Function and relation

EE320

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Agenda

- Review concepts
- Linear function
- Non-linear function
- Application

Review concepts

- Relation: mapping between two sets of variables.
  - One-to-one
  - Many-to-one
  - One-to-many
- Function is two special classes of relation
  - One-to-one
  - Many-to-one

Review concepts

- Domain and range of function/relation
- For example: $y = \sqrt{x} - 3$

Domain = $x \geq 0$

Range = $y \geq -3$

Review concepts

- Suppose
  - $y =$ quantity supplied (q) and
  - $x =$ price per unit (p)
- Would the domain of the function still be the same as before?
  - Domain and range that makes the function mathematically meaningful.
  - Domain and range that makes the function meaningful in economics.

Review concepts

- What if “X” is 16 ➔ “Y” is 1 unit. ➔ Fine.
- But, what happen to the quantity of “y” if “x” is 4? ➔ Y = -1 unit. ➔ ???
- Economically, this is not!
  - Quantity of product cannot be a NEGATIVE number.
  - The lowest it can go is “ZERO”.
- You would argue that price should be greater than 9 $(x \geq 9)$ to ensure that quantity of production is under a justified range.

Review concepts

- Alternatively, define the economic function properly.
- For example, it might be more natural to define the supply function as follow.

$$y = \begin{matrix} \sqrt{x} - 3 & x \geq 9 \\ 0 & 0 \leq x < 9 \end{matrix}$$

$$y = \min\{0, \sqrt{x} - 3\}\,, \qquad x \geq 0$$

Review concepts

- Operations
  - Addition
  - Subtraction
  - Division
  - Multiplication
  - Composite
  - Inverse

Review concepts

Suppoe $f(x) = 2x - 5$ and $g(x) = \frac{1}{x}$

Find:

a. $f + g = 2x - 5 + \frac{1}{x} \quad ; x \neq 0$

b. $f * g = \frac{2x-5}{x}; x \neq 0$

c. $g \circ f = \frac{1}{2x-5} \quad ; \; x \neq \frac{5}{2}$

d. $f^{-1}(x) = \frac{x+5}{2}$

e. $f^{-1} \circ f = x$

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Linear function

A linear function takes the form:

$$y = a + mx$$

$$a = y - intercept \ \ (x = 0, y = a)$$

$$m = slope$$

Linear function

- What is the slope?
  - a change in “y” with respect to a change in “x”.
  - Standard notation refers to $\frac{\Delta y}{\Delta x}$ as the slope of a function.
  - Generally, slope can be varied with respect to changing values of X.
  - But for the linear function, it’s not.
  - $\frac{\Delta y}{\Delta x} = m$ ➔ constant!

Linear function

- With the constant slope, suppose we know a line goes through two points (A and B), we can derive the equation representing that line, using the two-step procedure.
  – Steps 1: Deriving slope
  – Steps 2: Solve for “a” by using slope and one of the two points given.

Linear equation

- Suppose on the x-y plane, a line goes through A(1,4) and B(5,0). What is the equation of that line?

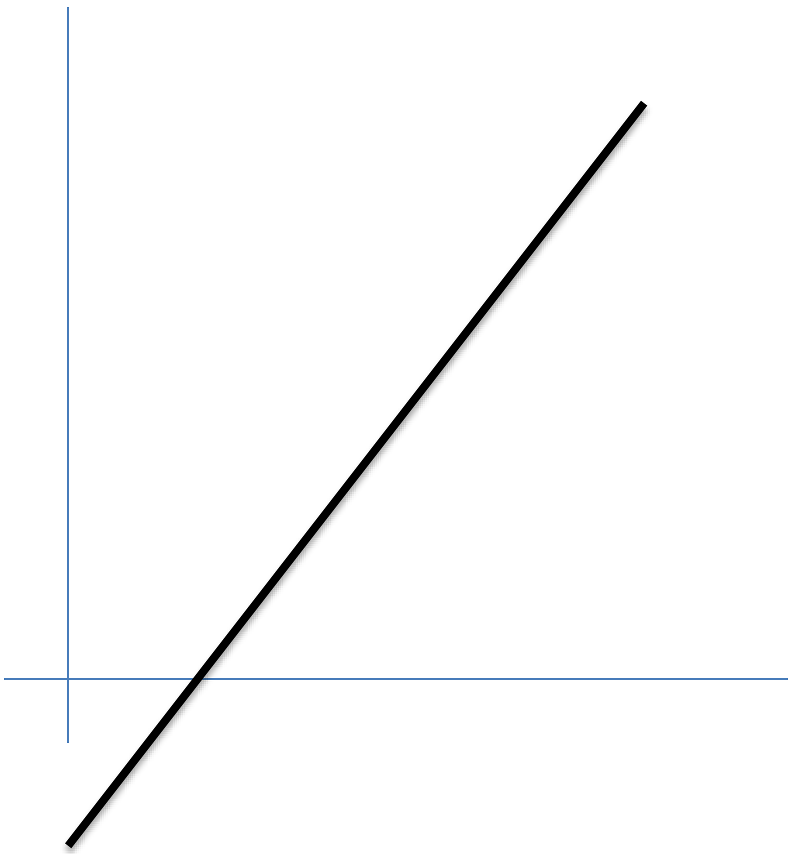
Slope = $-1 = \frac{0-4}{5-1} = -\frac{4}{4}$

