

FIXED INCOME INSTRUMENT 2

EE431

Semester 2/2017

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TOPICS

- ~~Bond pricing and yield to maturity~~
- **Bond returns and Interest rate risk.**
- Bond price and interest rate determination
- Risk and term structure of interest rates

READING LISTS

Required text:

Mishkin Ch 4 – Ch 6 (required!)

Fabozzi Ch 2 - Ch 5 (optional, a bit more finance)

A supplement note was uploaded on the BE Moodle.

BOND RETURNS

- What do we earn from bond holding?
- Most usually refers to *interest rate* or *yield-to-maturity*.
 - Annual rate of earning that an investor can earn if the investor holds it until the maturity date.
- Investors can and may sell bond before it gets matured.
- Bond return is often defined as *holding-period return*.
 - Net profit from the investment over a certain period of investment horizon.

HOLDING-PERIOD RATE OF RETURN (RER)

Assume (i) **one-period** investment horizon: $t \rightarrow t+1$

(ii) Coupon bond with $c\%$ per period coupon rate.

Definition: Rate of return on one-period investment on bond is

$$RE R_t = \frac{P_{t+1} + \frac{c}{100} FV - P_t}{P_t} = \frac{\frac{c}{100} FV}{P_t} + \frac{P_{t+1} - P_t}{P_t}$$

HOLDING-PERIOD RATE OF RETURN

$\frac{\frac{c}{100}FV}{P_t}$: Current yield from coupon

- Certain value!

$\frac{P_{t+1} - P_t}{P_t}$: Change in bond price

- Future resell price (P_{t+1}) is uncertain!
 - “+ Cap Gain: if $P_{t+1} > P_t$
 - “- Cap Loss: if $P_{t+1} < P_t$

Returns on bond holding is uncertain.

We call “price risk” or “market-price risk”

What is the factor associating with this price risk?

INTEREST RATE RISK

➤ Following the bond pricing, we know that

$$P_t = V_t = \frac{CF_1}{1 + r_T} + \frac{CF_2}{(1 + r_T)^2} + \dots + \frac{CF_T}{(1 + r_T)^T}$$

➤ How does r_T affect the price?

- First observation: Bond price can be volatile due to the change in the market sentiment/trend of interest rate.

➤ Volatility in the market price, and hence the implied returns, can be attributed to the movement in market interest rate, **and hence called “interest rate risk”**.

EXAMPLE 1: MEASURING RATE OF RETURN

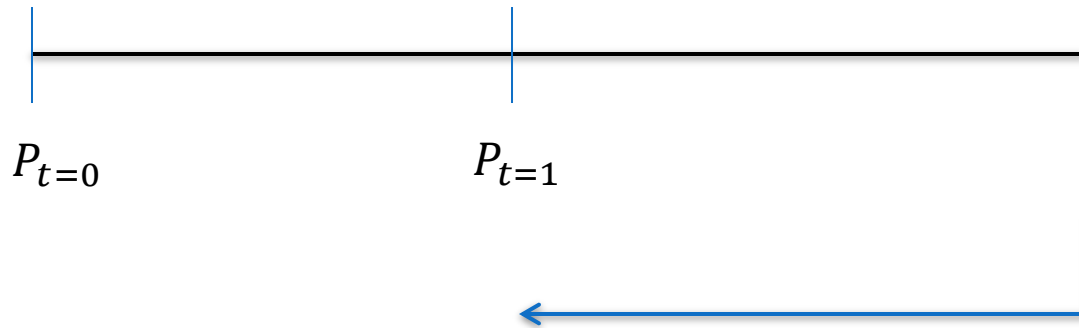
Question A 2-year bond with \$100 Face value is set to pay 1% of coupon rate once a year. Suppose that current market interest rate is 1%.

What is the one-year rate of return on bond if market interest rate is rising 2% next year.

EXAMPLE 1: INTEREST RATE RISK

$$CF_1 = 1$$

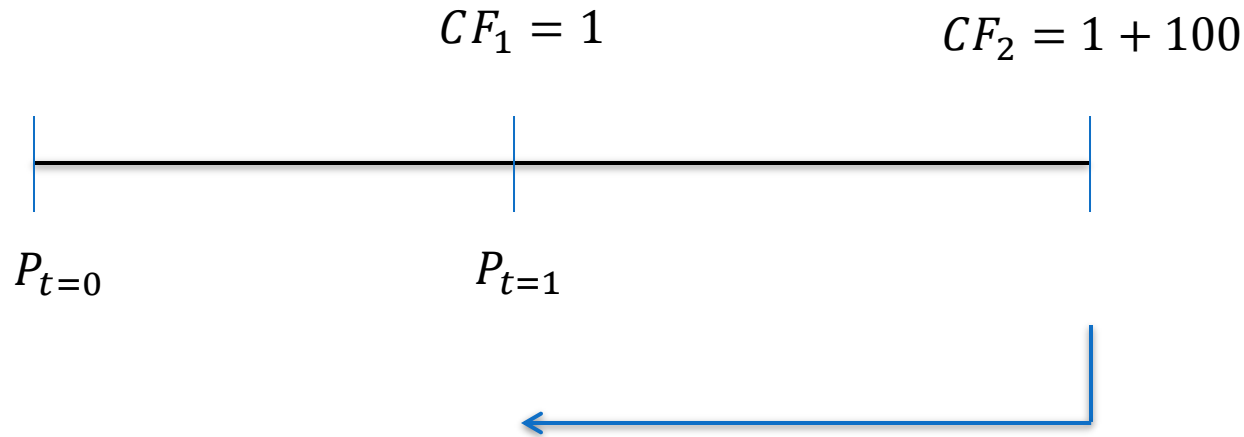
$$CF_2 = 1 + 100$$



Price at time $t = 1$

$$V_{t=1} = \frac{(1 + 100)}{(1 + 0.02)} = 99.01$$

EXAMPLE: INTEREST RATE RISK



$$P_{t=1} = \frac{1 + 100}{1 + 0.02} = 99.01 \Rightarrow \text{Cap Loss} = -0.09\%$$

