

EE415/418 Midterm (Solutions)

- 1.a) First, you should draw the game tree and make an explicit assumption about the game structure. Then, solving this game backward, you can reach the conclusion about what to be played in each stage by both players. This is what you need to specify in your answer, that is player 1's strategies in both stage, and player 2's strategies in both stage. Then, you end up concluding that following your answer, the game will end at the first period, and that defines the outcome of the game.

SPE is "player 2 demands δ_2 in the 1st period (δ_2 is player 2's discount factor) and will reject any offer less than δ_2 ; player 1 accepts an offer that is greater than or equal $1-\delta_2$, and will offer $1-\delta_2$ for himself.

SPE's outcome is $(1-\delta_2, \delta_2)$

Note that if we assume the game ends with $(s, 1-s)$ if no agreement is reached at the end of period two, then SPNE's outcome will be $(1-\delta_2(1-s), \delta_2(1-s))$. Clearly, if we assume that s is 0, we got the above answer.

Note : This shows the last mover advantage situation . Player 2 has advantage from being the last person who makes an offer. The game ends right after player 1 makes a decision.

As for the NE (that is not SPNE as you found above), the idea is that this must involve some move that is not credible (it is not NE in some stage game). In this game, we have multiple pure Nash equilibria. For example,

NE is "player 1 offers x for $x \in [0, 1]$ for himself and rejects any offer that less than x ; player 2 will reject any offer less than $1-x$ and will offer x for himself.

NE's outcome is $(x, 1-x)$ for $x \in [0, 1]$.

- 1.b) NE and SPE outcomes are $(1, 0)$.
 NE = SPE = $\left\{ \begin{array}{l} \text{Player 1 offers 1 for himself; rejects any offer less than 1.} \\ \text{Player 2 accepts any offer } \geq 0. \end{array} \right\}$

Note : assume that player 1 is the first mover.

This shows the first mover advantage.

In this game, there is only one-stage game or subgame, thus it is the whole game.

- 2.a) The strategy that that supports the joint monopoly outcome is the trigger strategy

“Play $\frac{q_m}{2} = \frac{1-c}{4}$ in the first period. Play $\frac{q_m}{2} = \frac{1-c}{4}$ in current period if collusion is observed in all previous periods, otherwise play $q_c = \frac{1-a}{3}$ ”

The immediate payoff from collusion is

$$\pi_{collude} = \left(1 - c - \frac{1-c}{2}\right) \frac{1-c}{4} = \frac{(1-c)^2}{8}.$$

If one of the players wants to deviate, then his problem is

$$\max_{q_i} \left(1 - c - \frac{1-c}{4} - q_i\right) q_i.$$

The FOC is given by

$$q_i^* = \frac{3}{8}(1-c).$$

Thus, the immediate payoff from deviation from collusion is

$$\pi_{deviate} = \left(1 - c - \frac{1-c}{4} - \frac{3}{8}(1-c)\right) \frac{3}{8}(1-c) = \frac{9}{64}(1-c)^2.$$

If one of the players deviate, Cournot equilibrium is played in all later periods, both players get in each period

$$\pi_c = \left(1 - c - \frac{2}{3}(1-c)\right) \frac{1}{3}(1-c) = \frac{(1-c)^2}{9}.$$

If deviation is optimal then the PV of payoffs is

$$V_{deviate} = \pi_{deviate} + \delta V_{punish} = \frac{9}{64}(1-c)^2 + \frac{\delta}{1-\delta} \frac{(1-c)^2}{9}.$$

If collusion is optimal then the PV of payoffs is

$$V_{collude} = \pi_{collude} + \delta V_{collude} \Rightarrow V_{collude} = \frac{1}{1-\delta} \frac{(1-c)^2}{8}.$$

Collusion is optimal if and only if

$$\begin{aligned} V_{collude} \geq V_{deviate} &\Rightarrow \frac{1}{1-\delta} \frac{(1-c)^2}{8} \geq \frac{9}{64}(1-c)^2 + \frac{\delta}{1-\delta} \frac{(1-c)^2}{9} \\ &\Rightarrow 72 \geq 81(1-\delta) + 64\delta \Rightarrow \delta \geq \frac{9}{17}. \end{aligned}$$

Thus, the joint monopoly outcome can be supported as a subgame perfect equilibrium in this infinitely repeated game only if $\delta \geq \frac{9}{17}$.

- 2.b) If $\delta < \frac{9}{17}$, then collusive outcome with lower aggregate profit can still be achieved by other trigger strategies

“Play q^* in the first period. Play q^* in current period if collusion is observed in all previous periods, otherwise play $q_c = \frac{1-a}{3}$ ”

where $q^* > \frac{q_m}{2} = \frac{1-c}{4}$.

Alternatively, one can achieve the joint monopoly outcome by using stick-and-carrot strategy

“Play $\frac{q_m}{2} = \frac{1-c}{4}$ in the first period. Play $\frac{q_m}{2} = \frac{1-c}{4}$ in current period if the outcome in the previous round has been $\left(\frac{q_m}{2}, \frac{q_m}{2}\right)$ or (x, x) , otherwise play x .”

- 3.a) One way to interpret the concept of mixed strategy Nash equilibrium is that a mixed strategy Nash equilibrium are mutual best responses when players' are uncertain about his opponent strategies. Since the player is uncertain, he conjectures some kind of belief about his opponents' pure strategy. If all players' are rational and rationality is a common knowledge, by rationality, everyone's belief must be consistent and is correctly predictable by his opponents. Given his belief about his opponents' pure strategies, taking into account the consistent beliefs conjectured by his opponent, and knowing that his opponents know his belief, he chooses his best responses which may involve uncertainty. If everybody's strategy profiles are mutual best response, then it is Nash equilibrium and no one has the incentive to deviate (from their strategies and beliefs).
- 3.b) SPNE is self-enforcing because it requires everyone to play NE in every subgame even if the SPNE predicts that some subgame will never be reached. Thus, in every subgames, if ever reached, everybody will play according to SPNE.

In contrast to SPNE, in NE that is not SPNE, at some point of the game - or the subgame, someone will have the incentive to deviate from NE if the NE does not induce the player to play NE in that subgame, e.g. NE predicts that the player will play a strictly dominated strategy in the subgame. It is now clear that such NE is not self-enforcing because once the subgame is reached, the player will not play according to NE. SPNE is self-enforcing, implying that there are no incredible threat or moves as parts of SPNE.