Linear economics model: Static and Comparative static Equilibrium analysis

EE320

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What we have seen in previous lecture

- How to characterize relationship of economic variables using mathematical function/equation.
- Properly define the function/equation.

Where we are headed? Now…

- Write down some basic linear economic models (in mathematical form)
  - Set of system of equations.
  - Interlink among variables in the model.
- Solve for the solution of the model
  - (Simultaneous) Solution to the system of equations
- Analyze behavior of the endogenous equilibrium solutions

Topic for applications/examples

- Micro-market equilibrium model
  - Single market equilibrium (Partial analysis)
  - Multi-market equilibrium (General analysis)
- Macroeconomic model
  - Keynesian cross
  - IS-LM model

Review: system of equations

- Comprises of N-unknown variables **and** M-equations.
  - Equations could be linear, nonlinear, or mixed.
  - But we now focus on linear system.
- Set of N-unknown variables that simultaneously satisfies all the M equations, at the same time, is called solution of the system.

Review: System of equations: solution

- For now, we will only focus on
  - 2 x 2 case
  - 3 x 3 case
- These two cases are solvable by pencil and pen.
- For any larger scale of the system of equations, we have to apply matrix algebra to solve for the solution. (next chapter)

Review: System of equations: solution

- At least, make sure you know how to solve for the solution of 2 x 2 case.

- Example:

  $X + Y = 7$

  $3X - 6y = -15;$

Answer: $X = 4$ and $Y = 3$

Review: Solving for the solution of system of equations

- Types of solution
- **Example:**
  - System 1: $2x + 3y = 7$ and $x - y = 5$.
  - System 2: $y = -2x + 6$ and $4x + 2y = 12$.
  - System 3: $x + 2y = 14$ and $3x + 6y = 8$

- System 1 has a __Unique_solution.
- System 2 has a _multiple_ solution.
- System 3 _NO solution___.

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Market (partial) equilibrium model

- What is market equilibrium model?
- A simple illustrative example.
- Individual v.s. market demand/supply.
- Government intervention and some analysis.

What is market equilibrium model?

- The model used to predict equilibrium price and output.
  – At what price, people trade?
  – How much do they trade?
- Two concepts: partial vs general
  – One particular market, ignore the rest.
  – All the interlinked markets are taken into consideration at the same time.

Partial

$$Q_d^x = a - bP_x + cP_y$$
$$Q_s^x = d + eP_x$$

P_y are given exogenously

General

$$Q_d^x = a - bP_x + cP_y$$
$$Q_s^x = d + eP_x$$

$$Q_d^y = f - gP_y + hP_x$$
$$Q_s^y = j + kP_y$$

(P_x, P_y) are solved for together.

Market (partial) equilibrium model

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Partial equilibrium model

Suppose that *market* demand and supply equation can be given by the following two equations:

$$Q_d = 30 - 5Y - 3P$$

and $P = 2 + Q_s$

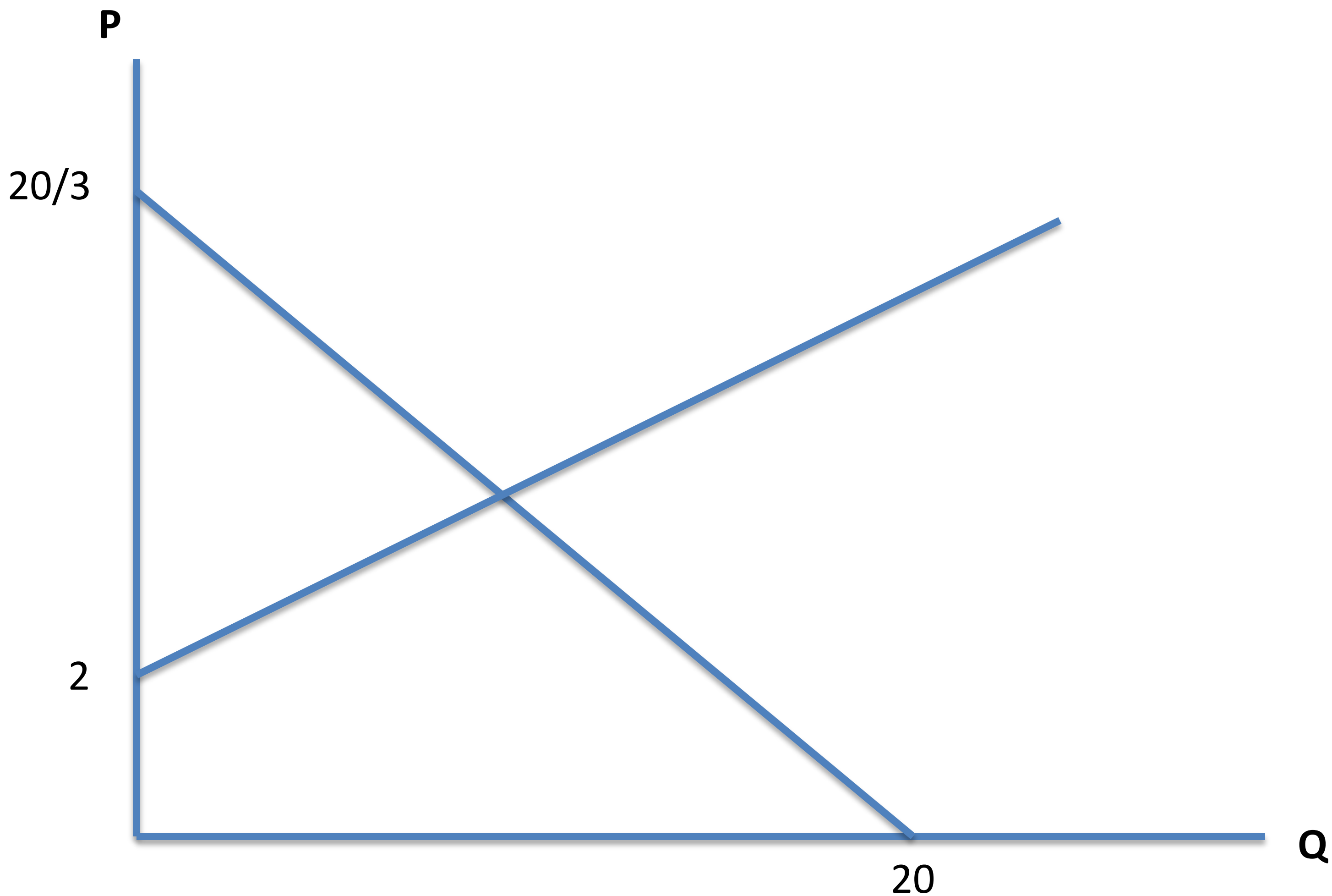
a. Graph the demand and supply curve for Y = 2
b. Is the product in this question normal/inferior goods?
c. At what level of income, does the demand function coincide with the equation $Q_d = 10 - 3P$?
d. Find the market equilibrium for quantity of output and price, using the demand equation given in “c”.
e. What happen to equilibrium output/price when income is $1 higher than the level associated to that in “c”?