$$\begin{aligned} \text{Rate of return} &= \text{Current yield} + \text{Cap Loss} \\ &= 1\% + (-0.09\%) \\ &= 0.01\% \end{aligned}$$

EXAMPLE 1: MEASURING RATE OF RETURN

- What if lower market rate to 0%?
- With the same bond, if market interest rate is lowering to 0% next year, bond returns would be

Current yield: 1%; Capital gain = 1%

Bond return = 2%.

- We can notice that uncertainty (risk) in bond returns is really associated with movements/volatilities in market interest rate.

BEHAVIOR OF INTEREST RATE RISK

Uncertainty in bond returns depends on...

1. Interest rate risk “*increases*” with “Time to maturity: TTM” because *long-term bond price is more fluctuate than short-term bond price.*

EXAMPLE 2: FLUCTUATIONS OF PRICE OF LONG-TERM BOND

Question The same fund manager considers changing his investment plan to long-term investment with **an 11-year bond** paying 1% coupon. Suppose that the market interest rate of 1%. Calculate the one-year holding return if the interest rate increases to 2% next year.

Note: everything the same as before, but only TTM increases from 2 years to 11 years.

EXAMPLE 2: INTEREST RATE RISK

Current price _____

Price in next year if interest rate increases to 2%

$$P_{t=1} = 1 * PVIFA(2\%, 10) + 100 * PVF(2\%, 10)$$

One-year rate of return =

THE LONGER TERM BOND; THE HIGHER INTEREST RATE RISK

Table 2 One-Year Returns on Different-Maturity 10%-Coupon-Rate Bonds When Interest Rates Rise from 10% to 20%

(1) Years to Maturity When Bond Is Purchased	(2) Initial Current Yield (%)	(3) Initial Price (\$)	(4) Price Next Year* (\$)	(5) Rate of Capital Gain (%)	(6) Rate of Return (2 + 5) (%)
30	10	1,000	503	-49.7	-39.7
20	10	1,000	516	-48.4	-38.4
10	10	1,000	597	-40.3	-30.3
5	10	1,000	741	-25.9	-15.9
2	10	1,000	917	-8.3	+1.7
1	10	1,000	1,000	0.0	+10.0

*Calculated using Equation 3.

MEASURING THE INTEREST RATE RISK

- Financial economists have developed some indicators that can be used to measure the degree of interest rate risk.
- A commonly used approach to measure the interest rate risk is the **Macaulay Duration (MacD)**.
 - Developed by Frederick Macaulay (1938)
- The measure highlights at measuring the **sensitivity of bond price to interest rate changes**.
 - Bond returns are largely driven by price changes.

MEASURING THE INTEREST RATE RISK: MACD

➤ Suppose that bond is matured T-period from now.

$$\text{MacD} = \sum_{t=1}^T t \frac{PV_t}{P_{t=0}} = 1 \frac{PV_1}{P_{t=0}} + 2 \frac{PV_2}{P_{t=0}} + \dots + T \frac{PV_T}{P_{t=0}}$$

- PV_t = present value of the cash flow receivable t-period from now.
- $P_{t=0}$ = current price

➤ Note that $\frac{PV_1}{P_{t=0}} + \frac{PV_2}{P_{t=0}} + \dots + \frac{PV_T}{P_{t=0}} = 1 \rightarrow \frac{PV_t}{P_{t=0}}$ is weight associated to period t payment.

MEASURING THE INTEREST RATE RISK

- Following the MacD formula, the value varies with “T”

$$\text{MacD} = \sum_{t=1}^T t \frac{PV_t}{P_{t=0}} = 1 \frac{PV_1}{P_{t=0}} + 2 \frac{PV_2}{P_{t=0}} + \dots + T \frac{PV_T}{P_{t=0}}$$

- One can see that $\frac{PV_T}{Price} = \frac{C+FV}{price}$ is the largest term!
- Therefore, “T” will be receiving the highest weight when one sums up the number of periods of payment.

MEASURING THE INTEREST RATE RISK

Why higher MacD implies higher?