$\mathrm{a} = 5 \implies \mathrm{y} = -\mathrm{x} + \mathrm{a};\ \ \mathrm{a} = \mathrm{y} + \mathrm{x} = 0 + 5$

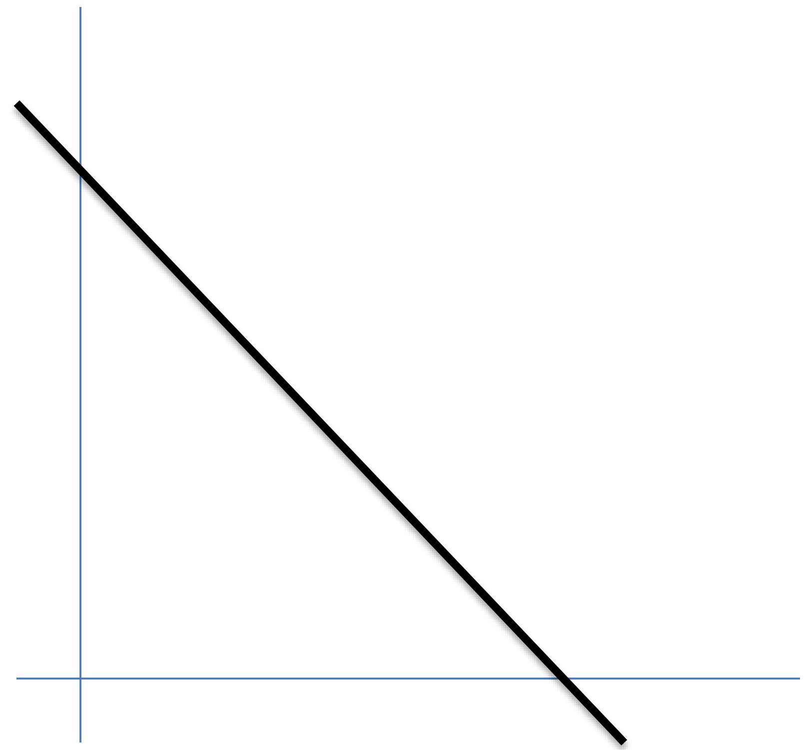
$y = -x + 5$

Linear function

Positive slope



Negative slope



Linear function in economics

- Demand/Supply function
- Production/Cost function
- Consumption function

Linear function: demand function

- Suppose $Q = 20 - P - Y$ as a market demand equation, where Q is quantity, P is price and Y is income.
  - Domain set.
  - Plot the demand curve for Y = 5, and compare with the demand curve for Y = 10.

Linear function: demand function

Suppose a market demand equation

$$Q = 20 - P - Y$$

where

Q is quantity,

P is price and

Y is income.

- Question 1
  - Domain set.
- Question 2
  - Plot the demand curve for Y = 5, and compare with the demand curve for Y = 10.

Linear function: demand function

- Question 1:

$$Q = 20 - (P + Y) \geq 0$$
$$(P + Y) \leq 20, \qquad P \geq 0 \;\; and \; Y \geq 0$$

