

A Review of Some Statistical Concepts

Summation and Product Operators

The Greek capital letter Σ (sigma) is used to indicate summation. Thus,

$$\sum_{i=1}^n x_i = x_1 + x_2 + \dots + x_n$$

Some of the important properties of the summation operator Σ are

1. $\sum_{i=1}^n k = nk$, where k is constant.

$$\sum_{i=1}^n k = \underbrace{k + k + \dots + k}_{n \text{ term}}$$

$$= nk$$

e.g. $\sum_{i=1}^4 3 = (4)(3) = 12$
 $= 3 + 3 + 3 + 3 = 12$

2. $\sum_{i=1}^n kx_i = k \sum_{i=1}^n x_i$, where k is constant.

$$\sum_{i=1}^n kx_i = kx_1 + kx_2 + \dots + kx_n$$

$$= k(x_1 + x_2 + \dots + x_n)$$

$$\sum_{i=1}^n x_i$$

$$= k \sum_{i=1}^n x_i$$

3. $\sum_{i=1}^n (a + bx_i) = na + b \sum_{i=1}^n x_i$, where a and b are constants and where use is made of properties 1 and 2 above.

$$\sum_{i=1}^n (a + bx_i) = na + b \sum_{i=1}^n x_i$$

$\underbrace{\hspace{10em}}_{1^{st} \text{ property}}$
 $\underbrace{\hspace{10em}}_{2^{nd} \text{ property}}$

$$= na + b (x_1 + x_2 + x_3 + \dots + x_n)$$

$\underbrace{\hspace{15em}}_{\sum x_i}$

4. $\sum_{i=1}^n (x_i + y_i) = \sum_{i=1}^n x_i + \sum_{i=1}^n y_i$

Additivity

$$= (x_1 + y_1) + (x_2 + y_2) + \dots + (x_n + y_n)$$

1^{st}
 2^{nd}
 \dots
 $n \text{ term}$
 n^{th}

$$= (x_1 + x_2 + \dots + x_n) + (y_1 + y_2 + \dots + y_n)$$

$$= \sum_{i=1}^n x_i + \sum_{i=1}^n y_i$$

x_i	5	8	6	9
y_i	4	6	8	9

$$1 \quad \sum_{i=1}^4 x_i$$

$$2 \quad \sum_{i=1}^4 x_i^2$$

$$3 \quad \sum_{i=1}^4 x_i y_i$$

$$4 \quad \sum_{i=1}^4 (x_i + y_i)$$

$$5 \quad \sum_{i=1}^4 (3x_i + 5y_i)$$

x_i	5	8	6	9
y_i	4	6	8	9

$$1 \quad \sum_{i=1}^4 x_i = 5 + 8 + 6 + 9$$

$$2 \quad \sum_{i=1}^4 x_i^2 = 5^2 + 8^2 + 6^2 + 9^2$$

$$3 \quad \sum_{i=1}^4 x_i y_i = (5)(4) + (8)(6) + (6)(8) + (9)(9)$$

$$4 \quad \sum_{i=1}^4 (x_i + y_i) = (5+4) + (8+6) + (6+8) + (9+9)$$

$$5 \quad \sum_{i=1}^4 (3x_i + 5y_i) = (3(5) + 5(4)) + (3(8) + 5(6)) + \dots$$

$$\left(\sum x_i\right)^2 \neq \sum x_i^2$$

Example

x_i	5	8	6	9
y_i	4	6	8	9

$$6. \sum_{i=1}^{10} 5$$

$$7. \sum_{i=1}^4 (x_i + y_i)^2 =$$

Example

x_i	5	8	6	9
y_i	4	6	8	9

$$6. \sum_{i=1}^{10} 5 = nk = 10(5)$$

$$7. \sum_{i=1}^4 (x_i + y_i)^2 =$$

$$(x_1 + y_1)^2 + (x_2 + y_2)^2 + \dots$$

Example

x_i	5	8	6	9
y_i	4	6	8	9

$$1 \quad \sum_{i=1}^4 x_i = x_1 + x_2 + x_3 + x_4 = 28$$

$$2 \quad \sum_{i=1}^4 x_i^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2 = 206$$

$$3 \quad \sum_{i=1}^4 x_i y_i = x_1 y_1 + x_2 y_2 + x_3 y_3 + x_4 y_4 = 197$$

$$4 \quad \sum_{i=1}^4 (x_i + y_i) = (x_1 + y_1) + \dots + (x_4 + y_4) = \sum_{i=1}^4 x_i + \sum_{i=1}^4 y_i = 55$$

$$5 \quad \sum_{i=1}^4 (3x_i + 5y_i) = 3 \sum_{i=1}^4 x_i + 5 \sum_{i=1}^4 y_i = 219$$