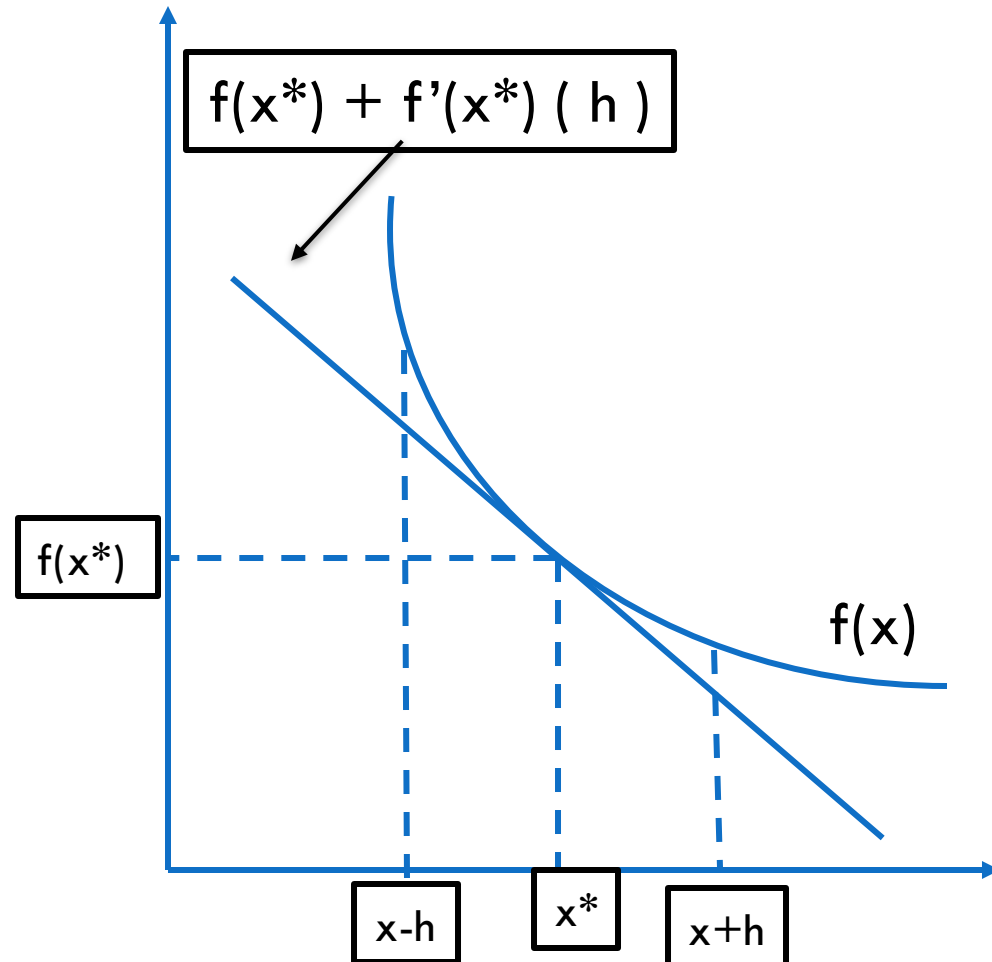
- MacD is an input (a parameter) of the **linear-approximation formula** that is used to measure/estimate the % change in bond price, with respect to market interest rate.
- See the derivation.

LINEAR APPROXIMATION

Let $f(x)$ be a differentiable function, with a known value of $f(x^*)$ at $x = x^*$.

$$f(x^*+h) \approx f(x^*) + f'(x^*) (h)$$

$$\Delta y \approx f'(x^*) \Delta x$$



MEASURING THE INTEREST RATE RISK

Following the Bond pricing, consider a coupon bond

$$P = \frac{C}{1+r} + \frac{C}{(1+r)^2} + \dots + \frac{C+FV}{(1+r)^T} \quad 1$$

$$\frac{dP}{dr} = -1 \frac{C}{(1+r)^2} - 2 \frac{C}{(1+r)^3} + \dots - T \frac{C+FV}{(1+r)^{T+1}} \quad 2$$

$$\frac{dP}{dr} = \left\{ -1 \frac{C}{(1+r)} - 2 \frac{C}{(1+r)^2} + \dots - T \frac{C+FV}{(1+r)^T} \right\} \times \frac{1}{1+r} \quad 3$$

$$\frac{dP}{dr} = (-1 * PV_1 - 2 * PV_2 + \dots - T * PV_T) \times \frac{1}{1+r} \quad 4$$

MEASURING THE INTEREST RATE RISK

$$\Delta P \approx \left[(-1 * PV_1 - 2 * PV_2 + \dots - T * PV_T) \times \frac{1}{1+r} \right] \Delta r$$

$$\frac{\Delta P}{P_{t=0}} \approx \left(-1 * \frac{PV_1}{P_{t=0}} - 2 * \frac{PV_2}{P_{t=0}} + \dots - T * \frac{PV_T}{P_{t=0}} \right) \times \frac{\Delta r}{1+r}$$

$$\frac{\Delta P}{P_{t=0}} \approx - \frac{(MacD)}{1+r} \Delta r$$

*negative sign \rightarrow inverse relationship between “r” and “P”

MEASURING THE INTEREST RATE RISK

- $\frac{(MacD)}{1+r}$ is called the modified duration, or ModD.
- ModD is the correct measure of **elasticity of bond price to market interest rate.**
- Higher MacD → Higher ModD
 - MacD cannot inform investors about the size of changes.

EXAMPLE 3: MACD AND MODD

Coupon rate	=	8.00%
Term	=	5 years (semi-annual interest payment)
Yield-to-maturity	=	8.00%
Price	=	100

Period (t)	Cash flow	PVCF	t x PVCF _t
1	\$4.0	3.8462	3.8462
2	4.0	3.6982	7.3964
3	4.0	3.5560	10.6680
4	4.0	3.4192	13.6769
5	4.0	3.2877	16.4385
6	4.0	3.1613	18.9675
7	4.0	3.0397	21.2777
8	4.0	2.9228	23.3821
9	4.0	2.8103	25.2931
10	104.0	70.2586	702.5867
Total		100.0000	843.5331
Macaulay duration =			

EXAMPLE

Following the table: Calculate the % change in price if the market interest rate drops from 8% to 7%

$$\frac{(MacD)}{1 + r} \Delta r$$

$$1 + r = \underline{\hspace{2cm}}$$

$$\Delta r = \underline{\hspace{2cm}}$$

BEHAVIOR OF INTEREST RATE RISK

Depends on...

