

## Exercise 2

1. Suppose Michael purchases only two goods, fish (F) and chips (C).
  - a. What is the relationship between  $MRS_{FC}$  and the marginal utilities  $MU_F$  and  $MU_C$ ?
  - b. Draw a typical indifference curve for the case in which the marginal utilities of both goods are positive and the marginal rate of substitution of fish for chips is diminishing. Using your graph, explain the relationship between the indifference curve and the marginal rate of substitution of fish for chips.
  - c. Suppose the marginal rate of substitution of fish for chips is constant. In this case, are fish and chips perfect substitutes or perfect complements?
  - d. Suppose that Michael always wants two orders of fish along with every sachet of chips. Draw a typical indifference curve. In this case, are fish and chips perfect substitutes or perfect complements?
  
2. Suppose a consumer's preferences for two goods can be represented by the Cobb–Douglas utility function  $U = Ax^\alpha y^\beta$ , where  $A$ ,  $\alpha$ , and  $\beta$  are positive constants. The marginal utilities are  $MU_x = \alpha Ax^{\alpha-1} y^\beta$  and  $MU_y = \beta Ax^\alpha y^{\beta-1}$ .
  - a. Is the assumption that more is better satisfied for both goods?
  - b. Does the marginal utility of  $x$  diminish, remain constant, or increase with more  $x$ ?
  - c. What is  $MRS_{XY}$ ? Interpret the value.
  - d. Is  $MRS_{XY}$  diminishing, constant, or increasing as the consumer substitutes  $x$  for  $y$  (buy more  $x$  and less  $y$ ) along an indifference curve?
  
3. At an optimal interior basket, interpret the following condition:  $\frac{MU_x}{P_x} = \frac{MU_y}{P_y}$ .
  
4. At a corner point solution, state the condition where the consumer will only buy good  $Y$ . Explain why the consumer will choose to do so (i.e., interpret the condition).
  
5. Consider the case of two goods,  $X$  (Normal Good) and  $Y$  (Giffen Good). Suppose the price of  $Y$  rises. Draw a consumer choice diagram AND show the Substitution and Income Effects, based on both Hicksian and Slutskian Approaches. Derive Hicksian demand curve and Marshallian demand curve. Also, explain how the Substitution and Income Effects work in this case.

6. Consider the case of two goods, X and Y (composite good). Let  $U(X, Y) = XY$ .  $P_y = \$1$ ,  $P_x = \$4$ , and  $I = \$72$ . Suppose  $P_x$  increases to  $\$9$ .
- Derive the demand function for X, as a function of  $P_x$  and  $I$ .
  - From the demand function, is X a normal good or inferior good? Why?
  - From the demand function, is X an ordinary good or Giffen good? Why?
  - Find the numerical values of the SE and IE, using Hicksian Approach, and draw a consumer choice diagram to show the two effects.
  - Find the numerical values of the SE and IE, using Slutskian Approach, and draw a consumer choice diagram to show the two effects.
  - Under the Slutskian Approach, how does the consumer's income get "compensated" so that his purchasing power is unchanged?
7. **(Voucher and Cash Subsidy)** Paul consumes only two goods, pizza (P) and hamburgers (H) and considers them to be perfect substitutes, as shown by his utility function:  $U(P, H) = P + 4H$ . The price of pizza is  $\$3$  and the price of hamburgers is  $\$6$ , and Paul's monthly income is  $\$300$ . Knowing that he likes pizza, Paul's grandmother gives him a birthday gift certificate of  $\$60$  redeemable only at Pizza Hut. Though Paul is happy to get this gift, his grandmother did not realize that she could have made him exactly as happy by spending far less than she did. How much would she have needed to give him in cash to make him just as well off as with the gift certificate?

**Hints**

- First, show that Paul will only consume Hamburger to maximize his utility, i.e., a corner solution.
- With a voucher from his grandma, he is going to buy some pizza (he will buy  $60/3 = 20$  pizzas). Find his utility from consuming only hamburger from the money he has and 20 pizzas from the voucher.
- Find the cash subsidy that would allow him to reach the same utility.