$$If \; (P + Y) > 20, \qquad Q = 0$$

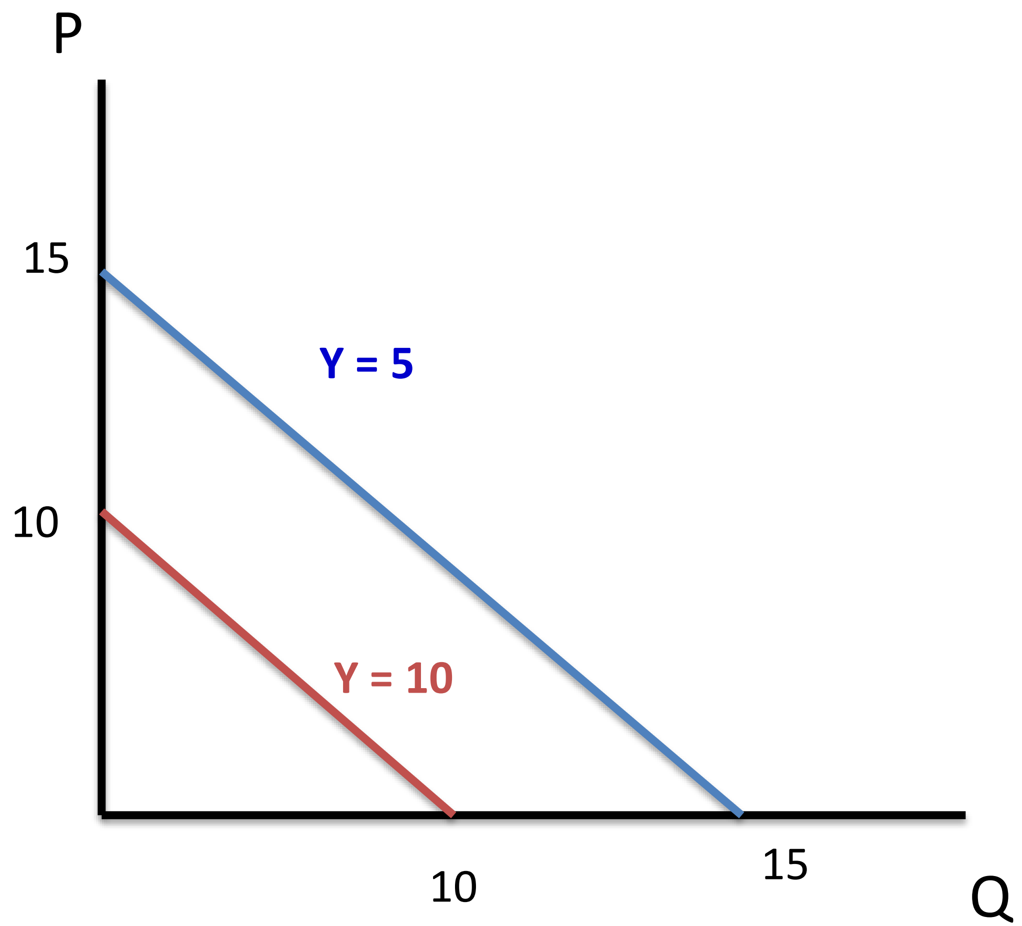
Linear function: demand function

- Question 2:
  - Y = 5 ➔

    P = 15 - Q

  - Y = 10 ➔

    P = 10 - Q



Linear function: demand function

- For fixed Y, what happen to quantity demanded as price increases?
  - Coefficient associated to price is _NEGATIVE___.
- For fixed P, what happen when income increase?
  - Normal/Inferior/Giffen goods?
  - Coefficient associated to income is __NEGATIVE___.
- NOTE: Giffen goods = upward sloping demand

Linear function: demand function

- Suppose $Q_x = 10 - P_x + bP_y$ where

  Q_x is quantity demanded for x.

  P_x is price of good x.

  P_y is price of good y.

Question 1: what could be summarized as the relationship between good x and good y under which b is positive, zero, and negative?

$$Q_x = 10 - P_x + bP_y$$

Using the delta approach:

$$\Delta\, Q_x = \Delta\left(10 - P_x + bP_y\right)$$

Recall that: $\Delta(A + B) = \Delta A + \Delta B$

$$\Delta\, Q_x = \Delta 10 - \Delta P_x + b\Delta P_y$$

Coefficient of linear equation?

Measuring the responsiveness of a variable to a change in another variable.

- Own-price effect:

$$\frac{\Delta Q_x}{\Delta P_x} = -1 \ < 0$$

- *Cross-price effect:*

$$\frac{\Delta Q_x}{\Delta P_y} = b \Rightarrow ??$$

Linear function: demand function

- b = positive ➔ an increase in P_y ➔ Q_x increases ➔ substitute product (coffee and cream)
- b = negative➔ an increase in P_y ➔ Q_x decrease ➔ complement product (I-phone vs Samsung)
- b = 0 ➔ an increase in P_y ➔ Q_x stay the same ➔ neither substitute nor complement (toilet paper and Ferrari)

Linear function: slope v.s. elasticity

- Suppose we know two points on the demand curve, namely $\mathrm{A}(Q_0, P_0)$ and $\mathrm{B}(Q_1, P_1)$
  - A is the initial point
  - B is the terminal point

- Elasticity of demand of demand is defined as

$$\frac{\%\Delta Q}{\%\Delta P} = \frac{Q_1 - Q_0}{P_1 - P_0} * \frac{P_0}{Q_0} = \frac{\Delta Q}{\Delta P} * \frac{P_0}{Q_0}$$