1. Interest rate risk “increases” with “Time to maturity: TTM”.
2. Interest rate risk falls when the initial level of interest rate is high.

BEHAVIOR OF INTEREST RATE

$$\frac{(MacD)}{1 + r \downarrow} \Rightarrow ModD \uparrow$$

- We already knew that long-term bond contains high price risk.
- *MacD* increases with TTM → higher Interest rate risk
- For a fixed “T”, lower “r” implies higher interest rate risk!

BOND PRICE FUNCTION HAS A DECLINING SLOPE

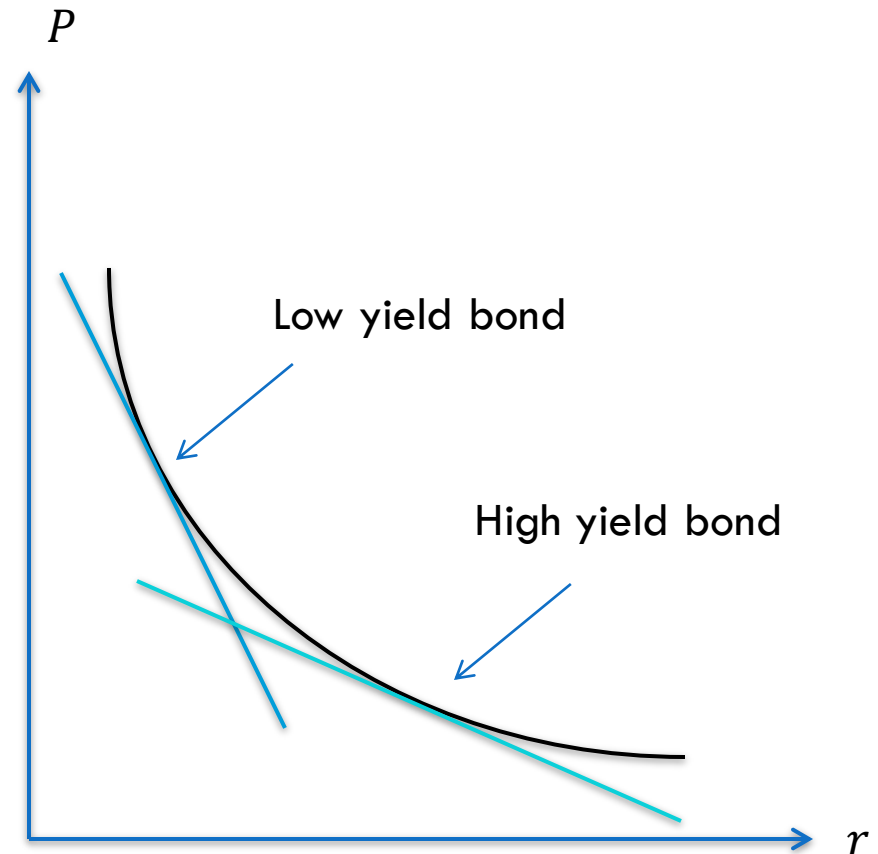
Again, bond pricing

$$P = \frac{1}{(1+r)^T}$$

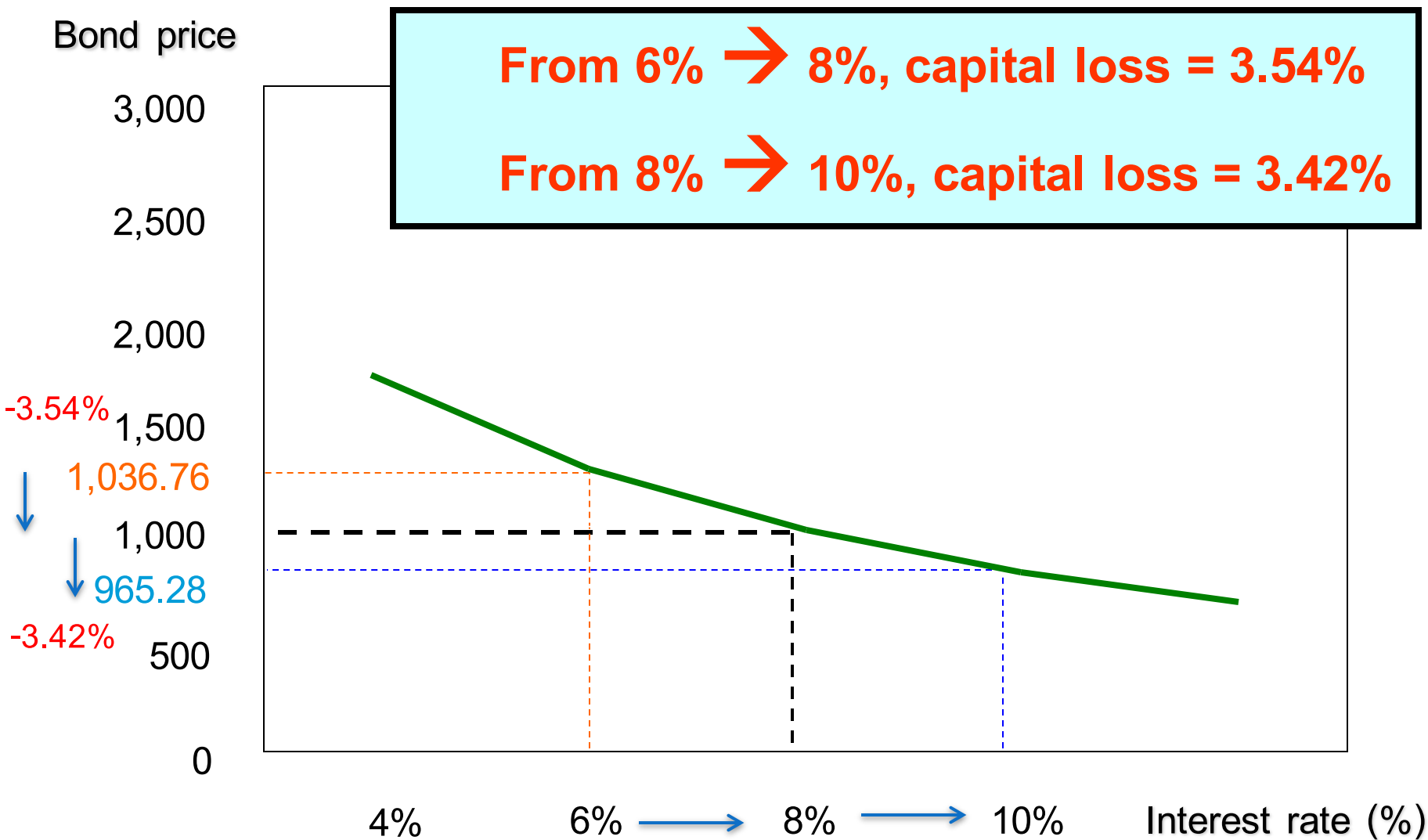