a. Graph the demand and supply curve when Y = 2

- Whenever you're asked to draw demand/supply curve, always rewrite any given form of demand/supply equations into the p-equal form.

- Demand ➔ $P = (20/3) - (1/3)Q$
- Supply ➔ $P = 2 + Q$

a.Graph the demand and supply curve when Y = 2



b.Is the product in this question normal/inferior goods?

$$Q_d = 30 - 5Y - 3P$$

- Negative coefficient means “inversely” relate.
- This is inferior goods.

c. At what level of income, does the demand function coincide with the equation $Q_d = 10 - 3P$?

Qd = 30 – 5Y - 3P

Qd = 10 - 3P

So, we must have that

30 – 5Y = 10

Income = 4.

d. Find the market equilibrium for quantity of output and price, using the demand equation given in “b”.

- A market is said to be in the equilibrium if $Qd = Qs$.
- Substituting “Qd” equation into “Qs”, we yield that:

$$P = 2 + 10 - 3p$$

$$4p = 12 \rightarrow p = 3.$$

$$Q = 1.$$

- Equilibrium output = $1 and Equilibrium price = 3 unit.

e. What happen to equilibrium output/price when income is $1 higher than the level associated to that in "b"?

Income is $1 higher. That is, income is now $5.

Demand equation is then given by,

$Qd = 30 - 5*5 - 3P$

$Qd = 5 - 3P$

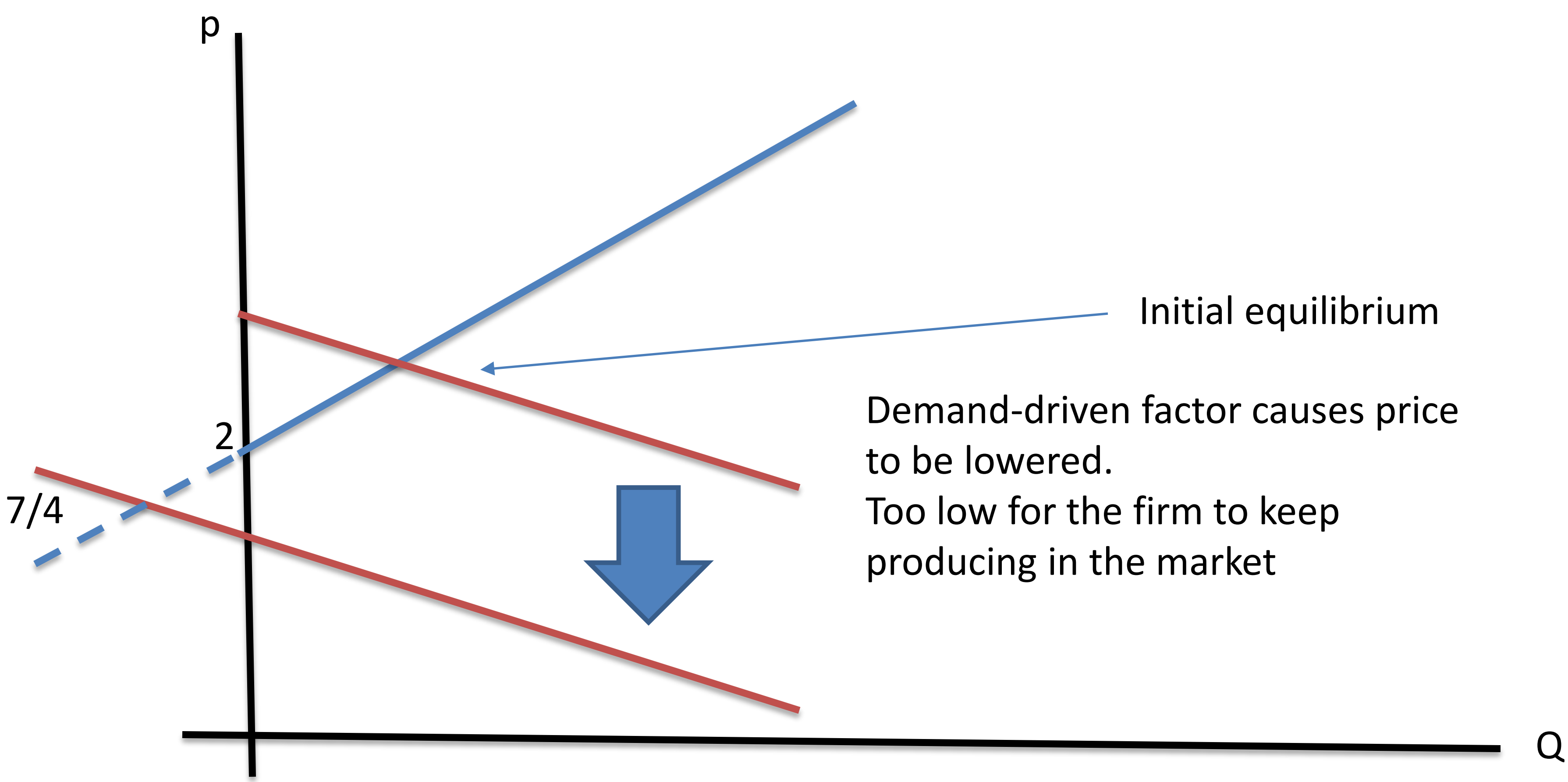
Now solve for the equilibrium.

