

Assignment 2: Due date: February 17, 2022 before 2.00 pm**Question 1 (30 Points)**

Score.....

At this moment, all of your assets are invested in asset A with the following return and risk characteristics:

$$E(r_A) = 10\%$$

$$\sigma_A = 10\% \quad \nearrow \quad \sigma_A^2 = 100 \%,^2 = 0.01$$

Another asset (call it "B") becomes available; the characteristics of B are as follows;

$$E(r_B) = 20\%$$

$$\sigma_B = 25\% \quad / \quad \sigma_B^2 = .25^2 = .0625$$

↗ perfect negative relationship.

. Furthermore, the correlation of A's and B's return patterns is -1.

Questions:

(1) Using MATLAB to write down the syntax (.m file) for determining the optimal weight (w) of asset A and B in order to achieve the lowest variance, or, in other words, determining the optimal weight for the minimum-variance portfolio.

(2) Find out the Expected return and its variance of the min-variance portfolio using the MATLAB.

(3) By reallocating your portfolio to include some of asset B, how much additional return could you expect to receive if you wanted to maintain your portfolio's risk at $\sigma_p = 10\%$. (Hint: Solve for W_B , not for the W_A).

Note: You must submit both the.m file and your answer in the next page's supplied space.

1) From MATLAB, optimized weight is

$$W_A^* = 0.7143$$

$$W_B^* = 0.2857$$

2) $\bar{R}_p = 1.1286$

$$\text{variance} = 7.0805e-19$$

3) σ_p can be computed by $\sqrt{\sigma_p^2}$. Let $\sigma_p = 0.1$

$$0.1^2 = 0.01(1 - W_B)^2 + 0.0625W_B^2 + 2(W_B - W_B^2)(-0.025)$$

$$0.01 = 0.01(1 - 2W_B + W_B^2) + 0.0625W_B^2 - 0.05W_B + 0.05W_B^2$$

$$= 0.01 - 0.02W_B + 0.01W_B^2 + 0.1125W_B^2 - 0.05W_B$$

$$0 = -0.07W_B + 0.1225W_B^2$$

$$0.07 = 0.1225W_B$$

$$W_B^* = 0.5714.$$

find R_p given W_B^* ,

$$R_p = 0.5714(20\%) + (1 - 0.5714)(10\%)$$

$$= 15.714\%$$

Hence, we can get additional 15.714% - 10%

= 5.714% by including some of asset B.

Define Syntax

```
%R_bar  
c = [1.1 1.2]
```

```
c = 1x2  
    1.1000    1.2000
```

```
%Var-Cov matrix V  
%first we find covariance from corr and variance  
cov = -0.25*0.1
```

```
cov = -0.0250
```

```
var_Ra = 0.1^2
```

```
var_Ra = 0.0100
```

```
var_Rb = 0.25^2
```

```
var_Rb = 0.0625
```

```
V = [var_Ra cov;  
     cov var_Rb]
```

```
V = 2x2  
    0.0100    -0.0250  
   -0.0250    0.0625
```

```
V_inv = inv(V)
```

Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND = 1.132881e-17.

```
V_inv = 2x2  
1017 x  
    7.2058    2.8823  
    2.8823    1.1529
```

```
H = [0.1^2 -0.0250; -0.0250 0.25^2] %variance should be written in squared form
```

```
H = 2x2  
    0.0100    -0.0250  
   -0.0250    0.0625
```

```
H_inv = inv(H)
```

Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND = 1.132881e-17.

```
H_inv = 2x2  
1017 x  
    7.2058    2.8823  
    2.8823    1.1529
```

```
e = ones(size(c))
```

```
e = 1x2  
    1    1
```

```
alpha = e*V_inv*transpose(c)
```

```
alpha = 1.5939e+18
```

```
sigma = c*V_inv*transpose(c)
```

```
sigma = 1.7988e+18
```

```
delta = e*V_inv*transpose(e)
```

```
delta = 1.4123e+18
```

```
R_bar = alpha/delta
```

```
R_bar = 1.1286
```

```
variance = 1/delta
```

```
variance = 7.0805e-19
```

```
std = sqrt(variance)
```

```
std = 8.4146e-10
```

```
%find w*
```

```
w_mv = variance*V_inv*transpose(e)
```

```
w_mv = 2x1  
    0.7143  
    0.2857
```

2.)