# EXTENSIONS OF THE TWO-VARIABLE LINEAR REGRESSION MODEL PART I



## SCALING AND UNITS OF MEASUREMENT

Consider the data given in Table 6.2, which refers to U.S. gross private domestic investment (GPDI) and gross domestic product (GDP) in billions as well as millions of (chained) 2000 dollars.

Suppose in the regression of GPDI on GDP one researcher uses data in billions of dollars but another expresses data in millions of dollars.

- **Will the regression results be the same in both cases?**
- **Do the units in which the regressand and regressor are measured make any difference in the regression results?**



$$Y_i = \hat{\beta}_1 + \hat{\beta}_2 X_i + \hat{u}_i$$

$$where\ Y = GDPI\ \ and\ \ X = GDP$$

$$Y_i^* = w_1 Y_i$$

$$X_i^* = w_2 X_i$$

Where $w_1$ and $w_2$ are constants, call the **Scale factors**



$$Y_i^* = \hat{\beta}_1^* + \hat{\beta}_2^* X_i + \hat{u}_i$$

$$where\ Y_i^* = w_1 Y_i,\ X_i^* = w_2 X_i,\ and\ \ \hat{u}_i^* = w_1 \hat{u}_i$$



$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X} \qquad \hat{\beta}_1^* = \bar{Y}^* - \hat{\beta}_2^* \bar{X}^*$$

$$\hat{\beta}_2 = \frac{\sum x_i y_i}{\sum x_i^2} \qquad \hat{\beta}_2^* = \frac{\sum x_i^* y_i^*}{\sum x_i^{*2}}$$

$$\operatorname{var}(\hat{\beta}_1) = \frac{\sum X_i^2}{n \sum x_i^2} \sigma^2 \qquad \operatorname{var}(\hat{\beta}_1^*) = \frac{\sum X_i^{*2}}{n \sum x_i^{*2}} \sigma^{*2}$$

$$\operatorname{var}(\hat{\beta}_2) = \frac{\sigma^2}{\sum x_i^2} \qquad \operatorname{var}(\hat{\beta}_2^*) = \frac{\sigma^{*2}}{\sum x_i^{*2}}$$

$$\hat{\sigma}^2 = \frac{\sum \hat{u}_i^2}{n-2} \qquad \hat{\sigma}^{*2} = \frac{\sum \hat{u}_i^{*2}}{n-2}$$



$$\hat{\beta}_2^* = \left(\frac{w_1}{w_2}\right) \hat{\beta}_2$$

$$\hat{\beta}_1^* = w_1 \hat{\beta}_1$$

$$\hat{\sigma}^{*2} = w_1^2 \hat{\sigma}^2$$

$$\operatorname{var}(\hat{\beta}_1^*) = w_1^2 \operatorname{var}(\hat{\beta}_1)$$

$$\operatorname{var}(\hat{\beta}_2^*) = \left(\frac{w_1}{w_2}\right)^2 \operatorname{var}(\hat{\beta}_2)$$

$$r_{xy}^2 = r_{x^* y^*}^2$$

# EXAMPLE

Gross Private Domestic Investment and GDP, United States, 1990-2005

$$Y_i = \hat{\beta}_1 + \hat{\beta}_2 X_i + \hat{u}_i$$
$$where\ Y_i = GPDI\ \ and\ \ X_i = GDP$$



**TABLE 6.2**
**Gross Private Domestic Investment and GDP, United States, 1990–2005 (Billions of chained [2000] dollars, except as noted; quarterly data at seasonally adjusted annual rates)**

*Source: Economic Report of the President*, 2007, Table B-2, p. 328.