$P = 2 + 5 - 3P$

$4p = 7 \Rightarrow p = 7/4 \Rightarrow Q = -1/4$

Market doesn't exit. Demand is getting too low after income has increased.

p = 7/4 ➔ Q = -1/4 ????



Demand-driven factor causes price to be lowered.
Too low for the firm to keep producing in the market

Market (partial) equilibrium model

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Individual vs market demand/supply

- Previous example: market demand/supply given.
- What if they are not given?
  - We have to derive the market demand/supply.
  - How then?
- If we know some information about individual demand/supply, we can then derive the equation for market demand/supply.

Market demand/supply

- Market demand:
  – Conceptually, it is the sum of the quantity demanded for each individual/buyer/consumer in the market.
- Market supply:
  – Conceptually, it is the sum of the quantity supplied for each producer/firm in the market.

Market demand/supply

- Two approaches:
  - Direction summation approach
  - Horizontal summation approach

Example: direct summation

Consumer 1: $Q_1^d = 3 - P$

Consumer 2: $Q_2^d = 2 - P$

Derive the market demand equation, given that Q^d is the total quantity demanded in the market.

Example: direct summation

- By definition $Q^d = Q_1^d + Q_2^d$.

$$Q^d = 3 - P + 2 - P.$$

$Q^d = 5 - 2P$ ➔ market demand equation

Example: direct summation

- DEAD WRONG!!!
- Why?
- Consider P = \$2.5

$$Q^d = 3 - P + 2 - P$$

$$Q^d = 3 - 2.5 + 2 - 2.5 = 0.5 + -0.5$$

- For the price above \$2, consumer#2 is not in the market.

Example: direct summation

Consumer 1: $Q_1^d = \begin{matrix} 3 - P & ; & 0 \leq P \leq 3 \\ 0 & ; & P > 3 \end{matrix}$

Consumer 2: $Q_2^d = \begin{matrix} 2 - P & ; & 0 \leq P \leq 2 \\ 0 & ; & P > 2 \end{matrix}$

Consumer 1 is **high** type.

Consumer 2 is relatively **low** type.

Example: direct summation

$$Q^d = \begin{cases} 0 & ; \quad P \geq 3 \Rightarrow 0\ consumer \\ 3 - P & ; \quad 2 \leq P < 3 \Rightarrow 1\ consumer \\ 5 - 2P & ; \quad 0 \leq P < 2 \Rightarrow 2\ consumers \end{cases}$$

- Summing them up is fine! But make sure you are summing everything up for the domain that makes sense for each individual demand equation.

Example: horizontal summation

Derive the market demand equation from the market demand curve.

- Draw individual demand curves
- Horizontal sum, for varied prices.

