

0 a). Profit: $tr - tc$

$$tr = Qp$$

$$q_1 = 80 - 2p_1 - 2(p_2 + p_3)$$

$$= 80 - 2p_1 - 2p_2 - 2p_3$$

$$p = 6.8$$

$$Q = 39.2$$

> Maximizing profit is $tr = 270$
 $tc = 39.2$
profit: 231.8

b). $p = 6.8$

c). $p = 7, Q = 38$

d.) Profit: $tr - tc$

$$= P(Q_1 + Q_2 + Q_3) - 240 \cdot 12P^*$$

2.)

$$\nabla L = 0 \rightarrow (x^*, y^*, \lambda^*)$$

$$U^* = U(x^*, y^*)$$

$$\lambda^* = \frac{dU^*}{db}$$

$$L(x, y, \lambda) = U(x, y) - \lambda(B(x, y) - b)$$

Step 1

Step 2

Step 3

$$\frac{\partial L}{\partial x} = \frac{\partial U(x, y)}{\partial x} - \lambda \left(\frac{\partial B(x, y)}{\partial x} \right) = 0$$

$$\textcircled{1} \quad 2y - \lambda(4) = 0$$

$$\textcircled{2} \quad 2x - \lambda(6) = 0$$

$$\textcircled{3} \quad 4x + 6y = 72$$

$$\therefore (x, y, \lambda) = (9, 6, 3)$$

$$\frac{\partial L}{\partial y} = \frac{\partial U(x, y)}{\partial y} - \lambda \left(\frac{\partial B(x, y)}{\partial y} \right) = 0$$

$$\text{mit } \textcircled{3} - 2 \cdot \textcircled{2}; (4x + 6y) - (4x - 12\lambda) = 72$$

$$\rightarrow \begin{cases} 6y + 12\lambda = 72 & \textcircled{4} \\ 2y - 4\lambda = 0 & \textcircled{1} \end{cases}$$

$$\frac{\partial L}{\partial \lambda} = \frac{\partial U(x, y)}{\partial \lambda} - \lambda \left(\frac{\partial B(x, y)}{\partial \lambda} \right) = 0$$

$$\text{mit } \textcircled{1} + 3 \cdot \textcircled{4}; \begin{cases} 6y + 12\lambda + 6y - 12\lambda = 72 \\ 12y = 72 \end{cases}$$

$$y = 6$$

$$\text{mit } y = 6 \text{ in } \textcircled{2}; 2(6) - 4\lambda = 0$$

$$12 = 4\lambda$$

$$\lambda = 3$$

$$\text{mit } \lambda = 3 \text{ in } \textcircled{2}; 2x - 6(3) = 0$$

$$x = 9$$



4.)

Question 4: Cost minimization problem

Consider a cost minimization problem where firm chooses for optimal combination of capital (K) and labor (L). Suppose that r and w are the prices per unit of capital and labor, respectively. And assume further that the production technology of this firm is given by $\sqrt{K} + L = Q$. Consider the following problem

- Solve for the optimal combination of capital and labor.
- State the condition under which both types of factor inputs are used by firm.
- Derive the cost function.

$$\text{Total cost equation } C = Kr + Lw$$

$$\mathcal{L} = Kr + Lw + \lambda(Q - \sqrt{K} - L)$$

$$\frac{\partial \mathcal{L}}{\partial L} = w - \lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial K} = r - \frac{1}{2}\lambda K^{-\frac{1}{2}} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = Q - \sqrt{K} - L = 0$$

$$\text{a) } \underline{L = -K^{\frac{1}{2}}}$$

$$\text{b) } \begin{aligned} Q &= L + K^{\frac{1}{2}} \\ Q &= -K^{\frac{1}{2}} + K^{\frac{1}{2}} \\ Q &= 0 \end{aligned}$$

$$\text{c) } C = K(-K^{\frac{1}{2}})r + -K^{\frac{1}{2}}w$$