Hyperbolic function

- Convex function

- $\frac{d^2P}{dr^2} < 0$



2-year bond with coupon rate = 8% and FV = 1000.



LIMITATIONS OF THE DURATION

- The formula stems from an **approximation!**
 - Good approximate for daily trade when yield does not change dramatically (small change in yield.)
- Duration is a **linear approximation**;
 - Linearly rising and falling; **symmetric**
 - Bond risk has the **asymmetric** feature.

ASYMMETRIC FEATURE IN BOND RETURNS

Definition:

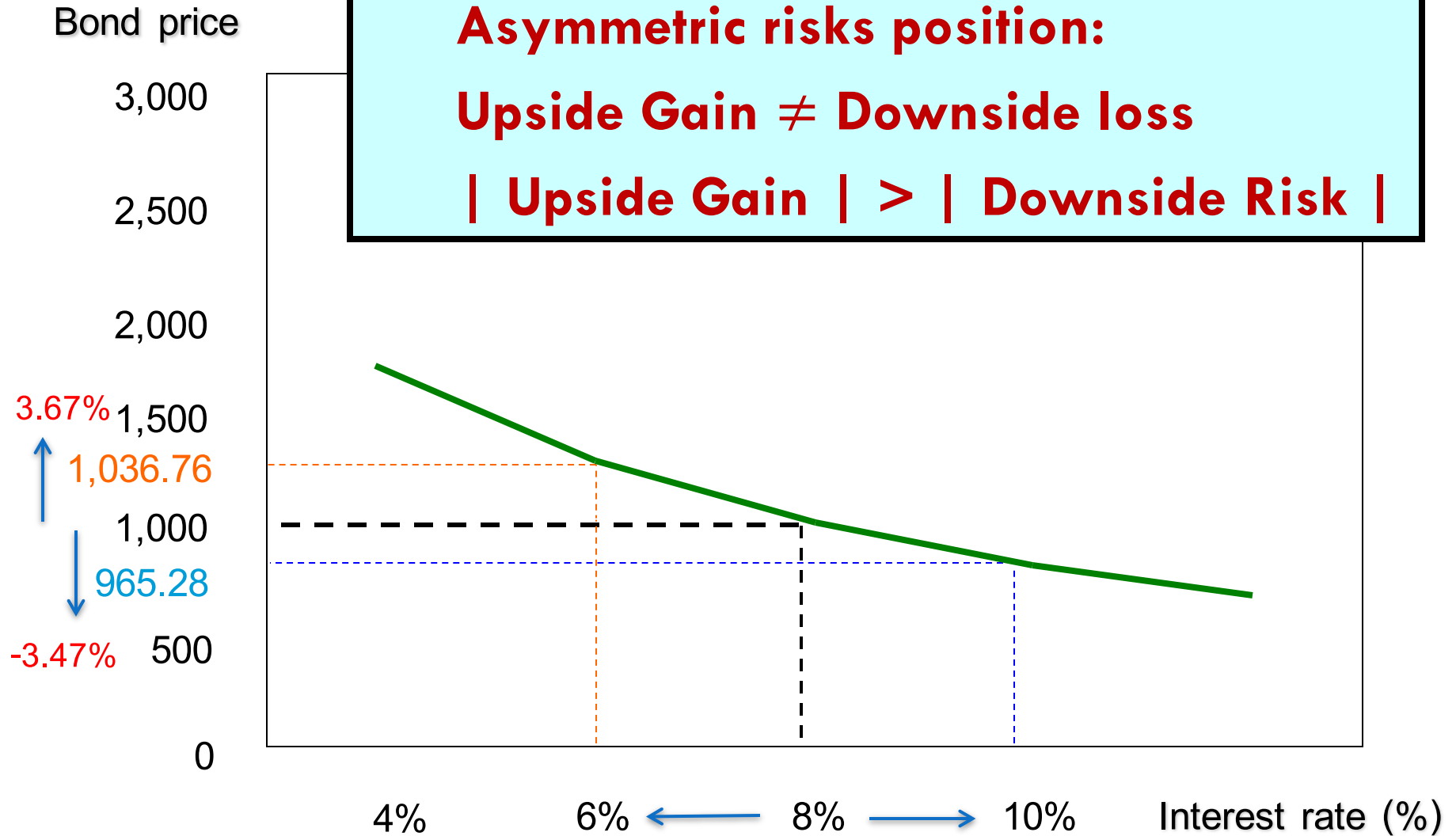
Upside gain occurs when interest rate falls.