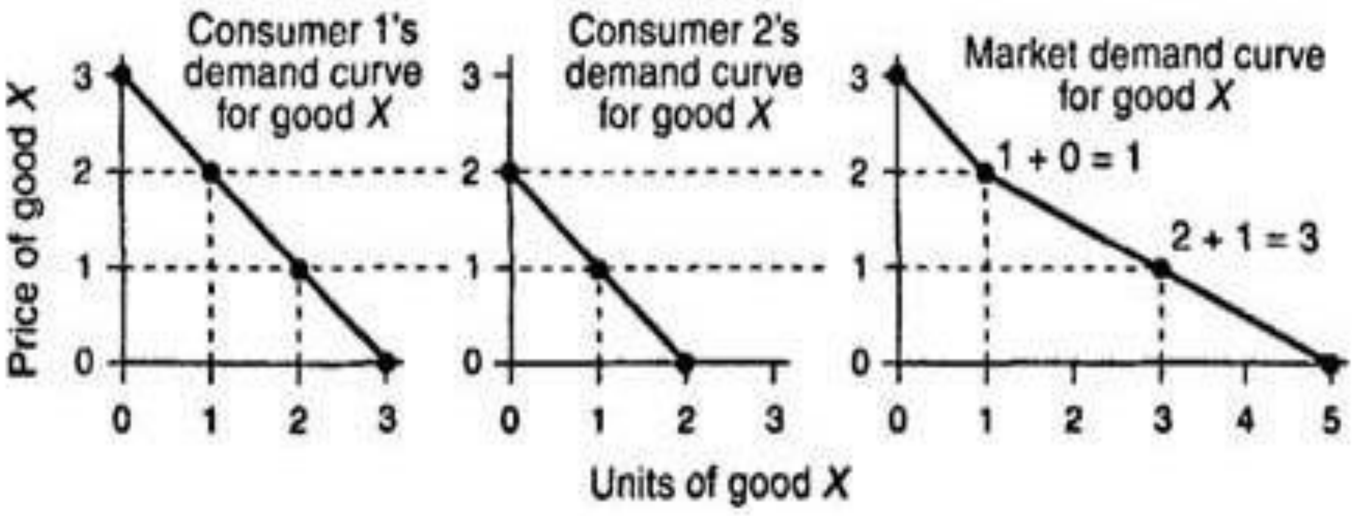
Example: horizontal summation

Consumer 1: $\mathrm{P} = 3 - Q_1^d$

Consumer 2: $\mathrm{P} = 2 - Q_2^d$

Example: horizontal summation

: $\mathbf{P = 3 - Q_1^d}$ $\mathbf{P = 2 - Q_2^d}$ *Kinked demand curve*



Derivation of the market demand curve from consumers' individual demand curves

Example: horizontal summation

- Q in between 0 and 1,
  slope is -1.

P = 3 – Q

$$P = \begin{cases} 3 - Q & ; \quad 0 \le Q < 1 \quad \Rightarrow 1 \; consumer \\ 2.5 - 0.5Q & ; \quad 1 \le Q < 5 \;\Rightarrow 2 \; consumers \end{cases}$$

- Q in between 1 and 5,
  slope is -1/2.

P = 2.5 - 0.5Q

Example: horizontal summation

- Notice that our market demand curve flatters out. That is, slope of the market demand curve, measured in the absolute term, decreases as Q increases.
- Intuitively, more consumers being attracted into the market

Example

Consider market supply equation with $P = Q^s$.

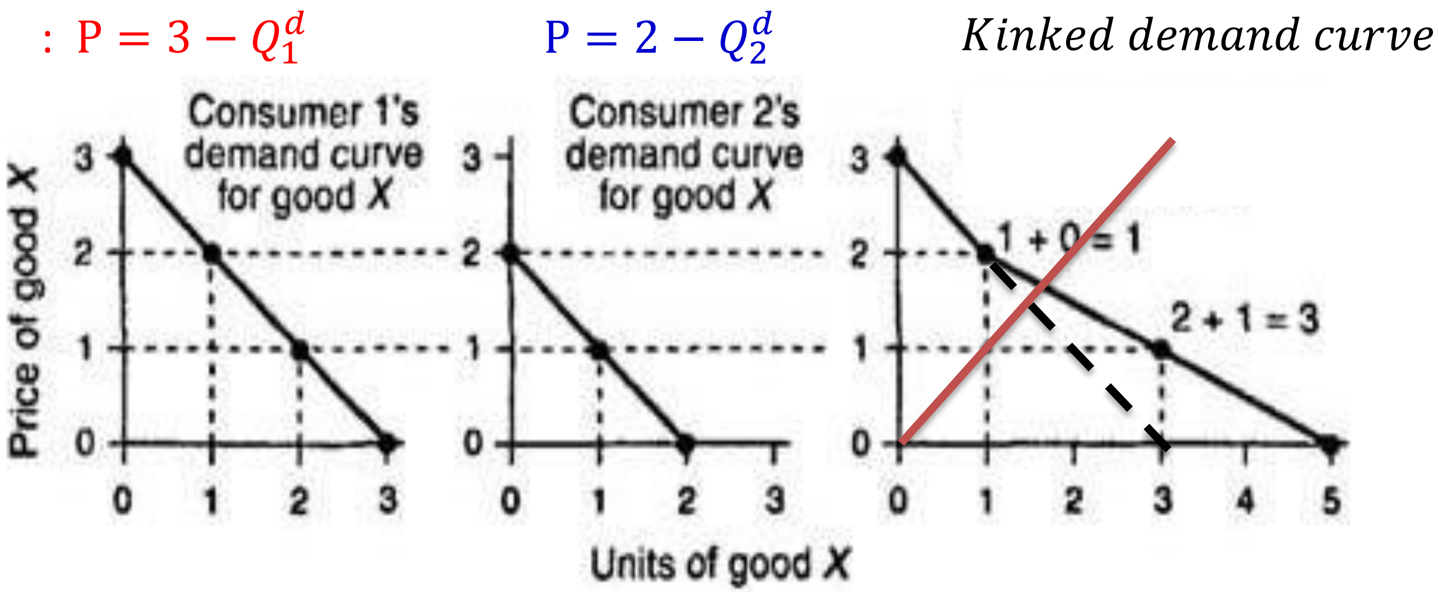
Using the two individual demand equations given before, and solve for market equilibrium.

