

Outline

1. Comparative Statics (Introduction)
2. Income Changes
3. Price Changes
4. Expenditure Minimization
5. Slutsky Equation

1 Comparative Statics (introduction)

- Nicholson, Ch. 5, pp. 145-155
- Utility maximization yields $x_i^* = x_i^*(p_1, p_2, M)$
- Quantity consumed as a function of income and price

- What happens to quantity consumed x_i^* as prices or income varies?

- Simple case: Equal increase in prices and income.

- $M' = tM, p'_1 = tp_1, p'_2 = tp_2.$

- Compare $x^*(tM, tp_1, tp_2)$ and $x^*(M, p_1, p_2).$

- What happens?

- Write budget line: $tp_1x_1 + tp_2x_2 = tM$

- Demand is homogeneous of degree 0 in \mathbf{p} and M :

$$x^*(tM, tp_1, tp_2) = t^0 x^*(M, p_1, p_2) = x^*(M, p_1, p_2).$$

- Consider Cobb-Douglas Case:

$$x_1^* = \frac{\alpha}{\alpha + \beta} M/p_1, x_2^* = \frac{\beta}{\alpha + \beta} M/p_2$$

- What is $\partial x_1^*/\partial M$?

- What is $\partial x_1^*/\partial p_1$?

- What is $\partial x_1^*/\partial p_2$?

- General results?

2 Income changes

- Income increases from M to $M' > M$.
- Budget line ($p_1x_1 + p_2x_2 = M$) shifts out:

$$x_2 = \frac{M'}{p_2} - x_1 \frac{p_1}{p_2}$$

- New optimum?

- Engel curve: $x_i^*(M)$: demand for good i as function of income M holding fixed prices p_1, p_2

- Does x_i^* increase with M ?

- Yes. Good i is *normal*

- No. Good i is *inferior*

3 Price changes

- Price of good i decreases from p_i to $p'_i > p_i$
- For example, decrease in price of good 2, $p'_2 < p_2$
- Budget line tilts:

$$x_2 = \frac{M}{p'_2} - x_1 \frac{p_1}{p'_2}$$

- New optimum?

- Demand curve: $x_i^*(p_i)$: demand for good i as function of own price holding fixed p_j and M

- Odd convention of economists: plot price p_i on vertical axis and quantity x_i on horizontal axis. Better get used to it!

4 Expenditure minimization

- Nicholson, Ch. 4, pp. 131-135; Ch. 5, pp. 155-157
- Solve problem **EMIN** (minimize expenditure):

$$\begin{aligned} \min & p_1x_1 + p_2x_2 \\ \text{s.t.} & u(x_1, x_2) \geq \bar{u} \end{aligned}$$

- Choose bundle that attains utility \bar{u} with minimal expenditure
- Ex.: You are choosing combination CDs/restaurant to make a friend happy
- If utility u strictly increasing in x_i , can maximize s.t. equality
- Denote by $h_i(p_1, p_2, \bar{u})$ solution to EMIN problem
- $h_i(p_1, p_2, \bar{u})$ is *Hicksian or compensated demand*

- Graphically:
 - Fix indifference curve at level \bar{u}
 - Consider budget sets with different M
 - Pick budget set which is tangent to indifference curve

- Optimum coincides with optimum of Utility Maximization!

- Formally:

$$h_i(p_1, p_2, \bar{u}) = x_i^*(p_1, p_2, e(p_1, p_2, \bar{u}))$$

- Expenditure function is expenditure at optimum
- $e(p_1, p_2, \bar{u}) = p_1 h_1(p_1, p_2, \bar{u}) + p_2 h_2(p_1, p_2, \bar{u})$
- $h_i(p_i)$ is *Hicksian or compensated demand* function
- Is h_i always decreasing in p_i ? Yes!
- Graphical proof: moving along a convex indifference curve
- (For non-convex indifferent curves, still true)

- Using first order conditions:

$$L(x_1, x_2, \lambda) = p_1x_1 + p_2x_2 - \lambda (u(x_1, x_2) - \bar{u})$$

$$\frac{\partial L}{\partial x_i} = p_i - \lambda u'_i(x_1, x_2) = 0$$

- Write as ratios:

$$\frac{u'_1(x_1, x_2)}{u'_2(x_1, x_2)} = \frac{p_1}{p_2}$$

- MRS = ratio of prices as in utility maximization!
- However: different constraint $\implies \lambda$ is different

- Example 1: Cobb-Douglas utility

$$\begin{aligned} \min & p_1x_1 + p_2x_2 \\ \text{s.t.} & x_1^\alpha x_2^{1-\alpha} \geq \bar{u} \end{aligned}$$

