Discussion handout 7 (Semester 1/2017)

Question 1: A two-product firm faces with the following demand and cost function:

$$Q_1 = 40 - 2p_1 - p_2; \quad Q_2 = 35 - p_1 - p_2; \quad C = Q_1^2 + 2Q_2^2 + 10$$

Consider the following problem.

a. Find the output levels that maximize the monopolist's profit. Determine the prices charged to each of the two products.
b. Use the second-order derivative test to confirm your result obtained in previous part.
c. What is the maximal profit?

Question 2 A profit-maximizing monopolist is selling its output in the UK and in the US. The demand functions are given by

$$p_1 = 100 - 2Q_1; \quad p_2 = 200 - Q_2$$

where country 1 is the UK, and 2 the US. The cost function of the monopolist is given by $C(Q) = Q^2$ where $Q = Q_1 + Q_2$. For simplicity, assume that both adopt the same common currency, so that prices set in each country are quoted in the same unit. Consider the following problem

a) Find the optimal prices and outputs for each country. Assume that the monopolist can do the third-degree price discrimination. Calculate the level of maximized profit obtained.

b) Suppose the monopolist cannot prevent the resell, find the optimal price that maximizes the profit of the monopolist. Determine the level of outputs sold in each of the two countries. Calculate the level of maximized profit obtained.

Question 3 Suppose a representative firm has the production function given by,

$$q = f(x_1, x_2, x_3, \dots, x_N) = A\left[\sum_{i=1}^{N} x_i^{\theta_i}\right]$$

where x_i is the amount of type-i input that the representative firm uses in the production. Assume further that the representative firm take factor prices (w_i) and market price of the product (p) as given. Consider the following problem.

a) Calculate the optimal combination of factor inputs that maximizes profit of the representation firm.

b) State the required assumption that ensure to global property of the solution obtained in a. (You may limit your analysis to the case N = 3.)

c) Calculate the optimal profit function for the representative firm.

d) Derive the supply equation for the representative firm.

e) How does market factor prices affect the supply of the representative firm?

f) (Optional) suppose that there are J firms in the industry. Each of the firm production's technology is identical to that of the representative firm. Describe the process to derive the market equilibrium price if market demand equation is given by $p = a - bQ^D$. (Hint: this question does not require that you need to go through the derivation process. The purpose is to only discuss the conceptual framework.)