- $Q^d = \begin{cases} 5 - 2P & ; \quad 0 \leq P < 2 \\ 3 - P & ; \quad 2 \leq P < 3 \\ 0 & ; \quad P \geq 3 \end{cases}$
- $P = Q^s$
- Segment 1: $2 \leq P < 3$

  3 – P = P ➔ P = 1.5 ➔ NO, $1.5 in not under this segment.
- Segement 2: $0 \leq P < 2$

  5 – 2P = P ➔ P = 5/3 ➔ YES, $5/3 is not under this segment.
- P* = $5/3 ➔ Q* = 5/3, Q1* = 4/3 and Q2*=1/3

Example: horizontal summation



Derivation of the market demand curve from consumers' individual demand curves

Market supply

A market has two firms, A and B. Each has the supply equation given by,

Firm A: $Qa = P - 15$

Firm B: $Qb = 0.5P - 5$

Using the direct summation approach to derive the market supply equation

Determining active domain set of price

- Active domain set:
- Firm a: P is greater than \$15. ➔ low-tech firm
- Firm b: P is greater than \$10. ➔ high-tech firm

$$Q^s = \begin{cases} 1.5P - 20 & ; \quad P > 15 \\ 0.5P - 5 & ; \quad 10 < P \leq 15 \\ 0 & ; \quad P \leq 10 \end{cases}$$

Market (partial) equilibrium model

- What is market equilibrium model?
- A simple illustrative example.
- Individual v.s. market demand/supply.
- Government intervention and some analysis.

Taxation

- Government needs revenue for financing their public projects/services.
- Types of tax
  - Unit tax ➔ \$t per each unit of production.
  - Ad-valorem tax ➔ t% of value.
- Administrative issue on taxation
  - Tax on consumer
  - Tax on producer

Taxation

- Economic issues: what is the effect of tax?
  - Tax affects market price/quantity.
  - How they split the tax cost? (tax burden)

Taxation

- With tax, price that consumer's paying will not be equal to price that producer's receiving.
- So, we need to make it clear from now that
  - $p^s = price\ that\ producer\ gets$
  - $p^d = price\ that\ consumer\ pays$
  - $p^s \neq p^d$ ➔ price gap *or* price wedges
- Behavior of buyers and suppliers would focus at their perceived prices.

Taxation

- Suppose that:
  - Demand: $p^d = a - bQ^d \quad ; \;\; a \geq 0, \quad b \leq 0.$
  - Supply : $p^s = c + dQ^s \quad ; \;\; d \geq 0.$
- Without tax, we solve for equilibrium by imposing that $Q^d = Q^s$.
- This would have been the same as we impose that $p^d = p^d$ since they both have the same perceived price.

Taxation

- As said before, when tax is introduced, government creates price gap.
- So, solving for equilibrium in the market, with tax imposed, would make sense only when (i) we impose the market clearing condition, i.e. $Q^d = Q^s$.
- Setting $p^d = p^s$ is WRONG.

Taxation

$$Q^d = Q^s = Q^*$$

$$p^d = a - bQ^*$$

$$p^s = c + dQ^*$$

- Still Unsolvable… 3 variables, but 2 equations.
- But, wait! Don't we know any relationship between p^d and p^s?

Introducing taxation

- The relationship between p^d and p^s depends on how tax is being imposed.
  - If the tax is imposed on consumer, it would be that: $p^d = p^s + t$.
  - If the tax is imposed on producer, it would be that: $p^s = p^d - t$.

Example

- Suppose that demand/supply equation can be given by

$$P^d = 20 - Q^d$$
$$P^s = 10 + Q^s$$

a. Find the market equilibrium.
b. Suppose that government imposes tax on producer equal to \$4 per unit, determine market equilibrium under taxation.
c. Redo the same exercise, but now suppose that tax is imposed on consumer for the same amount.
d. How much is the tax revenue that the government can collect in the equilibrium?
e. Calculate the tax burden.

Find the market equilibrium

$$P^d = 20 - Q^d$$

$$P^s = 10 + Q^s$$

$$20 - Q = 10 + Q$$

$$Q = 5 \Rightarrow P = 15$$

Suppose that government imposes tax on producer equal to $4 per unit, determine market equilibrium under taxation.

$P^S = P^d - 4$ ➔ $P^d = 20 - Q^d$ *--(1)*

$$P^d - 4 = 10 + Q^s \text{ --(2)}$$

Plut (1) into (2) and impose market clearing condition:

$$20 - Q - 4 = 10 + Q$$

$$Q = 3 \Rightarrow P^d = 17$$

$$P^s = P^d - 4 = 17 - 4 = 13$$

How much is the tax revenue that the government can collect in the equilibrium?

- Tax collected = tax*quantity

  = 4 * 3

  = $12

Calculate the tax burden.

- Pre-tax price is 15.
- After-tax
  - Consumer is paying $17. That's $2 more.
  - Producer is getting $13. That is $2 less.
- Thus, for $4 unit tax, each shares the burden equally. That is, the $12 of total tax revenue is economically borne upon by $6 of the consumer and $6 of the producer.