Downside loss occurs when interest rate rises.

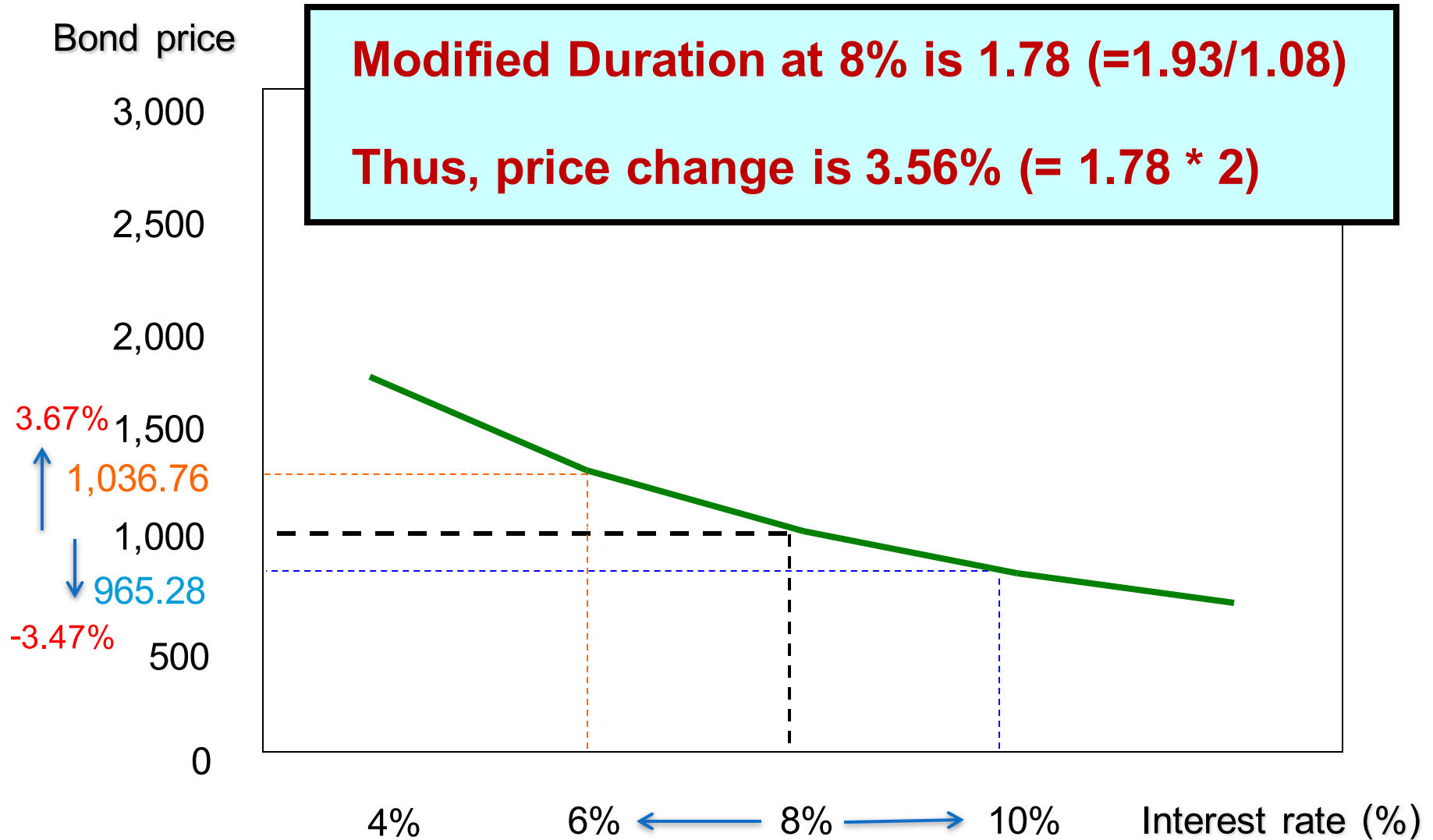
Asymmetric feature \Rightarrow returns under the upside gain is NOT equal to returns under the downside loss.

Coupon rate = 8% and FV = 1000.

