# 3. Autocorrelation

## The Model

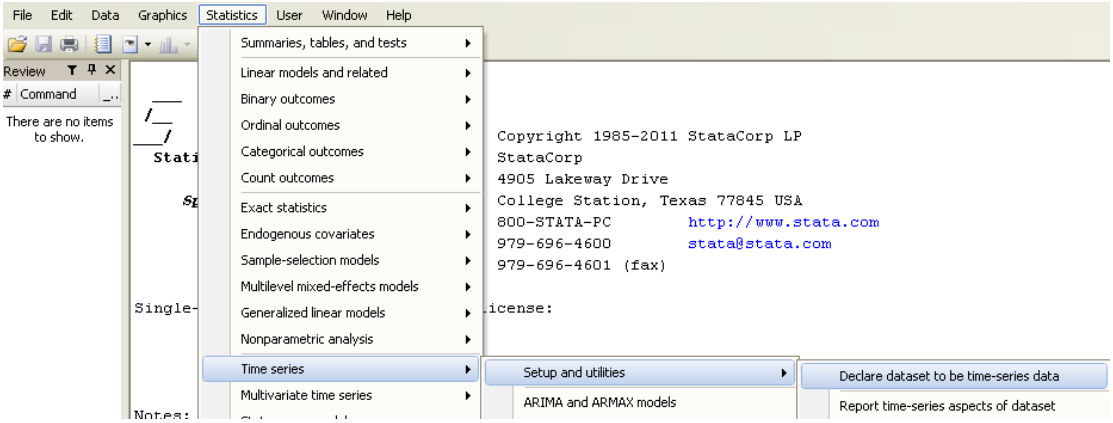
True relationship $Y_t = 1.3 + 0.7X_{1t} + 1.3X_{2t} + 1.1X_{3t} + 0.6X_{4t} + u_t$

where $u_t$ is random normally distributed error term and $u_t = 0.9u_{t-1} + \varepsilon_t$

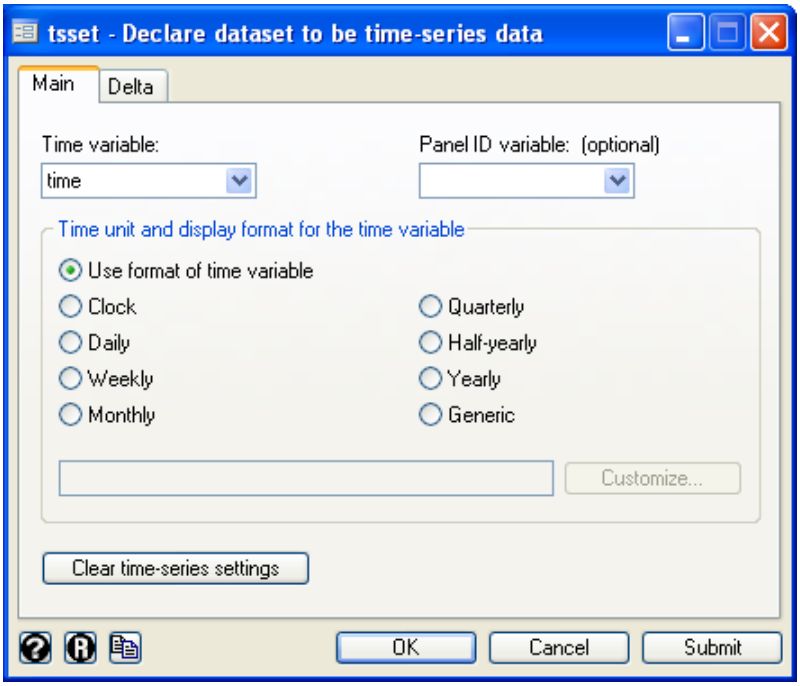
```
*Generate Data
set obs 200
g t=_n
tsset t
g u=0 in 1
replace u=0.9*l.u+rnormal(0,25) if t>1
g x1=rnormal(10,29)
mat C=(1, 0.8153\ 0.8153,1)
corr2data x2 x3, mean(0 0) sds(5 9) corr(C)
corr x2 x3
g x4=rnormal(-5,18)
g y=1.3+0.7*x1+1.3*x2+1.1*x3+0.6*x4+u
g x5=rnormal(7,21)
```

## Setting up data to be Time Series data

From menu bar, go to Statistics, choose Time series, select Setup & utilities, choose Declare dataset to be time-series data.