Tax on producers: general concept

Demand: $p^d = a - bQ^d \quad ; \; a \geq 0, \quad b \leq 0.$

Supply : $p^s = c + dQ^s \quad ; \quad d \geq 0.$

- $p^s = p^d - t$
- $p^s = p^d - t = c + dQ^s$
- $a - bQ^d \; - t = c + dQ^s$
- $Q^D = Q^S = Q^*$
- $Q^* = \frac{a-c-t}{b+d} = \frac{a-c}{b+d} - \frac{t}{b+d}$

Result 1: Market size contracts after the taxation.

Tax on producers

- Consumer's paying more

$$\Rightarrow p^d = \frac{ad + bc}{b + d} + \frac{b}{b + d}t$$

- Producer's getting less

$$\Rightarrow p^s = \frac{ad + bc}{b + d} - \frac{d}{b + d}t$$

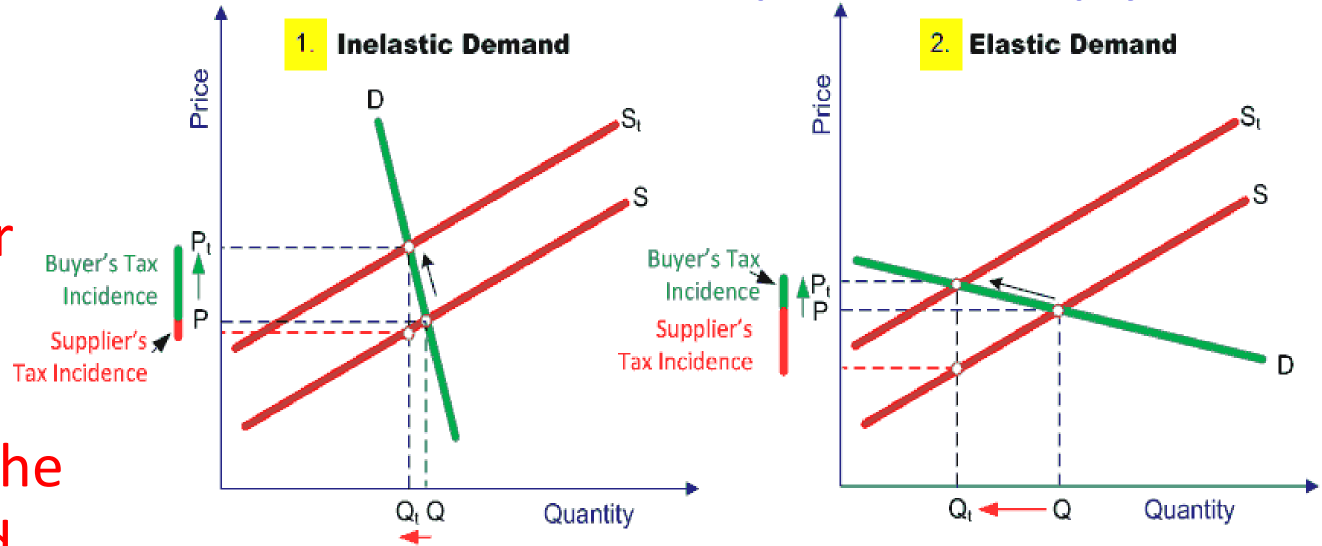
- Say t = $1, this means,
  - Fraction that consumers pay is $\frac{b}{b+d}$ cents.
  - Fraction that producers pay is $\frac{d}{b+d}$ cents.

Who pays more?

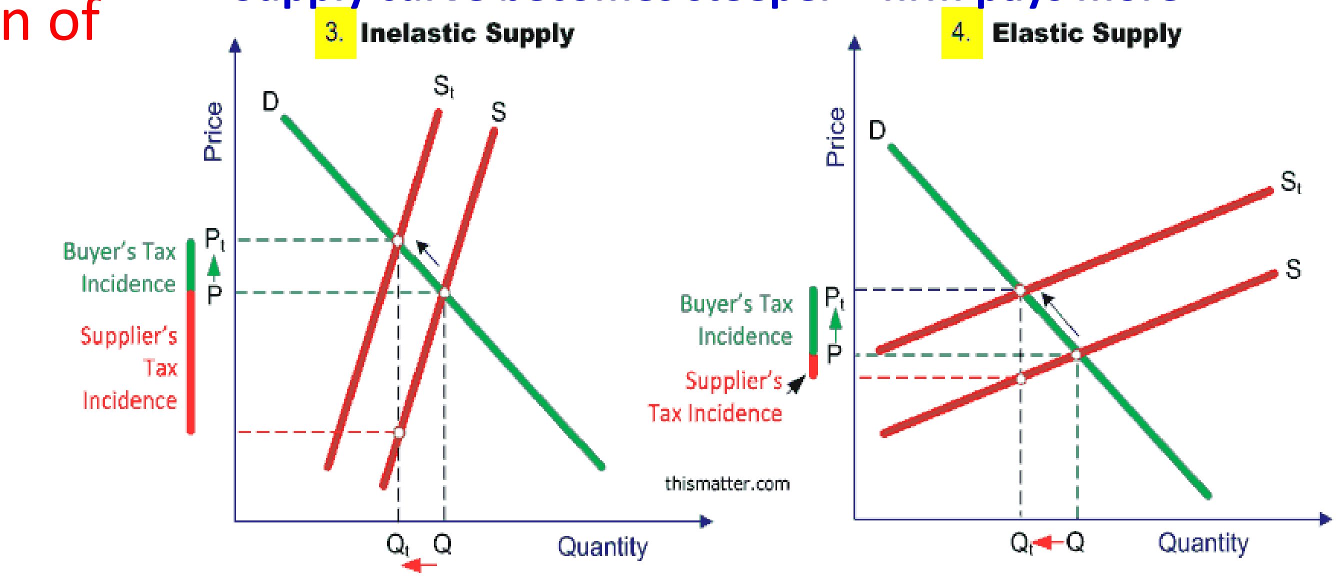
- As demand become steeper (high b), consumer's paying more!
  - Think about necessity products! The extreme cases is when "b = infinitiy". That is, demand curve is vertical line. Under this case, consumer takes all 100% of tax burden.
- As supply becomes steeper (high d), producer's paying more relatively.
  - Firm finds it difficult to adjust scale of the production.