Linear function: slope v.s. elasticity

- For a linear demand, $\frac{\Delta Q}{\Delta P}$ is constant.

$$\frac{\Delta Q}{\Delta P} = \frac{1}{slope\ of\ demand\ curve}$$

- Elasticity of demand is then

$$\frac{1}{slope\ of\ demand\ curve} \times \frac{P}{Q}$$

Linear function: slope v.s. elasticity

- Recall $Q = 20 - P - Y$ as a market demand equation, where Q is quantity, P is price and Y is income.
  - Suppose that Y = 10.
  - Calculate the elasticity of demand when
    - Q = 3
    - Q = 5
    - Q = 7

Linear function: slope v.s. elasticity

Q = 3 ➔ P =7 Q = 5 ➔ P =5 Q = 7 ➔ P =3

$\frac{\Delta Q}{\Delta P} =$ -1 ; constant slope

$$\varepsilon_d(Q = 3) = -\frac{7}{3}$$

$$\varepsilon_d(Q = 5) = -1$$

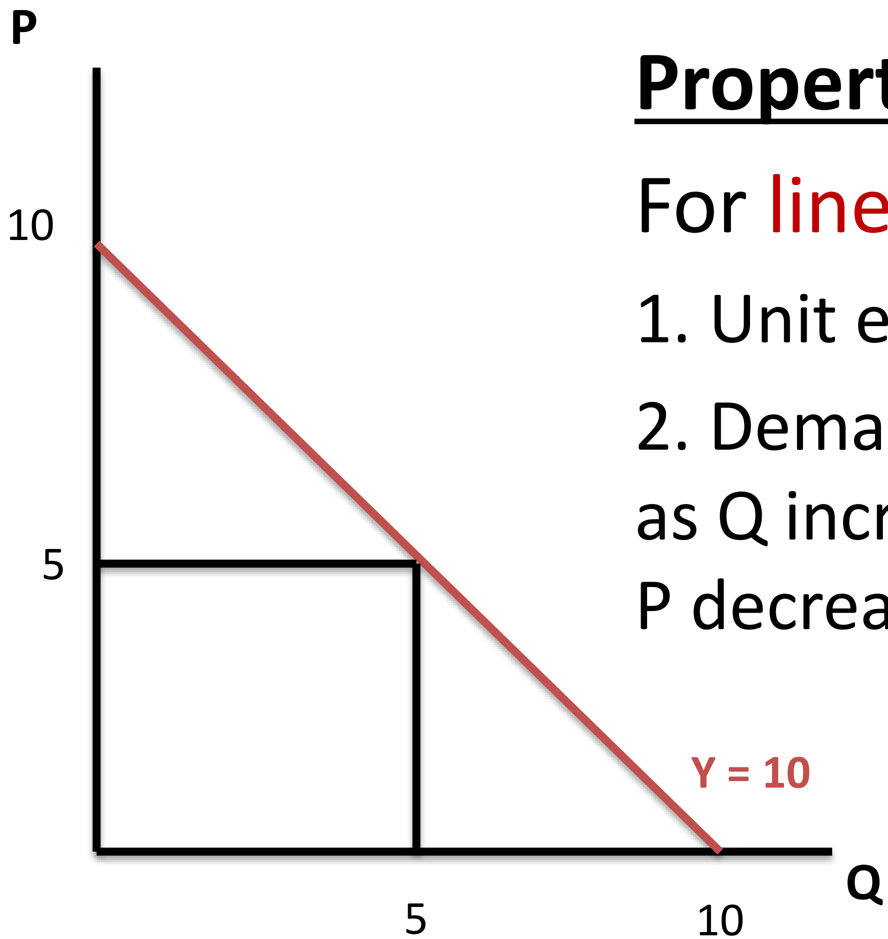
$$\varepsilon_d(Q = 7) = -\frac{3}{7}$$

Linear function: slope v.s. elasticity

- Own-price elasticity of demand is less than or equal to zero.
- Negative sign is used to refer to the inverse relationship between price and quantity demand.

$$\left|\frac{\%\Delta Q}{\%\Delta P}\right| : \quad \begin{array}{lcc} > 1 & ; & elastic \\ = 1 & ; & unit\ elastic \\ < 1 & ; & inelastic \end{array}$$

Linear function: slope v.s. elasticity



Property:

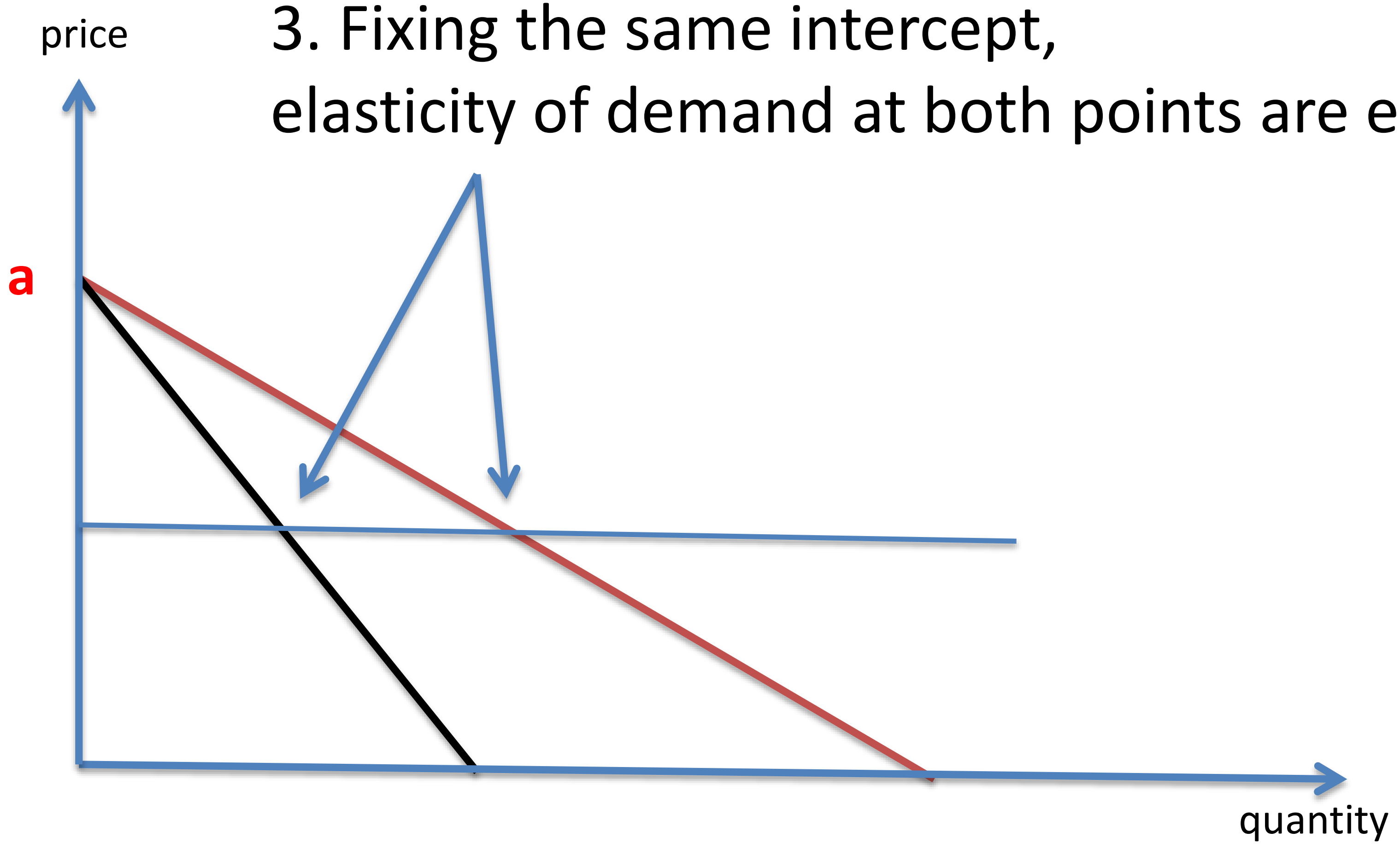
For linear demand curve:

1. Unit elasticity at the mid point.

2. Demand becomes more inelastic as Q increases, (correspondingly to P decreases)

Linear function: slope v.s. elasticity

3. Fixing the same intercept,
elasticity of demand at both points are equal!!



Linear function: slope v.s. elasticity

Proof: Suppose the equation for market demand curve is: $P = a - bQ; \qquad P \leq a$

b = slope of market demand curve

$$\frac{1}{slope\ of\ demand\ curve} \times \frac{P}{Q} = -\frac{1}{b} \times \frac{P}{Q} = -\frac{P}{a-P}$$

$$Q = \frac{(a-P)}{b}$$

Linear function: slope v.s. elasticity: Having fixed P = 5, now calculate income elasticity when Y = 5, 10 and 15.