From tsset – Declare dataset to be time-series data window, specify variable determine time (in this case – time) in Time variable: box, and select time frequency in Display format for the time variable (in this case Monthly), then, click OK.




```
. tsset t
        time variable:  t, 1 to 200
                delta:  1 unit
```

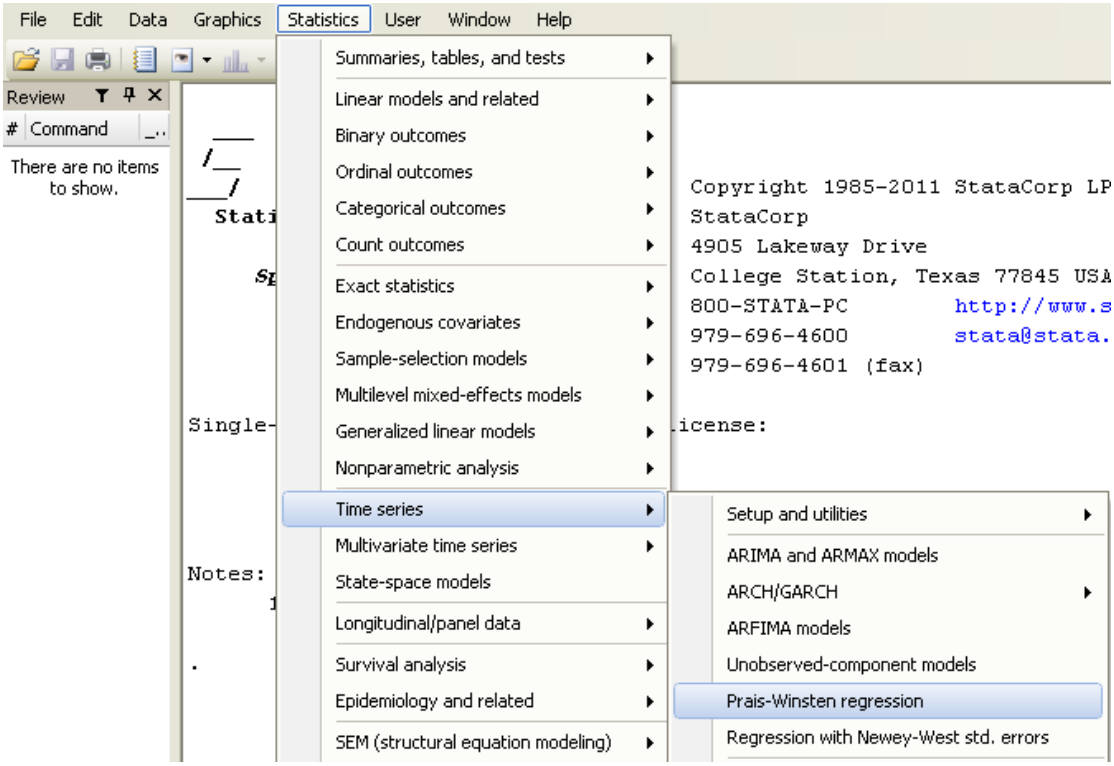
## Detecting Autocorrelation Problem

Model: $y_t = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + \beta_3 x_{3t} + \beta_4 x_{4t} + u_t$