Example

x_i	5	8	6	9
y_i	4	6	8	9

$$6. \sum_{i=1}^{10} 5 = 5 \times 10 = 50$$

$$7. \sum_{i=1}^4 (x_i + y_i)^2 = \sum_{i=1}^4 (x_i^2 + 2x_i y_i + y_i^2)$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$= \sum_{i=1}^n x_i^2 + 2 \sum_{i=1}^4 x_i y_i + \sum_{i=1}^4 y_i^2$$

$$= 797$$

The summation operator can also be extended to multiple sums. This, $\sum \sum$, The double summation operator, is defined as

$$\sum_{i=1}^n \sum_{j=1}^m x_{ij} = \sum_{i=1}^n (x_{i1} + x_{i2} + \dots + x_{im})$$

m term

$$= (x_{11} + x_{21} + \dots + x_{n1}) + (x_{12} + x_{22} + \dots + x_{n2}) + \dots + (x_{1m} + x_{2m} + \dots + x_{nm})$$

j=1 *j=2*

Some of the properties of $\sum \sum$ are

1. $\sum_{i=1}^n \sum_{j=1}^m x_{ij} = \sum_{j=1}^m \sum_{i=1}^n x_{ij}$; that is, the order in which the double summation is performed is interchangeable.

2. $\sum_{i=1}^n \sum_{j=1}^m x_i y_j = \sum_{i=1}^n x_i \sum_{j=1}^m y_j$.

The finite double series can be written as a product

of series

$$: x_1 y_1 + x_2 y_1 + \dots + x_n y_1 + x_1 y_2 + \dots + x_n y_2 + \dots + x_1 y_m + \dots + x_n y_m$$

↙ x_n y_1

①

②

$$= (x_1 + x_2 + \dots + x_n) y_1 + (x_1 + x_2 + \dots + x_n) y_2 + \dots$$

$$= \sum_{i=1}^n x_i (y_1 + y_2 + \dots + y_m) = \sum_{i=1}^n x_i \sum_{j=1}^m y_j$$

③

$$3. \sum_{i=1}^n \sum_{j=1}^m (x_{ij} + y_{ij}) = \sum_{i=1}^n \sum_{j=1}^m x_{ij} + \sum_{i=1}^n \sum_{j=1}^m y_{ij}$$

$$= \sum_{j=1}^m (x_{1j} + y_{1j}) + \sum_{j=1}^m (x_{2j} + y_{2j}) + \dots + \sum_{j=1}^m (x_{nj} + y_{nj}) \quad \text{①} \quad \text{n term}$$

$$= \sum_{j=1}^m (x_{1j} + x_{2j} + x_{3j} + \dots + x_{nj}) + \sum_{j=1}^m (y_{1j} + y_{2j} + \dots + y_{nj}) \quad \text{②}$$

$\underbrace{\sum_{i=1}^n x_{ij}} \qquad \underbrace{\sum_{i=1}^n y_{ij}}$

$$= \sum_{j=1}^m \sum_{i=1}^n x_{ij} + \sum_{j=1}^m \sum_{i=1}^n y_{ij} \quad \text{③}$$

$$\sum_{j=1}^m (x_{1j} + y_{1j}) + \sum_{j=1}^m (x_{2j} + y_{2j})$$

$$\sum_{j=1}^m (x_{1j} + x_{2j}) + \sum_{j=1}^m (y_{1j} + y_{2j})$$

$\underbrace{\sum_{i=1}^2 x_{ij}} \qquad \underbrace{\sum_{i=1}^2 y_{ij}}$