Main Idea: Whoever cannot adjust themselves instantaneously to the new situation would unfortunately have to bear a larger portion of the tax burden.

demand curve becomes steeper = consumer pays more



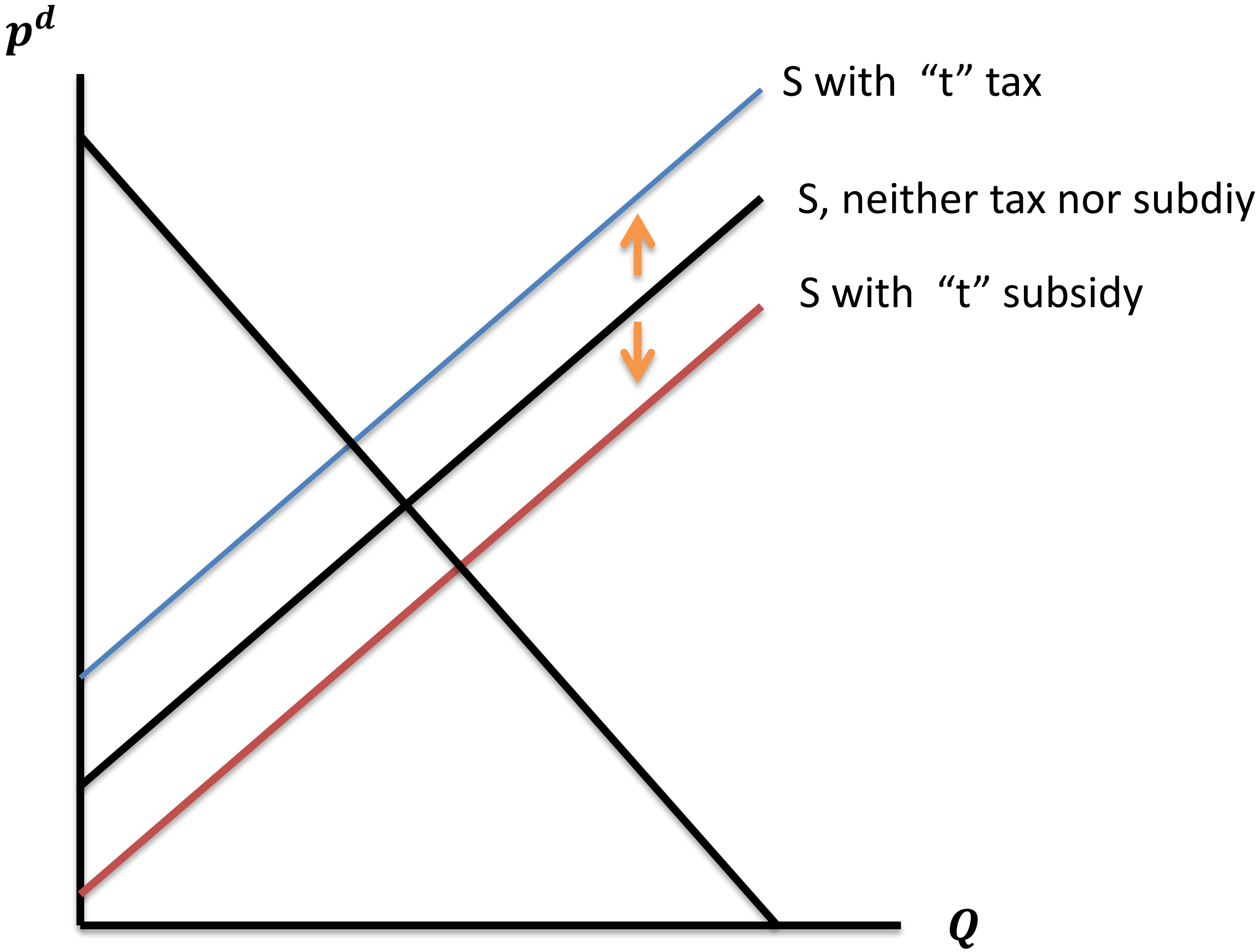
Supply curve becomes steeper = firm pays more



A thought experiment question: What about subsidy?

- What do we know about the effect of subsidy?
- Do we have to start everything from the beginning to derive the implications of subsidy program for market equilibrium?
- No, the result for subsidy would have been the same, because we can treat subsidy as negative tax. "t" is negative number!
- The burden concept would be changed into the benefit received, i.e. who's getting more benefit.