$Q = 15 - Y$

$\frac{\Delta Q}{\Delta Y} = -1$;

constant slope

Y = 5 ➔ Q = 10

Y = 10 ➔ Q = 5

Y = 15 ➔ Q = 0

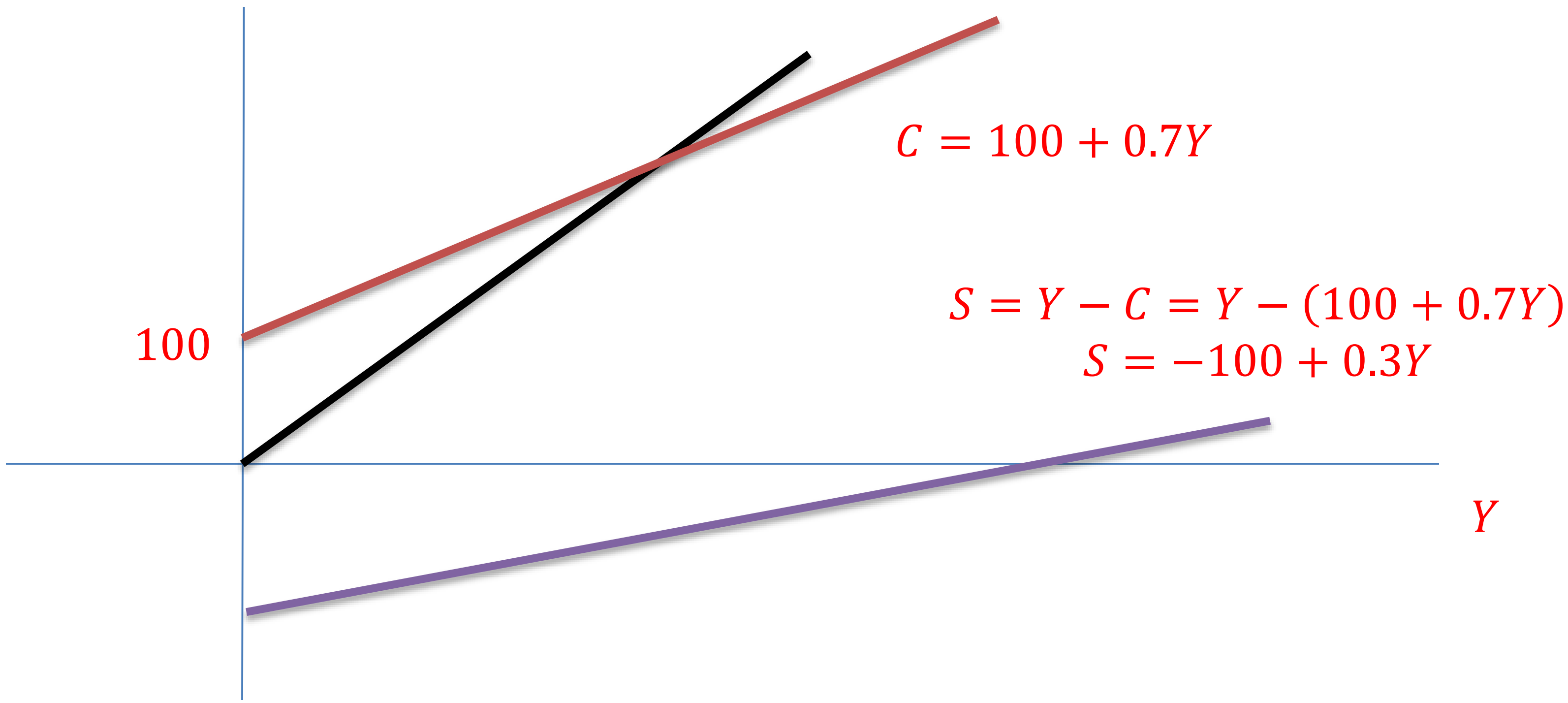
$$\varepsilon_Y(Y = 5) = -\frac{1}{2}$$

$$\varepsilon_Y(Y = 10) = -2$$

$$\varepsilon_Y(Y = 15) = -\infty \Rightarrow perfect\ elastic$$

Linear function: consumption function

- Suppose that $C = 100 + 0.7Y$
  - Plot the consumption function
  - Derive the saving function



Linear function: aggregate expenditure function

- Suppose that $I = 200 + 0.1Y - 100 * r$
  - Plot the investment function if $r = 0.1$ (10%).
  - Derive the Aggregate Expenditure function
  - Qualitatively, what happen to AE function if r increases?

$$I = 200 + 0.1Y - 100 * (0.1) = 190 + 0.1Y$$

Linear function: aggregate expenditure function

- Aggregate expenditure function
  - A function that defines behavior of aggregate spendings.
  - Aggregate spending/expenditure
    "C + I + G + X – M"

- $AE(Y, r) = 300 + 0.8Y - 100 * r$

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Drawbacks of linearity concept

- Constant slope is a big deal of the problem.
- SES Panel survey shows that rich tend to consume less out of each additional unit of income they have earned.
  - Thus, MPC should be decreasing as income increase.
- So, neither the slope of consumption function should be constant nor the form of consumption function could be truly represented by a linear equation.

