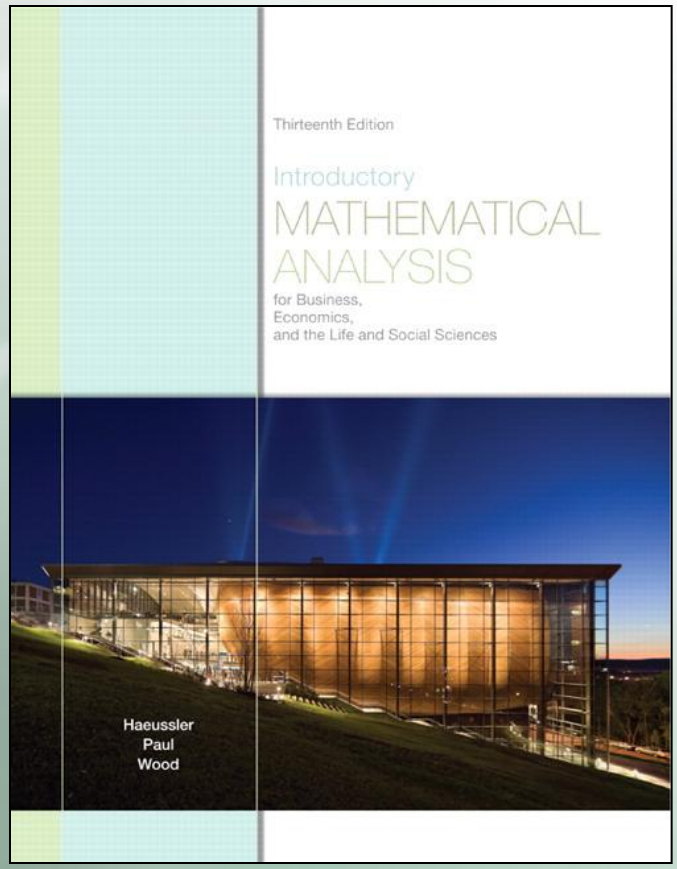
# INTRODUCTORY MATHEMATICAL ANALYSIS

## For Business, Economics, and the Life and Social Sciences

## Chapter 10
## Limits and Continuity

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# Chapter Objectives

- To study limits and their basic properties.
- To study one-sided limits, infinite limits, and limits at infinity.
- To study continuity and to find points of discontinuity for a function.
- To develop techniques for solving nonlinear inequalities.

# Chapter Outline

10.1) Limits

10.2) Limits (Continued)

10.3) Continuity

10.4) Continuity Applied to Inequalities

# 10.1 Limits

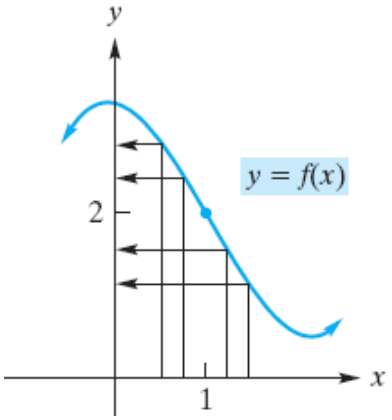
- The limit of $f(x)$ as $x$ approaches $a$ is the number $L$, written as

$$\lim_{x \to a} f(x) = L$$

## Example 1 – Estimating a Limit from a Graph

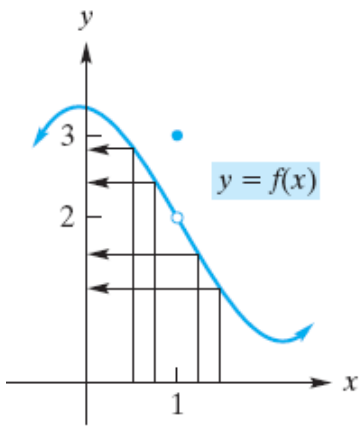
*a. Estimate* $\lim_{x \to 1} f(x)$ *from the graph.*

Solution:




*b. Estimate* $\lim_{x \to 1} f(x)$ *from the graph.*

Solution:



# Properties of Limits

1. $\lim_{x \to a} f(x) = \lim_{x \to a} c = c$ where $c$ is a constant

2. $\lim_{x \to a} x^n = a^n$ for any positive integer $n$

3. $\lim_{x \to a} [f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$

4. $\lim_{x \to a} [f(x) \cdot g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$