- Lagrangean =

- F.o.c.:

- Solution: $h_1^* =$, $h_2^* =$

- $\partial h_i^*/\partial p_i < 0$, $\partial h_i^*/\partial p_j > 0$, $j \neq i$

5 Slutsky Equation

- Nicholson, Ch. 5, pp. 160-163
- Now: go back to Utility Max. in case where p_2 increases to $p'_2 > p_2$
- What is $\partial x_2^* / \partial p_2$? Decompose effect:
 1. Substitution effect of an increase in p_i
 - $\partial h_2^* / \partial p_2$, that is change in EMIN point as p_2 decreases
 - Moving along an indifference curve
 - Certainly $\partial h_2^* / \partial p_2 < 0$

2. Income effect of an increase in p_i

- $\partial x_2^*/\partial M$, increase in consumption of good 2 due to increased income
- Shift out a budget line
- $\partial x_2^*/\partial M > 0$ for normal goods, $\partial x_2^*/\partial M < 0$ for inferior goods

- $h_i(p_1, p_2, \bar{u}) = x_i^*(p_1, p_2, e(p_1, p_2, \bar{u}))$
- How does the Hicksian demand change if price p_i changes?

$$\frac{dh_i}{dp_i} = \frac{\partial x_i^*(\mathbf{p}, e)}{\partial p_i} + \frac{\partial x_i^*(\mathbf{p}, e)}{\partial M} \frac{\partial e(\mathbf{p}, \bar{u})}{\partial p_i}$$

- What is $\frac{\partial e(\mathbf{p}, \bar{u})}{\partial p_i}$? Envelope theorem:

$$\begin{aligned} \frac{\partial e(\mathbf{p}, \bar{u})}{\partial p_i} &= \frac{\partial}{\partial p_i} [p_1 h_1^* + p_2 h_2^* - \lambda(u(h_1^*, h_2^*, \bar{u}) - \bar{u})] \\ &= h_i^*(p_1, p_2, \bar{u}) = x_i^*(p_1, p_2, e(p, \bar{u})) \end{aligned}$$

- Therefore

$$\frac{\partial h_i(\mathbf{p}, \bar{u})}{\partial p_i} = \frac{\partial x_i^*(\mathbf{p}, e)}{\partial p_i} + \frac{\partial x_i^*(\mathbf{p}, e)}{\partial M} x_1^*(p_1, p_2, e)$$

- Rewrite as

$$\frac{\partial x_i^*(\mathbf{p}, M)}{\partial p_i} = \frac{\partial h_i(\mathbf{p}, v(\mathbf{p}, M))}{\partial p_i} - x_i^*(p_1, p_2, M) \frac{\partial x_i^*(\mathbf{p}, M)}{\partial M}$$

- Important result! Allows decomposition into substitution and income effect

- Two effects of change in price:

1. Substitution effect negative: $\frac{\partial h_i(\mathbf{p}, v(\mathbf{p}, M))}{\partial p_i} < 0$

2. Income effect: $-x_i^*(p_1, p_2, M) \frac{\partial x_i^*(\mathbf{p}, M)}{\partial M}$

- negative if good i is normal ($\frac{\partial x_i^*(\mathbf{p}, M)}{\partial M} > 0$)

- positive if good i is inferior ($\frac{\partial x_i^*(\mathbf{p}, M)}{\partial M} < 0$)

- Overall, sign of $\frac{\partial x_i^*(\mathbf{p}, M)}{\partial p_i}$?

- negative if good i is normal

- it depends if good i is inferior

- Example 1 (ctd.): Cobb-Douglas. Apply Slutsky equation

- $x_i^* = \alpha M/p_i$

- $h_i^* =$

- Derivative of Hicksian demand with respect to price:

$$\frac{\partial h_i(\mathbf{p}, \bar{u})}{\partial p_i} =$$

- Rewrite h_i^* as function of m : $h_i(\mathbf{p}, v(\mathbf{p}, M))$

- Compute $v(\mathbf{p}, M) =$

- Substitution effect:

$$\frac{\partial h_i(\mathbf{p}, v(\mathbf{p}, M))}{\partial p_i} =$$

- Income effect:

$$-x_i^*(p_1, p_2, M) \frac{\partial x_i^*(\mathbf{p}, M)}{\partial M} =$$

- Sum them up to get

$$\frac{\partial x_i^*(\mathbf{p}, M)}{\partial p_i} =$$

- It works!