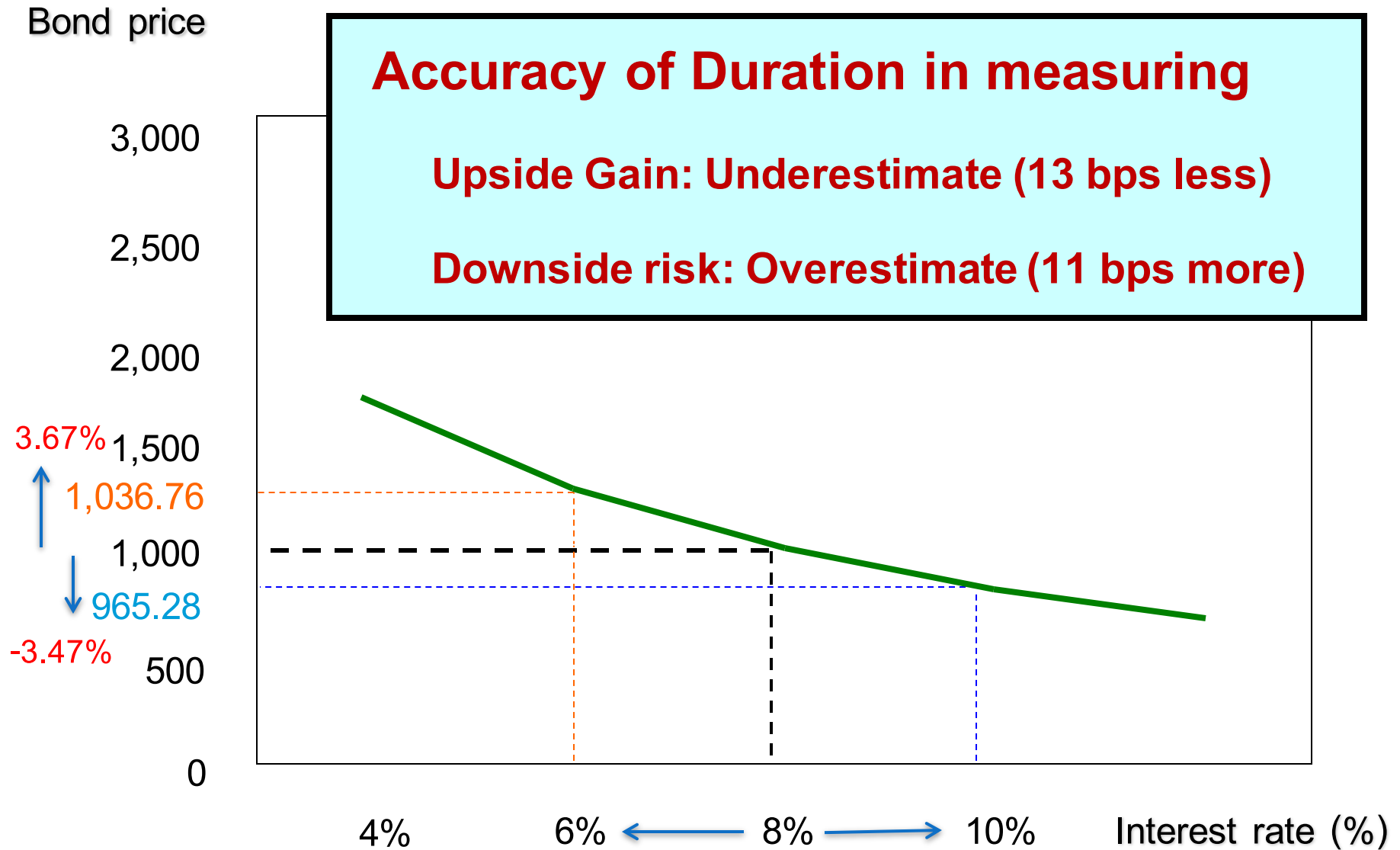
**Asymmetric risks position:
Upside Gain \neq Downside loss
| Upside Gain | $>$ | Downside Risk |**