| Year | GPDIBL | GPDIM | GDPB | GDPM |
|---|---|---|---|---|
| 1990 | 886.6 | 886,600.0 | 7,112.5 | 7,112,500.0 |
| 1991 | 829.1 | 829,100.0 | 7,100.5 | 7,100,500.0 |
| 1992 | 878.3 | 878,300.0 | 7,336.6 | 7,336,600.0 |
| 1993 | 953.5 | 953,500.0 | 7,532.7 | 7,532,700.0 |
| 1994 | 1,042.3 | 1,042,300.0 | 7,835.5 | 7,835,500.0 |
| 1995 | 1,109.6 | 1,109,600.0 | 8,031.7 | 8,031,700.0 |
| 1996 | 1,209.2 | 1,209,200.0 | 8,328.9 | 8,328,900.0 |
| 1997 | 1,320.6 | 1,320,600.0 | 8,703.5 | 8,703,500.0 |
| 1998 | 1,455.0 | 1,455,000.0 | 9,066.9 | 9,066,900.0 |
| 1999 | 1,576.3 | 1,576,300.0 | 9,470.3 | 9,470,300.0 |
| 2000 | 1,679.0 | 1,679,000.0 | 9,817.0 | 9,817,000.0 |
| 2001 | 1,629.4 | 1,629,400.0 | 9,890.7 | 9,890,700.0 |
| 2002 | 1,544.6 | 1,544,600.0 | 10,048.8 | 10,048,800.0 |
| 2003 | 1,596.9 | 1,596,900.0 | 10,301.0 | 10,301,000.0 |
| 2004 | 1,713.9 | 1,713,900.0 | 10,703.5 | 10,703,500.0 |
| 2005 | 1,842.0 | 1,842,000.0 | 11,048.6 | 11,048,600.0 |

*Note:* GPDIBL = gross private domestic investment, billions of 2000 dollars.
GPDIM = gross private domestic investments, millions of 2000 dollars.
GDPB = gross domestic product, billions of 2000 dollars.
GDPM = gross domestic product, millions of 2000 dollars.

GPDIBL = Gross private domestic investment, billions of 2000 dollars
GPDIM = Gross private domestic investment, millions of 2000 dollars
GDPB = Gross domestic product, billions of 2000 dollars
GDPM= Gross domestic product, millions of 2000 dollars



# BOTH GPDI AND GDP IN BILLIONS OF DOLLARS:

$$\widehat{GPDI}_t = -926.090 + 0.2535 GDP_t$$
$$se \quad = (116.358) \quad (0.0129)$$
$$r^2 = 0.9648$$

GPDI in billions of dollars → millions of dollars

GDP in billions of dollars → millions of dollars



$$w_1 = 1000$$
$$w_2 = 1000$$

$$w_1 \hat{\beta}_1 = 1000 * -926.090 = -926{,}090$$

$$\frac{w_1}{w_2} \hat{\beta}_2 = \frac{1000}{1000} * 0.2535 = 0.2535$$

$$\widehat{GPDI}_t = -926{,}090 + 0.2535 GDP_t$$
$$se \quad = (116{,}358) \quad (0.0129)$$
$$r^2 = 0.9648$$



# BOTH GPDI AND GDP IN MILLIONS OF DOLLARS:

$$\widehat{GPDI}_t = -926{,}090 + 0.2535 GDP_t$$
$$se \quad = (116{,}358) \quad (0.0129)$$
$$r^2 = 0.9648$$

GPDI in millions of dollars → billions of dollars

GDP in millions of dollars → billions of dollars



$$w_1 = \frac{1}{1000}$$
$$w_2 = \frac{1}{1000}$$
$$w_1 \hat{\beta}_1 = \frac{1}{1000} * -926{,}090 = -926.090$$

$$\frac{w_1}{w_2} \hat{\beta}_2 = \frac{\frac{1}{1000}}{\frac{1}{1000}} * 0.2535 = 0.2535$$

$$\widehat{GPDI}_t = -926.090 + 0.2535 GDP_t$$
$$se \quad = (116.358) \quad (0.0129)$$
$$r^2 = 0.9648$$

## BOTH GDPI AND GDP IN BILLIONS OF DOLLARS:

$$\widehat{GPDI}_t = -926.090 + 0.2535GDP_t$$
$$se \quad = (116.358) \quad (0.0129)$$
$$r^2 \ = 0.9648$$

GDPI in billions of dollars ⟶ billions of dollars

GDP in billions of dollars ⟶ millions of dollars



$$w_1 = 1$$
$$w_2 = 1000$$

$$w_1\hat{\beta}_1 = 1 * -926.090 = -926.090$$

$$\frac{w_1}{w_2}\hat{\beta}_2 = \frac{1}{1000} * 0.2535 = 0.0002535$$

$$\widehat{GPDI}_t = -926.090 + 0.0002535GDP_t$$
$$se \quad = (116.358) \quad (0.0000129)$$
$$r^2 \ = 0.9648$$



## BOTH GPDI AND GDP IN MILLIONS OF DOLLARS:

$$\widehat{GPDI}_t = -926{,}090 + 0.2535GDP_t$$
$$se \quad = (116{,}358) \quad (0.0129)$$
$$r^2 \ = 0.9648$$

