EE 431/ EE438 Economics of Financial Markets and Institution
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Assignment 6: Suggested Solutions.

1. *The demand for loan and deposit be given by* $L = \frac{1}{r_L^{\frac{1}{\alpha}}}$ and $D = r_D^{\frac{1}{\varepsilon}}$, *where L is the loan demand,* r_L *is the lending interest rate, D is the deposit demand, and* r_D *is the deposit rate. In addition, let q be the probability of default. Meaning even though the bank lend out L amount of loans, it only recovers (1-q)L of the loan back.*
   a. *Before doing anything, what do you think the default rate will do to the lending rate?*
   Higher default rate should lead to higher lending rate. Since banks make less on each dollar loaned out, it needs to charge more to make enough to cover the loss income from non-performing loans.

   b. *Setup the monopolist problem and solve for the profit-maximum lending rate the monopolist will charge. What is the relationship of default rate and the lending rate?*

The profit of a monopolist looks as follows: $\pi = r_L(1-q)L(r_L) + r_B B - r_D D(r_D)$ where the balance sheet identity is $L(r_L) + B = (1-k)D(r_D)$. The monopolist substitutes the identity for *B* in the profit and chooses the interest rates to maximize profits:

$$\max_{r_L, r_D} r_L(1-q)L(r_L) + r_B\left((1-k)D(r_D) - L(r_L)\right) - r_D D(r_D)$$

Substituting the demand functions in, we get:

$$\max_{r_L, r_D} r_L \frac{(1-q)}{r_L^{\frac{1}{\alpha}}} + r_B\left((1-k)r_D^{\frac{1}{\varepsilon}} - \frac{1}{r_L^{\frac{1}{\alpha}}}\right) - r_D r_D^{\frac{1}{\varepsilon}}$$

Differentiating with respect to r_L to get:

$$(1-q)(1-\frac{1}{\alpha})r_L^{-\frac{1}{\alpha}} + \frac{r_B}{\alpha r_L^{1+\frac{1}{\alpha}}} = 0 \Rightarrow r_L = \frac{r_B}{(1-\alpha)(1-q)}.$$

As expected, higher *q* leads to higher lending rate.

   c. *Solve for the deposit rate charge by the monopolist bank.*

Differentiating with respect to r_D to get:

$$\frac{r_B(1-k)}{\varepsilon} r_D^{\frac{1}{\varepsilon}-1} - \left(1+\frac{1}{\varepsilon}\right) r_D^{\frac{1}{\varepsilon}} = 0 \Rightarrow r_D = \frac{(1-k)r_B}{1+\varepsilon}.$$

2. *Re-derive and complete the analysis of the Cournot competition in banking. Particular, find the expression of the deposit rate and the margin of intermediation (interest spread between lending rate and deposit rate).*

We start from what's already in the lecture:

$$\pi_1 = r_L L_1 + r_B \left(D_1(1-k) - L_1\right) - r_D D_1$$

Use loan and deposit demand, to rewrite:

$$\pi_1 = \left(b - a(L_1 + L_2)\right) L_1 + r_B \left(D_1(1-k) - L_1\right) - \left(d + c(D_1 + D_2)\right) D_1$$

Differentiation yields:

$$b - aL_2 - r_B = 2aL_1$$

and

$$r_B(1-k) - d - cD_2 = 2cD_1$$

After re-writing the equations, we get:

$$L_1^* = \frac{b - aL_2 - r_B}{2a} \text{ and } D_1^* = \frac{r_B(1-k) - d - cD_2}{2c}.$$

And by symmetry of the firm's structure

$$L_2^* = \frac{b - aL_1 - r_B}{2a} \text{ and } D_2^* = \frac{r_B(1-k) - d - cD_1}{2c}.$$

Solving together we find that:

$$L_1^* = L_2^* = \frac{b - r_B}{3a}$$

$$\text{so that } L^* = L_1^* + L_2^* = \frac{2\left(b - r_B\right)}{3a}$$

$$r_L = b - a\left(\frac{2\left(b - r_B\right)}{3a}\right)$$

$$r_L = \frac{b}{3} + \frac{2}{3} r_B.$$

For deposits, we also find that:

$$D_1^* = D_2^* = \frac{(1-k)r_B - d}{3c}$$

$$\text{so that } D^* = D_1^* + D_2^* = \frac{2\left((1-k)r_B - d\right)}{3c}$$

$$r_D = d + c\left(\frac{2\left((1-k)r_B - d\right)}{3c}\right)$$

$$r_D = \frac{d}{3} + \frac{2}{3}(1-k) r_B.$$

Subtracting the two interest rate equations, we arrive at the margin for intermediation:

$$r_L - r_D = \frac{b - d}{3} + \frac{2}{3} k r_B.$$