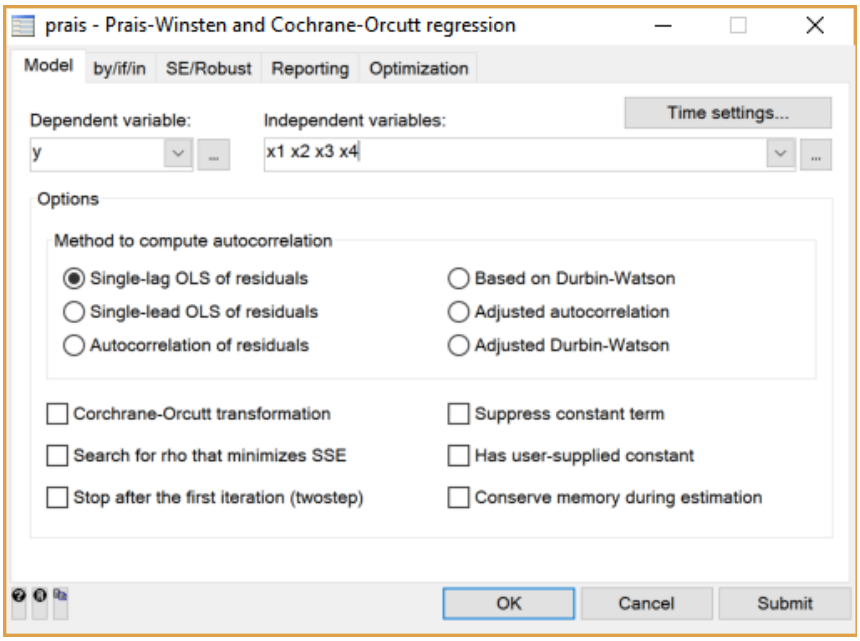
```
. reg y x1 x2 x3 x4

      Source |       SS           df       MS      Number of obs   =       200
-------------+----------------------------------   F(4, 195)       =     21.77
       Model |  138152.596         4   34538.149   Prob > F        =    0.0000
    Residual |  309425.267       195  1586.79624   R-squared       =    0.3087
-------------+----------------------------------   Adj R-squared   =    0.2945
       Total |  447577.863       199  2249.13499   Root MSE        =    39.835

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          x1 |   .6323429   .0971257     6.51   0.000     .4407912    .8238946
          x2 |   1.357717   .9774565     1.39   0.166    -.5700267    3.285461
          x3 |   1.110347   .5430026     2.04   0.042     .0394349    2.181259
          x4 |   .5426169   .1622647     3.34   0.001     .2225979    .8626359
       _cons |  -15.82143   3.050376    -5.19   0.000    -21.83739   -9.805466
------------------------------------------------------------------------------

. estat dwatson

Durbin-Watson d-statistic(  5,   200) =  .4402098
```

## Solving Problem using Cochrane-Orcutt Technique

To solve problem using Cochrane-Orcutt technique, from menu bar, go to Statistics, choose Time series, select Prais-Winsten regression.




From prais – Prais-Winsten and Cochrane-Orcutt regression window, specify dependent and independent variables in Dependent variable: and Independent variables: boxes, and check on Corchrane-Orcutt transformation box, then, click OK.



```
. prais y x1 x2 x3 x4, rhotype(reg) corc

Iteration 0:  rho = 0.0000
Iteration 1:  rho = 0.7812
Iteration 2:  rho = 0.7901
Iteration 3:  rho = 0.7902
Iteration 4:  rho = 0.7902

Cochrane-Orcutt AR(1) regression -- iterated estimates

      Source |       SS           df       MS      Number of obs   =       199
-------------+----------------------------------   F(4, 194)       =    114.09
       Model |  277660.151         4  69415.0376   Prob > F        =    0.0000
    Residual |  118032.419       194   608.41453   R-squared       =    0.7017
-------------+----------------------------------   Adj R-squared   =    0.6956
       Total |  395692.569       198  1998.44732   Root MSE        =    24.666

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          x1 |   .6639764   .0496668    13.37   0.000     .5660201    .7619327
          x2 |   1.172564   .4832696     2.43   0.016     .2194266      2.1257
          x3 |   1.484829   .2700012     5.50   0.000     .9523148    2.017344
          x4 |   .6383948   .0771094     8.28   0.000     .4863144    .7904752
       _cons |  -15.19655   8.348466    -1.82   0.070    -31.66196    1.268855
-------------+----------------------------------------------------------------
         rho |   .7901594
------------------------------------------------------------------------------
Durbin-Watson statistic (original)    0.440210
Durbin-Watson statistic (transformed) 1.784009
```

## Cochrane-Orcutt Iterative Process

```
. reg y x1 x2 x3 x4

      Source |       SS           df       MS      Number of obs   =       200
-------------+----------------------------------   F(4, 195)       =     21.77
       Model |  138152.596         4   34538.149   Prob > F        =    0.0000
    Residual |  309425.267       195  1586.79624   R-squared       =    0.3087
-------------+----------------------------------   Adj R-squared   =    0.2945
       Total |  447577.863       199  2249.13499   Root MSE        =    39.835

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          x1 |   .6323429   .0971257     6.51   0.000     .4407912    .8238946
          x2 |   1.357717   .9774565     1.39   0.166    -.5700267    3.285461
          x3 |   1.110347   .5430026     2.04   0.042     .0394349    2.181259
          x4 |   .5426169   .1622647     3.34   0.001     .2225979    .8626359
       _cons |  -15.82143   3.050376    -5.19   0.000    -21.83739   -9.805466
------------------------------------------------------------------------------

. predict uhat1, resid

. reg uhat1 l.uhat1, nocon

      Source |       SS           df       MS      Number of obs   =       199
-------------+----------------------------------   F(1, 198)       =    305.60
       Model |  187529.598         1  187529.598   Prob > F        =    0.0000
    Residual |  121502.905       198  613.651034   R-squared       =    0.6068
```

```
-------------+----------------------------------   Adj R-squared   =    0.6048
       Total |  309032.503       199  1552.92715   Root MSE        =    24.772

------------------------------------------------------------------------------
       uhat1 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       uhat1 |
         L1. |   .7812105   .0446883    17.48   0.000     .6930843    .8693366
------------------------------------------------------------------------------

. mat rho1=e(b)

. mat list rho1

symmetric rho1[1,1]
            L.
         uhat1
y1   .78121046

. sca rho1=el(rho1,1,1)

. sca list rho1
      rho1 =  .78121046

. g ys1=y-rho1*l.y
(1 missing value generated)

. g x1s1=x1-rho1*l.x1
(1 missing value generated)

. g x2s1=x2-rho1*l.x2
(1 missing value generated)

. g x3s1=x3-rho1*l.x3
(1 missing value generated)

. g x4s1=x4-rho1*l.x4
(1 missing value generated)

. g x0s1=1-rho1

. reg ys1 x1s1 x2s1 x3s1 x4s1 x0s1, nocon

      Source |       SS           df       MS      Number of obs   =       199
-------------+----------------------------------   F(5, 194)       =     90.91
       Model |   276621.15         5  55324.2299   Prob > F        =    0.0000
    Residual |   118057.18       194  608.542164   R-squared       =    0.7009
-------------+----------------------------------   Adj R-squared   =    0.6932
       Total |  394678.329       199  1983.30819   Root MSE        =    24.669

------------------------------------------------------------------------------
         ys1 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        x1s1 |   .6638128    .049881    13.31   0.000     .5654341    .7621914
        x2s1 |    1.17403   .4854826     2.42   0.017     .2165287    2.131532
        x3s1 |   1.483359    .271218     5.47   0.000     .9484449    2.018274
        x4s1 |   .6382096   .0774535     8.24   0.000     .4854507    .7909686
        x0s1 |  -15.22359   8.009481    -1.90   0.059    -31.02043    .5732476
------------------------------------------------------------------------------

. g uhat2=y-(-15.22359+0.6638128*x1+1.17403*x2+1.483359*x3+0.6382096*x4)
```

```
Iteration 0:  rho = 0.0000
Iteration 1:  rho = 0.7812
Iteration 2:  rho = 0.7901
Iteration 3:  rho = 0.7902
Iteration 4:  rho = 0.7902
```

```
. reg uhat2 l.uhat2, nocon

      Source |       SS           df       MS      Number of obs   =       199
-------------+----------------------------------   F(1, 198)       =    324.33
       Model |   193340.87         1   193340.87   Prob > F        =    0.0000
    Residual |  118032.456       198  596.123516   R-squared       =    0.6209
-------------+----------------------------------   Adj R-squared   =    0.6190
       Total |  311373.327       199  1564.69008   Root MSE        =    24.416

------------------------------------------------------------------------------
       uhat2 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       uhat2 |
         L1. |   .7901458   .0438746    18.01   0.000     .7036243    .8766674
------------------------------------------------------------------------------

. mat rho2=e(b)

. sca rho2=el(rho2,1,1)

. g  ys2=y- rho2*l.y
(1 missing value generated)

. g x1s2=x1-rho2*l.x1
(1 missing value generated)

. g x2s2=x2-rho2*l.x2
(1 missing value generated)

. g x3s2=x3-rho2*l.x3
(1 missing value generated)

. g x4s2=x4-rho2*l.x4
(1 missing value generated)

. g x0s2=1-rho2

. reg ys2 x1s2 x2s2 x3s2 x4s2 x0s2, nocon

      Source |       SS           df       MS      Number of obs   =       199
-------------+----------------------------------   F(5, 194)       =     91.73
       Model |  279058.811         5  55811.7623   Prob > F        =    0.0000
    Residual |   118032.42       194  608.414536   R-squared       =    0.7028
-------------+----------------------------------   Adj R-squared   =    0.6951
       Total |  397091.231       199  1995.43332   Root MSE        =    24.666

------------------------------------------------------------------------------
         ys2 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        x1s2 |   .6639762   .0496672    13.37   0.000     .5660192    .7619331
        x2s2 |   1.172566   .4832729     2.43   0.016     .2194224    2.125709
        x3s2 |   1.484827    .270003     5.50   0.000      .952309    2.017345
        x4s2 |   .6383945   .0771099     8.28   0.000     .4863131    .7904759
        x0s2 |   -15.1966   8.347929    -1.82   0.070    -31.66095    1.267755
------------------------------------------------------------------------------

. g uhat3=y-(-15.1966+0.6639768*x1+1.172566*x2+1.484827*x3+0.6383945*x4)
```

```
Iteration 0:  rho = 0.0000
Iteration 1:  rho = 0.7812
Iteration 2:  rho = 0.7901
Iteration 3:  rho = 0.7902
Iteration 4:  rho = 0.7902
```

```
. reg uhat3 l.uhat3, nocon

      Source |       SS           df       MS      Number of obs   =       199
-------------+----------------------------------   F(1, 198)       =    324.36
       Model |  193359.729         1  193359.729   Prob > F        =    0.0000
    Residual |  118032.418       198  596.123324   R-squared       =    0.6210
-------------+----------------------------------   Adj R-squared   =    0.6190
       Total |  311392.147       199  1564.78466   Root MSE        =    24.416

------------------------------------------------------------------------------
       uhat3 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       uhat3 |
         L1. |   .7901594   .0438732    18.01   0.000     .7036406    .8766782
------------------------------------------------------------------------------

. mat rho3=e(b)

. sca rho3=el(rho3,1,1)

. g  ys3=y- rho3*l.y
(1 missing value generated)

. g x1s3=x1-rho3*l.x1
(1 missing value generated)

. g x2s3=x2-rho3*l.x2
(1 missing value generated)

. g x3s3=x3-rho3*l.x3
(1 missing value generated)

. g x4s3=x4-rho3*l.x4
(1 missing value generated)

. g x0s3=1- rho3

. reg ys3 x1s3 x2s3 x3s3 x4s3 x0s3, nocon

      Source |       SS           df       MS      Number of obs   =       199
-------------+----------------------------------   F(5, 194)       =     91.73
       Model |  279062.524         5  55812.5048   Prob > F        =    0.0000
    Residual |  118032.418       194  608.414528   R-squared       =    0.7028
-------------+----------------------------------   Adj R-squared   =    0.6951
       Total |  397094.942       199  1995.45197   Root MSE        =    24.666

------------------------------------------------------------------------------
         ys3 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        x1s3 |   .6639764   .0496668    13.37   0.000     .5660201    .7619327
        x2s3 |   1.172564   .4832696     2.43   0.016     .2194266    2.125701
        x3s3 |   1.484829   .2700012     5.50   0.000     .9523147    2.017344
        x4s3 |   .6383948   .0771094     8.28   0.000     .4863144    .7904752
        x0s3 |  -15.19655   8.348466    -1.82   0.070    -31.66196    1.268855
------------------------------------------------------------------------------

. g uhat4=y-(-15.19655+0.6639764*x1+1.172564*x2+1.484829*x3+0.6383948*x4)
```

```
Iteration 0:  rho = 0.0000
Iteration 1:  rho = 0.7812
Iteration 2:  rho = 0.7901
Iteration 3:  rho = 0.7902
Iteration 4:  rho = 0.7902
```

```
. reg uhat4 l.uhat4, nocon

      Source |       SS           df       MS      Number of obs   =       199
-------------+----------------------------------   F(1, 198)       =    324.36
       Model |  193359.749         1  193359.749   Prob > F        =    0.0000
    Residual |  118032.421       198  596.123337   R-squared       =    0.6210
-------------+----------------------------------   Adj R-squared   =    0.6190
       Total |   311392.17       199  1564.78477   Root MSE        =    24.416

------------------------------------------------------------------------------
       uhat4 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       uhat4 |
         L1. |   .7901594   .0438732    18.01   0.000     .7036406    .8766782
------------------------------------------------------------------------------

. mat rho4=e(b)

. sca rho4=el(rho4,1,1)

. g  ys4=y- rho4*l.y
(1 missing value generated)

. g x1s4=x1-rho4*l.x1
(1 missing value generated)

. g x2s4=x2-rho4*l.x2
(1 missing value generated)

. g x3s4=x3-rho4*l.x3
(1 missing value generated)

. g x4s4=x4-rho4*l.x4
(1 missing value generated)

. g x0s4=1- rho4
```

```
Iteration 0:  rho = 0.0000
Iteration 1:  rho = 0.7812
Iteration 2:  rho = 0.7901
Iteration 3:  rho = 0.7902
Iteration 4:  rho = 0.7902
```

```
. reg ys4 x1s4 x2s4 x3s4 x4s4

      Source |       SS           df       MS      Number of obs   =       199
-------------+----------------------------------   F(4, 194)       =    114.09
       Model |   277660.15         4  69415.0375   Prob > F        =    0.0000
    Residual |  118032.419       194  608.414531   R-squared       =    0.7017
-------------+----------------------------------   Adj R-squared   =    0.6956
       Total |  395692.569       198  1998.44732   Root MSE        =    24.666

------------------------------------------------------------------------------
         ys4 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        x1s4 |   .6639764   .0496668    13.37   0.000     .5660201    .7619327
        x2s4 |   1.172564   .4832696     2.43   0.016     .2194266      2.1257
        x3s4 |   1.484829   .2700012     5.50   0.000     .9523148    2.017344
        x4s4 |   .6383948   .0771094     8.28   0.000     .4863144    .7904752
       _cons |  -3.188854   1.751847    -1.82   0.070    -6.643965    .2662574
------------------------------------------------------------------------------

. estat dwatson

Durbin-Watson d-statistic(  5,   199) =  1.784009

. prais y x1 x2 x3 x4, rhotype(reg) corc

Iteration 0:  rho = 0.0000
Iteration 1:  rho = 0.7812
Iteration 2:  rho = 0.7901
Iteration 3:  rho = 0.7902
Iteration 4:  rho = 0.7902

Cochrane-Orcutt AR(1) regression -- iterated estimates

      Source |       SS           df       MS      Number of obs   =       199
-------------+----------------------------------   F(4, 194)       =    114.09
       Model |  277660.151         4  69415.0376   Prob > F        =    0.0000
    Residual |  118032.419       194   608.41453   R-squared       =    0.7017
-------------+----------------------------------   Adj R-squared   =    0.6956
       Total |  395692.569       198  1998.44732   Root MSE        =    24.666

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          x1 |   .6639764   .0496668    13.37   0.000     .5660201    .7619327
          x2 |   1.172564   .4832696     2.43   0.016     .2194266      2.1257
          x3 |   1.484829   .2700012     5.50   0.000     .9523148    2.017344
          x4 |   .6383948   .0771094     8.28   0.000     .4863144    .7904752
       _cons |  -15.19655   8.348466    -1.82   0.070    -31.66196    1.268855
-------------+----------------------------------------------------------------
         rho |   .7901594
------------------------------------------------------------------------------
Durbin-Watson statistic (original)    0.440210
Durbin-Watson statistic (transformed) 1.784009
```

b0=(-3.188854)/(1-0.7901594)=-15.19655