Accuracy of *Duration* in measuring risks

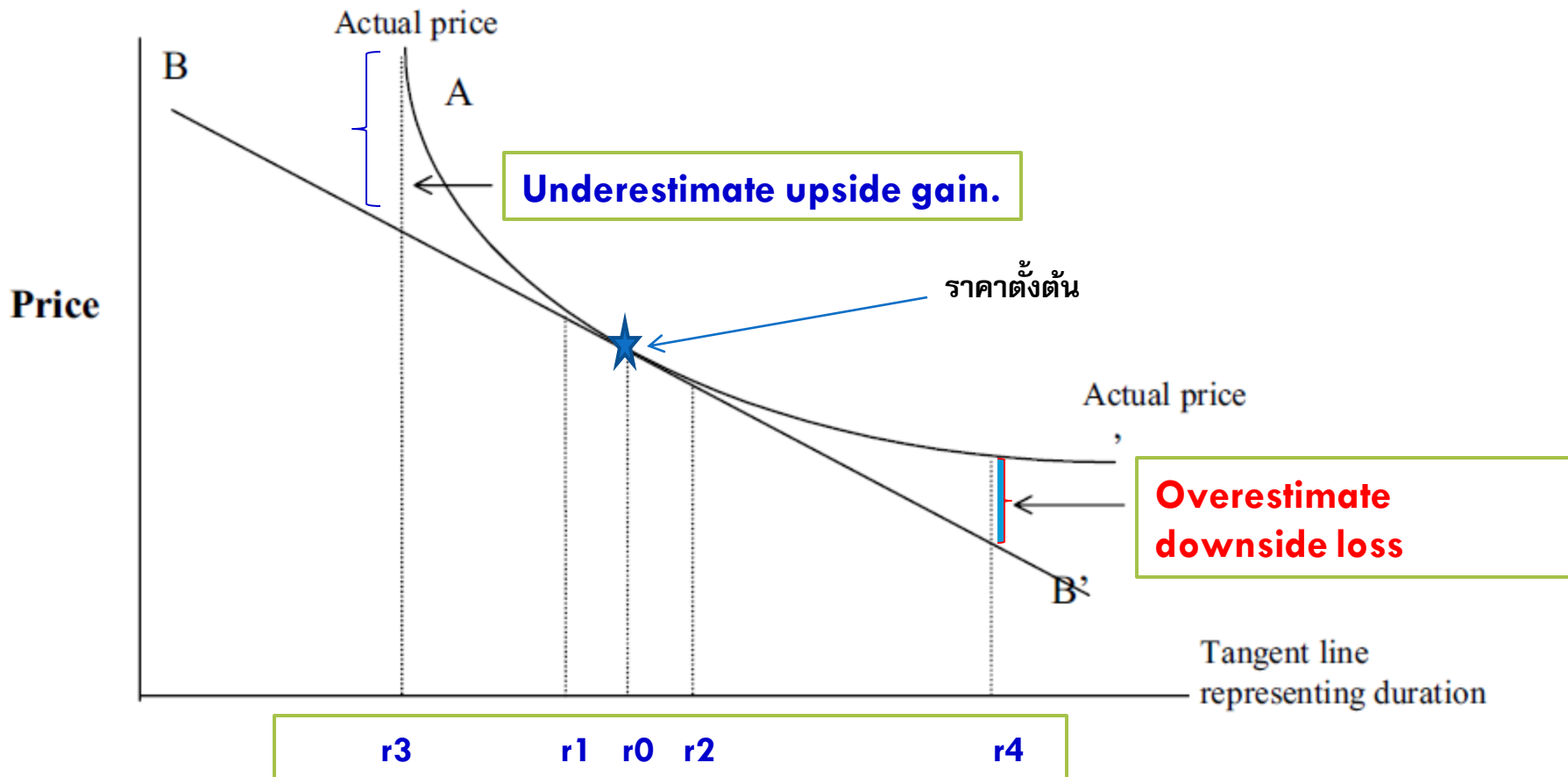


Accuracy of *Duration* in measuring risks



CONVEXITY AND PRICING ERRORS

Error in risk calculation: Slope of BB' is the Modified duration



A BETTER MEASURE OF RISK FORMULA

Using the higher order approximation; second-order Taylor approximation

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2} f''(x_0)(x - x_0)^2$$

$$\Delta y \approx \frac{dy}{dx} \Delta x + \frac{1}{2} \frac{d^2y}{dx^2} (\Delta x)^2$$

Intuitively, we approximate the original function with a quadratic function.

BOND PRICING APPROXIMATION

- Now treat the bond price formula as the function of “r”

$$P = f(r) = \frac{C}{1+r} + \frac{C}{(1+r)^2} + \dots + \frac{C + FV}{(1+r)^T}$$

$$\Delta P \approx \frac{dP}{dr} \Delta r + \frac{1}{2} \frac{d^2P}{dr^2} (\Delta r)^2$$

$$\frac{\Delta P}{P} \approx \frac{dP}{dr} * \frac{1}{P} \Delta r + \frac{1}{2} \frac{d^2P}{dr^2} * \frac{1}{P} (\Delta r)^2$$

BOND PRICING APPROXIMATION: CONVEXITY FORMULA

$$\frac{\Delta P}{P} \approx \frac{dP}{dr} * \frac{1}{P} \Delta r + \frac{1}{2} \frac{d^2 P}{dr^2} * \frac{1}{P} (\Delta r)^2$$

- $\frac{dP}{dr} * \frac{1}{P} = -\frac{(\text{MacD})}{1+r_0}$
- $\frac{d^2 P}{dr^2} * \frac{1}{P} = \text{convexity}$

$$\text{Convexity} = \frac{\sum_{t=1}^T t(t+1) \frac{PV_t}{\text{Price}}}{(1+r_0)^2}$$

$$= \frac{\left[1(2) \frac{PV_1}{P} + 2(3) \frac{PV_2}{P} + \dots + T(T+1) \frac{PV_T}{P} \right]}{(1+r_0)^2}$$

EXAMPLE: UPSIDE GAIN

Coupon rate	=	8.00%
Term	=	5 years (semi-annual interest payment)
Yield-to-maturity	=	8.00%
Price	=	100

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10	104.0	70.2586	702.5867
Total		100.0000	843.5331

Linear approximation

4.055%

Non-linear approximation

4.055% + 0.1009%

Upside gain = 4.1564

Macaulay duration =

EXAMPLE: DOWNSIDE LOSS

Coupon rate	=	8.00%
Term	=	5 years (semi-annual interest payment)
Yield-to-maturity	=	8.00%
Price	=	100

Period (t)	Cash flow	PVCF	t x PVCF _t
1	\$4.0	3.8462	3.8462
2	4.0	3.6982	7.3964
3	4.0	3.5560	10.6680
4	4.0	3.4192	13.6769
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8	4.0	2.9228	23.3821
9	4.0	2.8103	25.2931
10	104.0	70.2586	702.5867
Total		100.0000	843.5331

Linear approximation

4.055%

Non-linear approximation

-4.055% + 0.1009%

Upside gain = -3.9546%

Macaulay duration =

CONCLUSIONS

- Difference between YTM and bond returns
- Measuring market-price risk of bond
- Duration as the approximation of interest rate risk
- Convexity gives a more accurate approximate to the interest rate risk, both upside and downside perspective.