Final exam solutions

(c)

$$\left[\begin{array}{cccc|c} 3 & -6 & -3 & 0 & 3 \\ 0 & 0 & 1 & 2 & 1 \\ 6 & -12 & 2 & R & C \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -2 & -1 & 0 & 1 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & R-16 & C-14 \end{array}\right]$$

Inconsistent $R=16$ and $C \neq 14$

d) $C=14$

$$\left[\begin{array}{cccc|c} 1 & -2 & -1 & 0 & 1 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\underline{X} = \begin{bmatrix} 2 \\ 0 \\ 1 \\ 0 \end{bmatrix} + C_1 \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + C_2 \begin{bmatrix} -2 \\ 0 \\ -2 \\ 1 \end{bmatrix}, \quad C_1 \,\&\, C_2 \in \mathbb{R}$$

(2) a) a basis for Nul A is

$$\left\{ \begin{bmatrix} -1 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} \right\}$$

(b) a basis for Column space A (Col A) is

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 7 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ 6 \\ 28 \end{bmatrix} \right\}$$

$$\underline{X} = \begin{bmatrix} 3 \\ 8 \\ 0 \\ 0 \end{bmatrix} + C_1 \begin{bmatrix} -1 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + C_2 \begin{bmatrix} -1 \\ 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} \quad C_1, C_2 \in \mathbb{R}$$

① Past exam solutions

c)

$$\left[\begin{array}{cccc|c} 3 & -6 & -3 & 0 & 3 \\ 0 & 0 & 1 & 2 & 1 \\ 6 & -12 & 2 & R & C \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -2 & -1 & 0 & 1 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & R-16 & C-14 \end{array}\right]$$

Inconsistent $R = 16$ and $C \neq 14$

d) $C = 14$

$$\left[\begin{array}{cccc|c} 1 & -2 & -1 & 0 & 1 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\underline{x} = \begin{bmatrix} 2 \\ 0 \\ 1 \\ 0 \end{bmatrix} + c_1 \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} -2 \\ 0 \\ -2 \\ 1 \end{bmatrix}, \quad c_1 \& c_2 \in \mathbb{R}$$

② a) a basis for Nul A is

$$\left\{ \begin{bmatrix} -1 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} \right\}$$

b) a basis for Column space A (Col A) is

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 7 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ 6 \\ 28 \end{bmatrix} \right\}$$

c) $$\underline{x} = \begin{bmatrix} 3 \\ 8 \\ 0 \\ 0 \end{bmatrix} + c_1 \begin{bmatrix} -1 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} \quad c_1, c_2 \in \mathbb{R}$$

(3)

$$\underline{A} = \begin{bmatrix} 2 & 2 & 2 \\ 4 & 3 & 1 \\ -2 & -1 & 4 \end{bmatrix} \underset{\substack{R_2 - 2R_1 \\ R_3 - (-1)R_1}}{\sim} \begin{bmatrix} 2 & 2 & 2 \\ 0 & -1 & -3 \\ 0 & 1 & 6 \end{bmatrix} \underset{R_3 - (-1)R_2}{\sim} \begin{bmatrix} 2 & 2 & 2 \\ 0 & -1 & -3 \\ 0 & 0 & 3 \end{bmatrix}$$

$$U = \begin{bmatrix} 2 & 2 & 2 \\ 0 & -1 & -3 \\ 0 & 0 & 3 \end{bmatrix}, \quad L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix}$$

<u>Check</u>

$$\underline{L}\,\underline{U} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 2 & 2 \\ 0 & -1 & -3 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 2 \\ 4 & 3 & 1 \\ -2 & -1 & 4 \end{bmatrix}$$

$-2+3$

$$\underline{L}\,\underline{U}\,\underline{x} = \begin{bmatrix} 2 \\ -3 \\ 11 \end{bmatrix} \qquad \text{let } \underline{U}\,\underline{x} = \underline{y}$$

$\cancel{LUX} =$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 11 \end{bmatrix}$$

$y_1 = 2$

$y_2 = -7$

$y_3 = 6$

$$\underline{U}\,\underline{x} = \begin{bmatrix} 2 \\ -7 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 & 2 \\ 0 & -1 & -3 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -7 \\ 6 \end{bmatrix}$$

$x_3 = 3$

$x_2 = -2$

$x_1 = 0$

) Yes, because all rows contain pivots.

c) No, columns 4, 5, 6 and 7 in $\underline{B}$ are dependent and row operations preserve linear dependence and independence of columns. Hence, columns 4, 5, 6, 7 of $\underline{A}$ are dependent.

d) $$\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ -1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -2 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\}$$

e) $$\left\{ \begin{bmatrix} 1 \\ -3 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ 0 \\ 0 \end{bmatrix} \right\}$$

5 $(Ax=0)$ $17-11=6$

$(A^T y=0)$ $64-11=53$

b) 3

c) False