Non-linear function

- Two broad classes of non-linear functions commonly used.
  - Polynomial function
  - Exponential/Logarithm function

Polynomial function

Polynomial takes the form

$$y = a_0 + a_1 x + a_2 x^2 + \cdots + a_N x^N$$

N = 1 ➔ linear ➔ straight line

N = 2 ➔ Quadratic function ➔ Parabola

N = 3 ➔ Cubic function ➔ Don’t know, but who care

Quadratic function

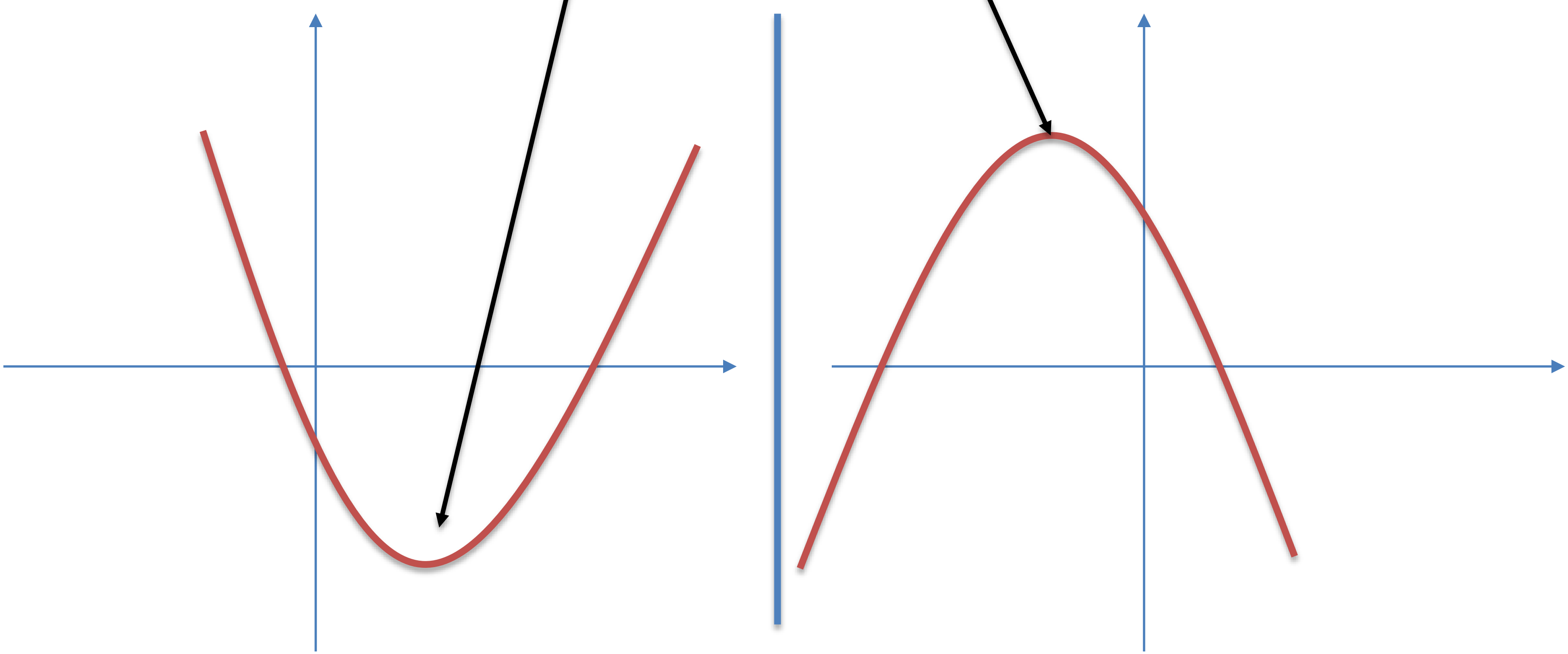
- Polynomial degree 2.
- Let's write the function in the form that you are used to.

$$y = ax^2 + bx + c$$

Quadratic function

$$x = -\frac{b}{2a}$$

$a > 0$ $a < 0$



U-shape, An upward-opened Inverted U-shape, A downward-opened

Quadratic function: revenue function/break even analysis

- A monopolist firm faces the market demand given by $P = 10 - Q$. Suppose the cost function $C(Q) = 4Q$.

Question 1 What is the revenue-maximizing level of output?