GDPI in millions of dollars ⟶ billions of dollars

GDP in millions of dollars ⟶ millions of dollars



$$w_1 = \frac{1}{1000}$$
$$w_2 = 1$$
$$w_1\hat{\beta}_1 = \frac{1}{1000} * -926{,}090 = -926.090$$

$$\frac{w_1}{w_2}\hat{\beta}_2 = \frac{\frac{1}{1000}}{1} * 0.2535 = 0.0002535$$

$$\widehat{GPDI}_t = -926.090 + 0.0002535GDP_t$$
$$se \quad = (116.358) \quad (0.0000129)$$
$$r^2 \ = 0.9648$$



## BOTH GDPI AND GDP IN BILLIONS OF DOLLARS:

$$\widehat{GPDI}_t = -926.090 + 0.2535GDP_t$$

$$se = (116.358) \quad (0.0129)$$

$$r^2 = 0.9648$$

GDPI in billions of dollars ⟶ millions of dollars

GDP in billions of dollars ⟶ billions of dollars



$$w_1 = 1000$$
$$w_2 = 1$$
$$\hat{\beta}_1^* = w_1\hat{\beta}_1 = 1000 * -926.090 = -926{,}090$$
$$\hat{\beta}_2^* = \left(\frac{w_1}{w_2}\right)\hat{\beta}_2 = \left(\frac{1000}{1}\right)0.2535 = 253.524$$

$$\widehat{GPDI}_t = -926{,}090 + 253.524GDP_t$$
$$se = (116{,}358) \quad (12.9465)$$
$$r^2 = 0.9648$$

## BOTH GPDI AND GDP IN MILLIONS OF DOLLARS:

$$\widehat{GPDI}_t = -926.090 + 0.2535 GDP_t$$
$$se = (116.358) \quad (0.0129)$$
$$r^2 = 0.9648$$

GDPI in millions of dollars ⟶ millions of dollars

GDP in millions of dollars ⟶ billions of dollars



$$w_1 = 1$$
$$w_2 = \frac{1}{1000}$$
$$w_1 \hat{\beta}_1 = 1 * -926{,}090 = -926{,}090$$
$$\frac{w_1}{w_2}\hat{\beta}_2 = \frac{1}{\frac{1}{1000}} * 0.2535 = 253.524$$
$$\widehat{GPDI}_t = -926{,}090 + 253.524 GDP_t$$
$$se = (116{,}358) \quad (12.9465)$$
$$r^2 = 0.9648$$



## REGRESSION THROUGH THE ORIGIN

$$Y_i = \beta_2 X_i + u_i$$



## REGRESSION THROUGH THE ORIGIN

$$\hat{\beta}_2 = \frac{\sum X_i Y_i}{\sum X_i^2}$$

$$\operatorname{var}(\hat{\beta}_2) = \frac{\sigma^2}{\sum X_i^2}, \; where \; \sigma^2 = \frac{\sum \hat{u}_i^2}{n-1}$$



## R-SQUARED FOR REGRESSION THROUGH ORIGIN MODEL

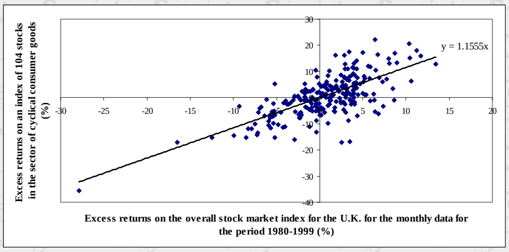
$$raw\ r^2 = \frac{(\sum X_i Y_i)^2}{\sum X_i^2 \sum Y_i^2}$$



## EXAMPLE

Table 6.1 (P.151) gives data on excess returns $Y_t$(%)on an index on 104 stocks in the sector of cyclical consumer goods and excess returns $X_t$ (%) on the overall stock market index for the U.K. for the monthly data for the period 1980-1999, for a total of 240 observations.

Excess returns refers to return in excess of return on a riskless asset. (Capital asset pricing model, CAPM)






The slope coefficient is highly significant

If the excess market rate goes up by 1 percentage point, the excess return on the index of consumer goods sector goes up by about 1.15 percentage points.

If a Beta coefficient is greater than 1, such a security is said to be volatile; it moves more than proportionately with the overall stock market index



# SOURCE

Gujarati, D.N. (2009) Basic Econometrics. 5th ed. Singapore, McGraw-Hill. (G)