

Partial market equilibrium

Demand: $Q^D = a - bP$
Supply: $Q^S = -c + dP$
Equilibrium: $Q^D = Q^S$

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Note: 1.) If we assume $Q^D = Q^S = Q$

2.) If we collect specific tax from supplier t baht per unit

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<u>Exercise:</u>

1.)

$Q^D = 24 - 4P$

$Q^S = 13P - 27$ $\qquad (P^* = 3, Q^* = 12)$

$Q^D = Q^S$

2.) Consider a simple model in which two commodities are related to each other

Good 1	Good 2
$Q_{d1} = Q_{s1}$	$Q_{d2} = Q_{s2}$
$Q_{d1} = a_0 + a_1P_1 + a_2P_2$	$Q_{d2} = c_0 + c_1P_1 + c_2P_2$
$Q_{s1} = b_0 + b_1P_1 + b_2P_2$	$Q_{s2} = d_0 + d_1P_1 + d_2P_2$

Find the equilibrium prices and quantities in good 1 and good 2

Simple Macroeconomic Model

Consider simple Keynesian crossing model:

(1.) $Y = C + I + G$

(2.) $C = a + bY_d$

(3.) $Y_d = Y - T$

(4.) $I = I_0$

(5.) $G = G_0$

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IS – LM

The Good Market:

$Y = C + I + G$

$C = C_0 + b(1 - t)Y$

$I = I_0 - er$

$G = G_0$

The Money Market:

$M_d = fY - \beta r$

$M_s = M_0$

$M_d = M_s$

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