Question 2 What is the break-even output?

Question 3 What is the profit-maximizing level of output?

- Question 1

$$R(Q) = P * Q$$
$$= (10 - Q)Q$$
$$= (10Q - Q^2)$$
$$Q^* = -\frac{b}{2a} = -\frac{10}{2(-1)} = 5\ units$$
$$P^* = 10 - Q^* = 10 - 5 = \$5\ \ per\ unit$$
$$R^* = \mathrm{R(Q^*)} = \mathrm{P^*Q^*} = 5 * 5 = \$25$$

- Question 2

$$\pi(Q) = P * Q - C(Q)$$

$$Q \;\; such\ that: \;\; \pi(Q) = 0 \Rightarrow Break - even$$

$$\pi(Q) = \; (10Q - Q^2) - 4Q = 6Q - Q^2$$

$$Q^{BR} = 0, 6$$

- Question 3

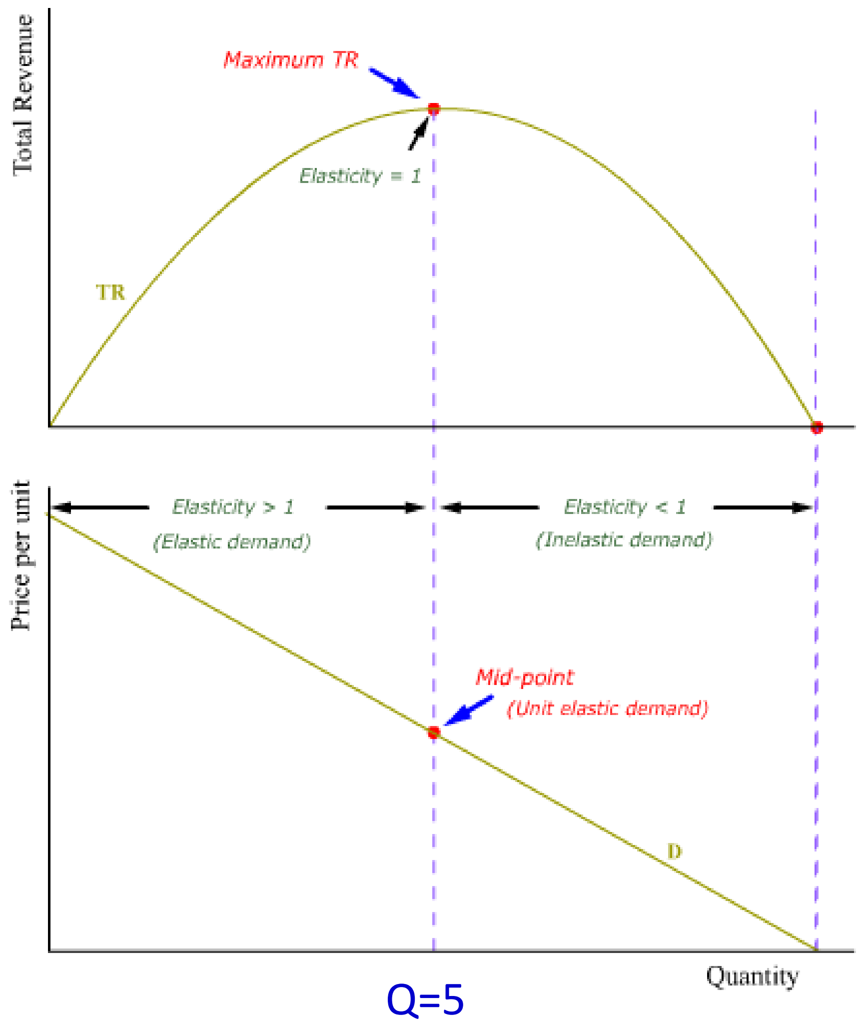
$$\pi(Q) = 6Q - Q^2$$

$$Q^{\pi *} = -\frac{\mathrm{b}}{2\mathrm{a}} = -\frac{6}{2(-1)} = 3 \ \text{units}$$

$$P^* = 10 - Q^{\pi *} = 7$$

$$\pi^* = \pi(Q^{\pi *}) = 18 - 9 = \$9 \ (= 21 - 12)$$

Quadratic function: revenue function/break even analysis



$$\left|\frac{\%\Delta Q}{\%\Delta P}\right| : \begin{array}{lcl} > 1 & ; & elastic \\ = 1 & ; & unit\ elastic \\ < 1 & ; & inelastic \end{array}$$

Revenue maximizing output:
unit elasticity

Profit-maximizing output:
under region with **elastic** demand

Exponential/Logarithm function

- Exponential: $y = a^x$

- Logarithm: $y = \log_a x$