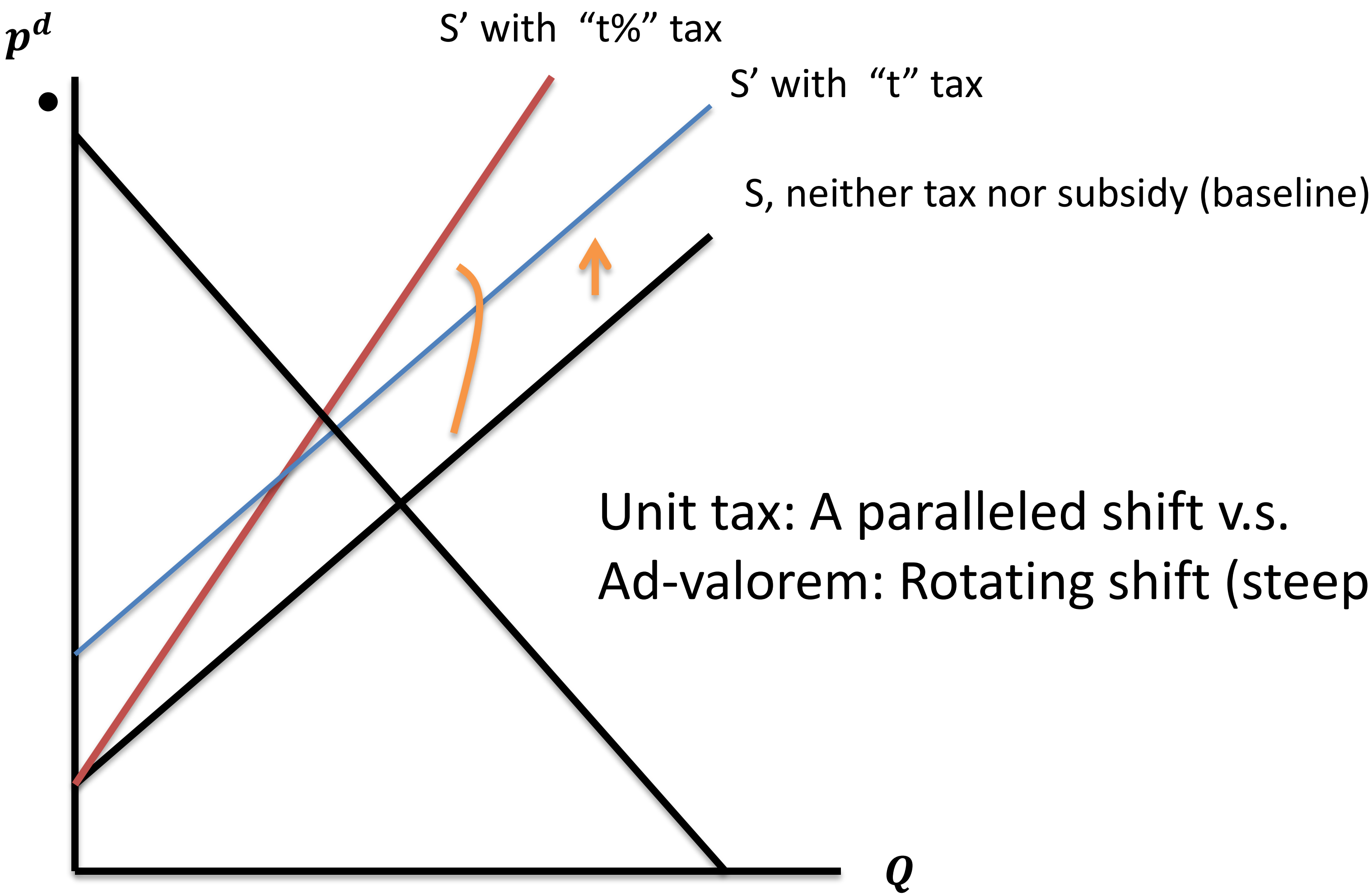
Tax v.s. Subsidy (unit-based)



Ad-valorem tax?

- What type of tax being calculated on Ad-valorem basis, i.e. by the value of goods?
- *Most* Ad-valorem taxes are imposed at the point of purchase, i.e. imposing on consumers
- If the tax is imposed on consumer, it would be that: $p^d = p^s\left(1 + \frac{t}{100}\right)$.

Unit tax v.s. Ad-valorem tax



Unit tax: A paralleled shift v.s.
Ad-valorem: Rotating shift (steeper)

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Multimarket Equilibrium

- So far, we have only analyzed the market equilibrium concept when we assume that there is only single market in the economy.
- In fact, economy comprises of so many markets.
- Equilibrium concept that we use so far can be extended to the more realistic environment where we have N markets.

Example

- Let demand and supply of market for goods A and B be

$$D_a = 20 - 3Pa - 2Pb$$
$$S_a = 12 + 2Pa + 5Pb$$
$$D_b = 4 - Pa - 3Pb$$
$$S_b = -1 + 2Pa + 4Pb$$

Answer the following questions:

- State equilibrium conditions of this multimarket model.
- Find the equilibrium price and quantity of the market for goods A and the equilibrium price and quantity of the market for goods B.

Example

Imposing two market clearing conditions:

(1) $D_a = S_a$

$$20 - 3P_a - 2P_b = 12 + 2P_a + 5P_b$$

$$5P_a + 7P_b = 8$$

(2) $D_b = S_b$

$$4 - Pa - 3Pb = -1 + 2Pa + 4Pb$$

$$3Pa + 7Pb = 5$$

$$Pa = \frac{3}{2}, \mathrm{P_b} = \frac{1}{14}$$