

FINAL 2011 MA217

1. Suppose that a house is situated in one corner of a rectangular plot of land 300 feet by 200 feet.

The contour plot of land is given by the equation $E(x, y) = 10^{-4} \left(x^2 - \frac{xy}{2} \right)$. The house is situated at the origin, the x -axis is the 300 feet length and the y -axis is the 200 feet width. It is desired to level this property at the level of the house ($E(x, y) = 0$).

- (a) How much fill has to be removed or brought in to accomplish this? **State clearly if the soil is to be removed or brought and state the unit clearly.**
- (b) In addition, what is the original average elevation of the properties from the level of the house ($E(x, y) = 0$). **State clearly if the original average elevation above or below the level of the house state the unit clearly.**

(4 marks)

5. To find a critical point of $f(x, y, z) = x^2 + xy + 2y^2 + z^2$ subjected to $x - 3y - 4z = 16$, a Lagrange multiplier equation can be setup as follow

$$L = x^2 + xy + 2y^2 + z^2 - \lambda \{x - 3y - 4z - 16\}$$

and hence consider the first derivatives

$$L'_x = 2x + y - \lambda = 0 \quad (1)$$

$$L'_y = x + 4y + 3\lambda = 0 \quad (2)$$

$$L'_z = 2z + 4\lambda = 0 \quad (3)$$

$$x - 3y - 4z = 16 \quad (4)$$

Equations (1) – (4) are a system of linear equation. **Use matrix** to solve this system of linear equations and show that your answer is correct.

(5 marks)

6. Find $\underline{\mathbf{A}}$ if $(\underline{\mathbf{A}}^{-1} - 3\underline{\mathbf{I}})^T = \begin{bmatrix} 5 & 10 & \frac{7}{2} \\ 3 & 1 & \frac{3}{2} \\ 1 & 1 & -\frac{5}{2} \end{bmatrix}$.

(5 marks)

7. Given $\underline{\mathbf{A}} = \begin{bmatrix} 1 & 4 & -3 & 2 & -6 & 8 \\ 3 & -5 & 7 & -9 & 1 & 2 \\ -1 & 3 & -4 & 3 & 2 & 1 \\ -3 & 6 & 5 & 4 & 9 & 8 \\ 1 & 0 & 2 & -1 & 5 & 9 \end{bmatrix}$, $\underline{\mathbf{B}} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & t^2 \end{bmatrix}$, $\underline{\mathbf{C}} = \begin{bmatrix} 0 & 0 & 8 & 2 \\ 0 & 0 & 1 & 0 \\ 7 & 1 & 2 & 5 \\ 2 & 0 & 0 & 1 \end{bmatrix}$,

$$\underline{\mathbf{D}} = \begin{bmatrix} 2 & 2 & 1 & 1 \\ -3 & 3 & 0 & 1 \\ 4 & 0 & 0 & 5 \\ 2 & 0 & 0 & 0 \end{bmatrix}$$

- (a) Determine $\det \underline{\mathbf{A}}$
- (b) What values of t which make the matrix $\underline{\mathbf{B}}$ singular?
- (c) If $t = 2$, find $\det \underline{\mathbf{B}}$, $\det (\underline{\mathbf{B}} \underline{\mathbf{B}}^T)$ and $\det (\underline{\mathbf{B}}^{-1} \underline{\mathbf{B}}^T)$
- (d) Determine $\det \underline{\mathbf{E}}$ if $\det (\underline{\mathbf{B}}^2 \underline{\mathbf{C}} \underline{\mathbf{D}}^{-1} \underline{\mathbf{E}}^3) = \frac{4}{9}$ where $t = 2$ for $\underline{\mathbf{B}}$.

(5 marks)

8. Given $\underline{\mathbf{A}} = \begin{bmatrix} 1 & 0 & 4 \\ 2 & 2 & 0 \\ 0 & 2 & -9 \end{bmatrix}$ and $(\underline{\mathbf{A}} \underline{\mathbf{B}})^T = \underline{\mathbf{I}}$,

- (a) Find matrix $\underline{\mathbf{B}}$.

(b) If $\underline{\mathbf{D}} = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 2 & 0 \\ 0 & -2 & -1 \end{bmatrix}$ and $\underline{\mathbf{D}}$ is invertible, solve $(\underline{\mathbf{A}} \underline{\mathbf{D}}^{-1})^T \underline{\mathbf{x}} = \begin{bmatrix} 2 \\ 3 \\ -4 \end{bmatrix}$ for $\underline{\mathbf{x}}$.

(5 marks)