5. $\lim_{x \to a} [cf(x)] = c \cdot \lim_{x \to a} f(x)$

# Properties of Limits

6. $\lim_{x\to a}\dfrac{f(x)}{g(x)}=\dfrac{\lim_{x\to a}f(x)}{\lim_{x\to a}g(x)}$ if $\lim_{x\to a}g(x)\neq 0$

7. $\lim_{x\to a}\sqrt[n]{f(x)}=\sqrt[n]{\lim_{x\to a}f(x)}$

## Example 3 – Applying Limit Properties 1 and 2

a. $\lim_{x\to 2}7=7;\qquad \lim_{x\to -5}7=7$

b. $\lim_{x\to 6}x^2=6^2=36$

c. $\lim_{t\to 2}t^4=(-2)^4=16$

# Example 5 – Limit of a Polynomial Function

*Find an expression for the polynomial function,*

$$f(x) = c_n x^n + c_{n-1} x^{n-1} + \ldots + c_1 x + c_0$$

Solution:

# Limits and Algebraic Manipulation

- If $f(x) = g(x)$ for all $x \neq a$, then

$$\lim_{x \to a} f(x) = \lim_{x \to a} g(x)$$

## Example 7 – Finding a Limit

*Find* $\lim_{x \to 1} \dfrac{x^2 - 1}{x + 1}$.

Solution:

## Example 9 – Finding a Limit

*If* $f(x) = x^2 + 1$, *find* $\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$.

Solution:

# 10.2 Limits (Continued)

## Infinite Limits

- Infinite limits are written as $\lim_{x \to -0^+} \frac{1}{x} = \infty$ and $\lim_{x \to -0^-} \frac{1}{x} = -\infty$.

## Example 1 – Infinite Limits

*Find the limit (if it exists).*

a. $\lim_{x \to -1^+} \frac{2}{x+1}$ b. $\lim_{x \to 2} \frac{x+2}{x^2-4}$

Solution:

# Example 3 – Limits at Infinity

*Find the limit (if it exists).*

a. $\lim_{x\to\infty} \frac{4}{(x-5)^3}$

b. $\lim_{x\to\infty} \sqrt{(4-x)}$

Solution:

## Limits at Infinity for Rational Functions

- If f (x) is a rational function,

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{a_n x^n}{b_m x^m} \quad \text{and} \quad \lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{a_n x^n}{b_m x^m}$$

# Example 5 – Limits at Infinity for Polynomial Functions

*Find the limit (if it exists).*

a. $\lim_{x\to-\infty}\left(x^3-x^2+x-2\right)=\lim_{x\to-\infty}x^3$

Solution:

b. $\lim_{x\to-\infty}\left(-2x^3+9x\right)=\lim_{x\to-\infty}-2x^3$

Solution:

# 10.3 Continuity

## Definition

- *f(x)* is ***continuous*** if three conditions are met:
  1. $f(x)$ exists
  2. $\lim_{x \to a} f(x)$ exists
  3. $\lim_{x \to a} f(x) = f(a)$

## Example 1 – Applying the Definition of Continuity

*a. Show that f(x) = 5 is continuous at 7.*

Solution:

*b. Show that g(x) = $x^2$ − 3 is continuous at −4.*

Solution:

# Example 3 – Discontinuities

*a. When does a function have **infinite discontinuity**?*

Solution:

*b. Find discontinuity for* $f(x)=\begin{cases}1 & \text{if } x>0\\ 0 & \text{if } x=0\\ -1 & \text{if } x<0\end{cases}$

Solution:

## Example 5 – Locating Discontinuities in Case-Defined Functions

*For each of the following functions, find all points of discontinuity.*

a. $f(x)=\begin{cases} x+6 & \text{if } x \geq 3 \\ x^2 & \text{if } x < 3 \end{cases}$

b. $f(x)=\begin{cases} x+2 & \text{if } x > 2 \\ x^2 & \text{if } x < 2 \end{cases}$

## Example 5 – Locating Discontinuities in Case-Defined Functions

Solution:

a.

## Example 5 – Locating Discontinuities in Case-Defined Functions

Solution:

b.

# 10.4 Continuity Applied to Inequalities

## Example 1 – Solving a Quadratic Inequality

*Solve* $x^2 - 3x - 10 > 0$.

Solution:

# Example 3 – Solving a Rational Function Inequality

$$Solve\ \frac{x^2 - 6x + 5}{x} \geq 0.$$

Solution: