Chapter 2

Mathematics and Economic Relations

Outline

Recap:

1. An economic model is a skeletal and simplified representation of the actual economy in which only primary factors and relationships relevant to our problem at hand is presented.

2. There are many benefits of using mathematics in the analysis within economics.

2.1 Ingredients of a Mathematical Model

If the economic model is mathematical, it will usually consist of a set of __equations__ designed to describe the structure of the model.

By relating a number of __variables__ to one another in certain ways, these equations give mathematical form to the set of analytical assumptions adopted.

Then, through application of the relevant mathematical operations to these equations, we may seek to derive a set of conclusions which logically follow from those assumptions.

In an economic model, it will consist of:

→ constant

→ variables

→ equations

→ *Variables*

A __variable__ is something whose magnitude can change and take on different values (so we use symbol, instead of using a specific number).

e.g. price __P__, profit __π__, revenue __TR__, cost __TC__, etc.

Variables whose solution values we seek from the model is called __endogenous variable__.

Variables which are assumed to be determined by forces external to the model and whose magnitudes are accepted as given is called exogenous variable.

Examples: Think of an economic model for market of rice.

Endogenous variables are P^*_{rice}, Q^*_{rice}

Exogenous variables are weather, Price of other commodities that are substitutes or complement etc.

→ *Constant*

A constant is a magnitude that does not change.

It can by numerical, e.g. 7, 8, etc.

Or, it can be represented by a symbol, e.g. a, b, c α, β, γ in case we haven't assigned the value to it yet.

If it is represented in a form of symbol, it will be called parametric constant or parameters

Note: Parameters closely resemble exogenous variables in that they are to be treated as "givens" in a model.

→*Equations*

Equation relates variables to one another. There are three types of equations.

- A Definitional equation sets up an identity between two alternate expressions that have exactly the same meaning.

The regular equals sign $=$ or the identical-equality sign $\equiv$ "identically equal to" can be used interchangeably.

Example, $\pi \equiv TR - TC$.

- A behavioral equation specifies the manner in which a variable behaves in response to changes in other variables.

Example,

$TC = 75 + 10Q$ TC depends linearly on Quantity Q↑, TC↑

$TC = 100 + Q^2$ " ——— " nonlinearly on Q

- A conditional equation states the requirement to be satisfied.

An equilibrium condition describes the prerequisite for the attainment of equilibrium.

Example,

$Q_d = Q_s$

$S = I$

$Y = DAE$

An optimization condition is derived from an optimization model.

Example,

$MC = MR$

Examples of economic models

An economic model for market equilibrium

$Q_d = Q_s$

$Q_d = a - bP$

$Q_s = -c + dP$

endogenous variables:

$Q_d = Q_s = Q^*$, P^*

$Y = DAE = C + I + G$ ① conditional eq.

$C = a + bY_d$ ② Beh

$Y_d = Y - T$ ③ Def

$I = I_0 + I_1 r$, $I_1 = 0$ ④ Beh

$G = G_0$ ⑤ Beh

2.2 Relation and Function

Set is a collection of items/objects. Each item or object in the set must share some properties/characteristic to the group/set where it belongs.

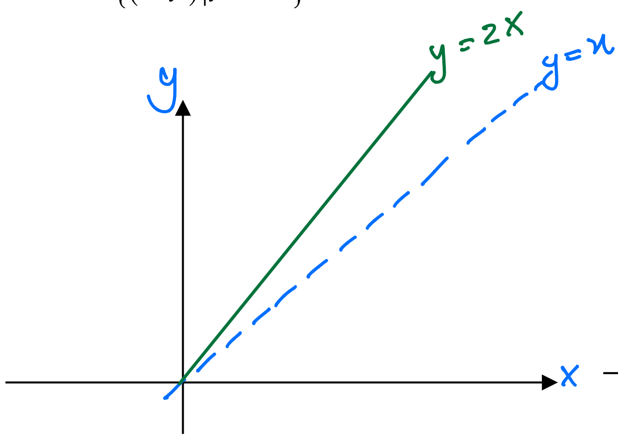
Element of one set (sets) can be associated with element of the other set, through a relation.

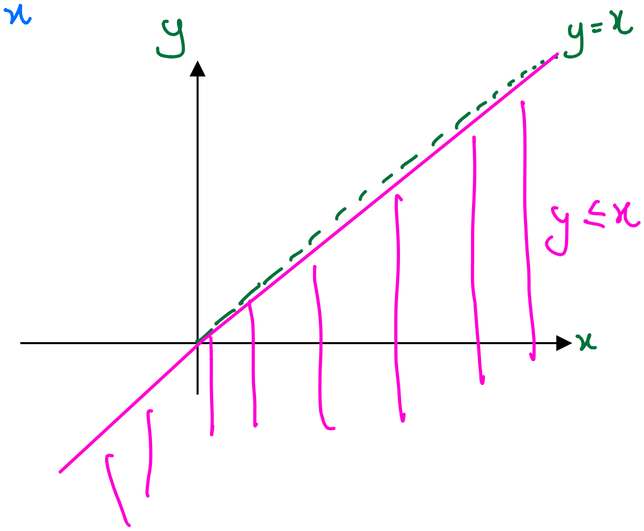
A **relation** represents a prescribed **mapping rule** between element in one set and element of the other set.

A **relation** between y and x is constituted of a collection of ordered pairs (x, y) that associates a y value with an x value.

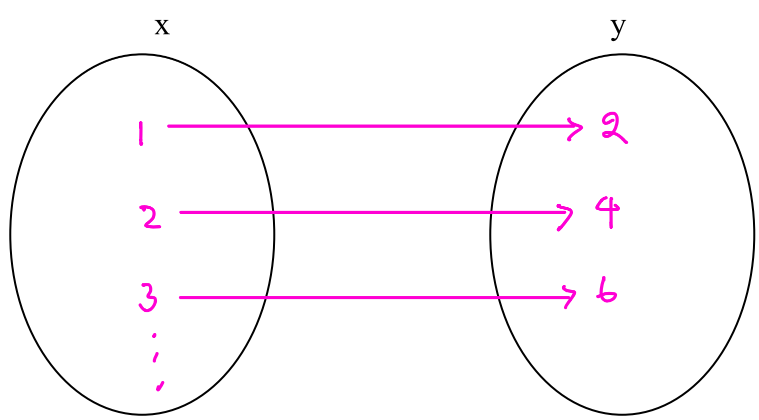
That is, given an x value, one or more y values will be specified by that relation.

$\{(x,y) | y = 2x\}$ $\{(x,y) | y \le x\}$





In the case of $\{(x,y) | y = 2x\}$, we have that:



For each x value, there exists only one corresponding y value. This is "One-to-One Relationship" or "One-to-One Mapping"

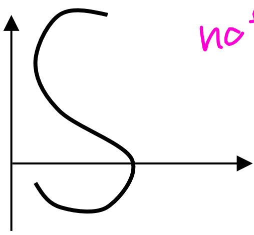
In this case, y is said to be a function of x, and this is denoted by $y = f(x)$ "y equals f of x".

We may write $f: x \to y$ to show that the function f is a mapping rule from set x to set y.

In $y = f(x)$, x is referred to as the argument of the function or "independent Variable"
y is called the value of the function or "dependent Variable".

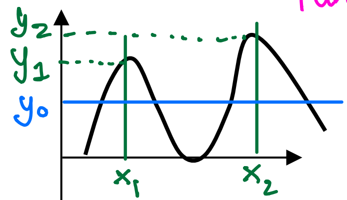
Test:

Is this following graph a function?



not function

(a)



function

(b)

Note: A function needs a unique y for each x, but the converse is not required. More than one x value may legitimately be associated with the same y value. That is, function is one-to-one or many-to-one relation, but not one-to-many relation.

2.3 Types of Function

Different types of function represent different rules of mapping.

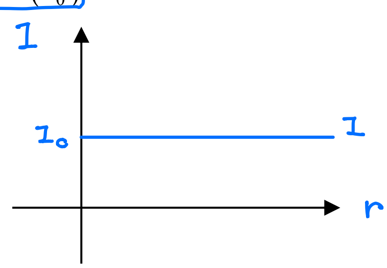
2.3.1 Constant Functions

A function whose range (the set of all values that the y variable can take) consists of only one element is called a constant function.

$$y = f(x) = c$$

$$y = c$$

Example: In a national-income model, if we assume that investment is exogenously determined and equal to autonomous investment (I_0) which takes value of 100 millionbaht, the graph is:



2.3.2 Polynomial Functions

A polynomial (multi-term) function of a single variable x has the general form of

$$\boxed{y = \sum_{i=0}^{n} a_i x^i}$$

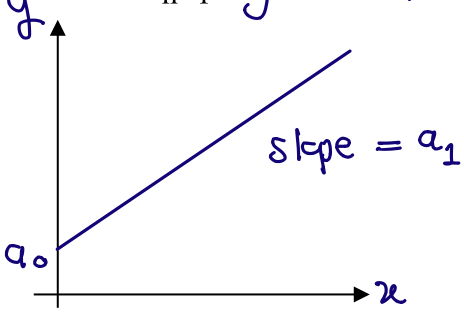
$$y = a_0 x^0 + a_1 x^1 + a_2 x^2 + a_3 x^3 + \ldots + a_n x^n$$

The highest power involved, the value of n is called the degree of the polynomial function.

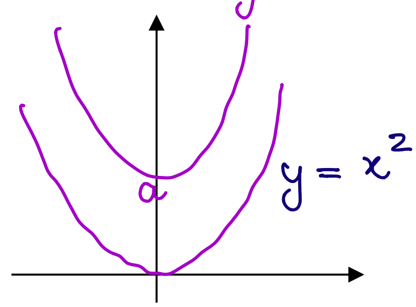
Depending on the value of the integer n, subclasses of polynomial function are:

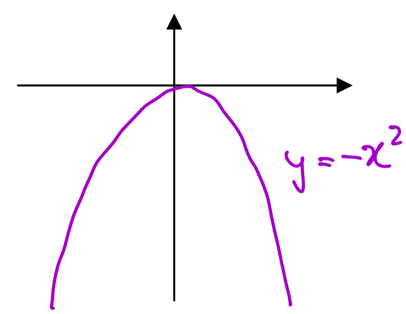
Degree n	Type of function	name
n=0	$y = a_0$	Constant
n=1	$y = a_0 + a_1 x$	Linear
n=2	$y = a_0 + a_1 x + a_2 x^2$	Quadratic
n=3	$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$	Cubic

n=1 $y = a_0 + a_1 x$

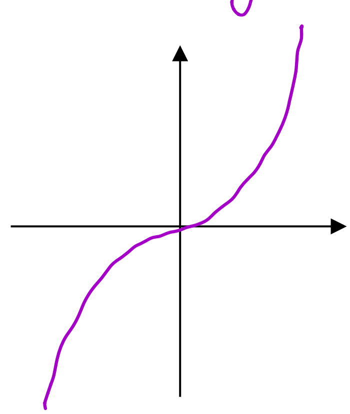


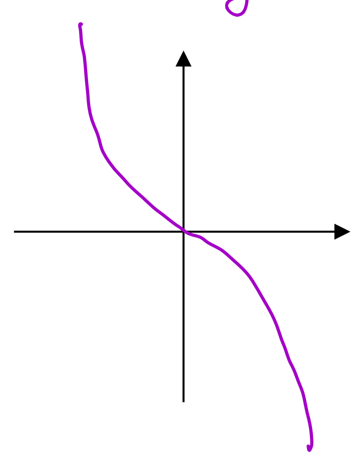
$y = x^2 + a$ n=2





$y = x^3$ n=3 $y = -x^3$





Note: The roots of a function are the x-intercepts. By definition, the y-coordinate of points lying on the x-axis is zero.

For $y = ax^2 + bx + c$, we can find the roots of a quadratic function, by using Quadratic formula $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

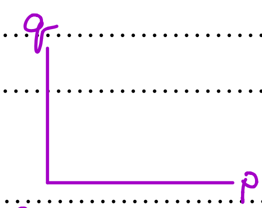
Example:

Suppose a research has a priori belief that the relationship between “q” and “p” is linear. From the survey, the research only knows that when p is equal to 1, q is equal to 4. Meanwhile, when p is equal to 5, q is then equal to 0. Find the equation of the line that represents the p-q relationship? What does the relationship represent?

$p = 1$, $q = 4$

$p = 5$, $q = 0$

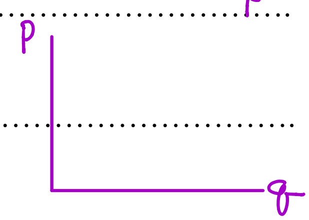
$q = -p + 5$



Note:

the “demand function” is in the form of $q = f(p)$

$q = -p + 5$



the “inverse demand function” is in the form of $p = g(q)$

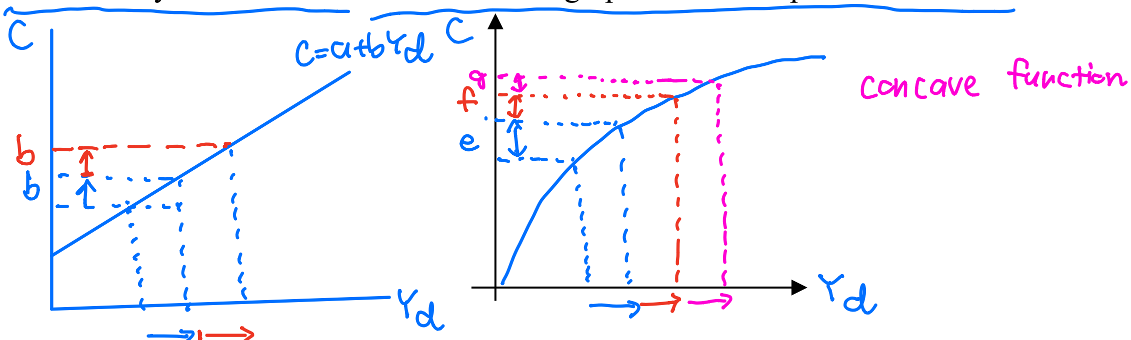
$p = -q + 5$

Example:

Consider Keynesian aggregate consumption, $C = a + bY_d$.

Does it make sense to assume that each individual household has a linear consumption function?

What if SES panel survey shows that the rich tend to consume less out of each additional unit of income they have earned? How should the graph for consumption function be?



Socioeconomic Survey

2.3.3 Exponential and Logarithmic Function

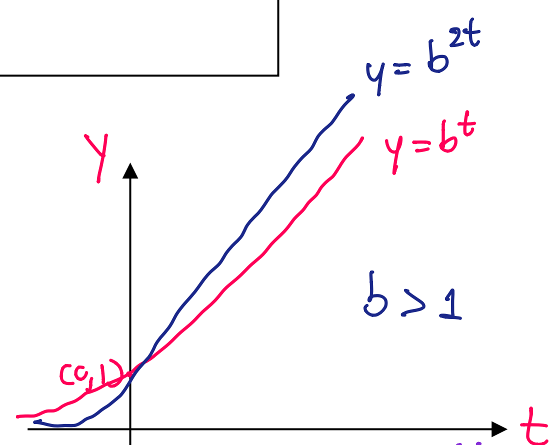
Exponential Function

$$\boxed{y = Ab^{rx}}$$

Example: draw the following graphs

$y = g(t) = b^t$

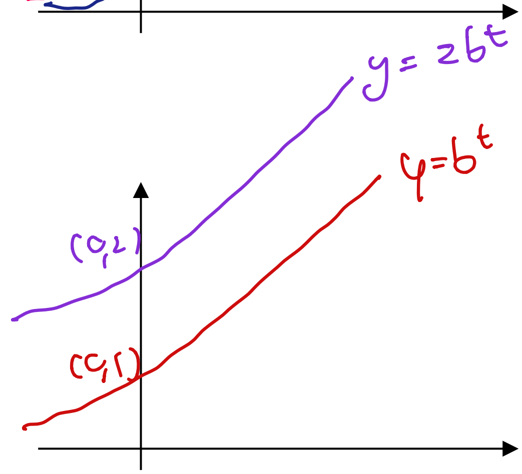
$y = f(t) = b^{2t}$



Example: draw the following graphs

$y = g(t) = b^t$

$y = f(t) = 2b^t$



If the base is the natural number e, we call it Natural exponential function:

$$y = e^t$$

, where $e = 2.71828...$

Common applications of exponential function are such as:

1. Growth calculation
$\boxed{X_t = (1+g)X_{t-1}}$ where g is growth rate, X_t is output at the t.
If we know output at the beginning period and growth rate, can we calculate output at time t?

X_0
$X_1 = (1+g)X_0$
$X_2 = (1+g)(1+g)X_0$
$X_3 = (1+g)(1+g)(1+g)X_0$
$\vdots$
$X_t = (1+g)^t X_0$

exponential function

2. Present value

$$PV = \frac{FV}{(1+r)^t}$$

\$10

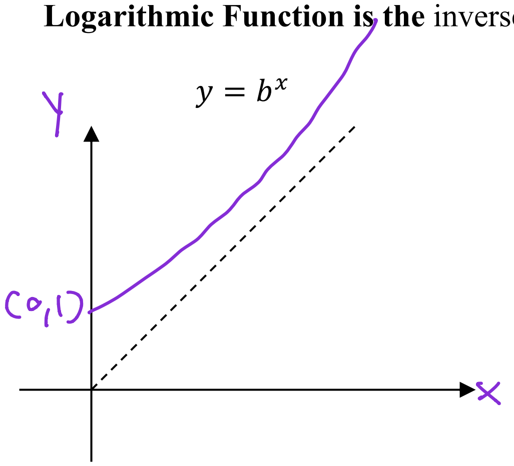
\$10 + r \$10

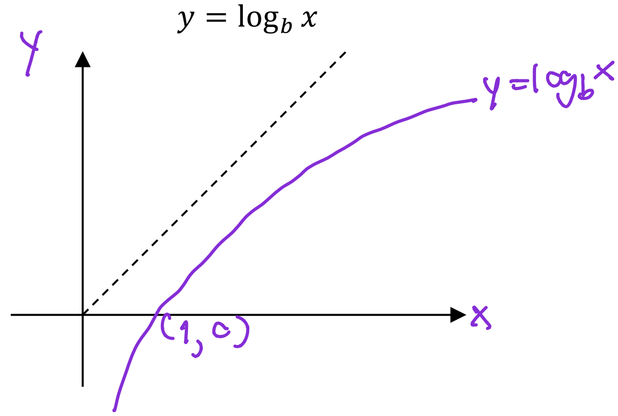
(1+r)\$10

$(1+r)^t$ \$10

Logarithmic Function

Logarithmic Function is the inverse function of exponential function.



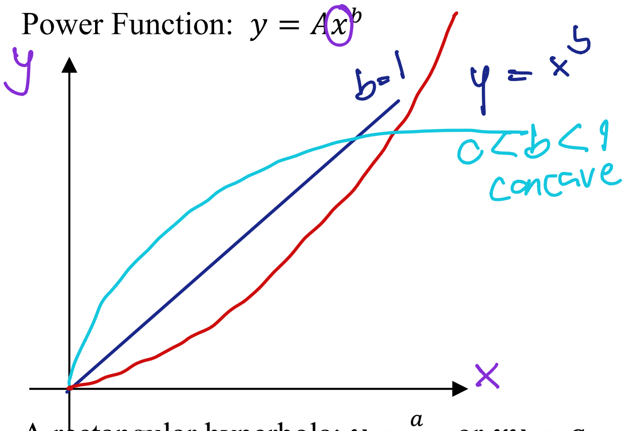


2.3.4 Other functions

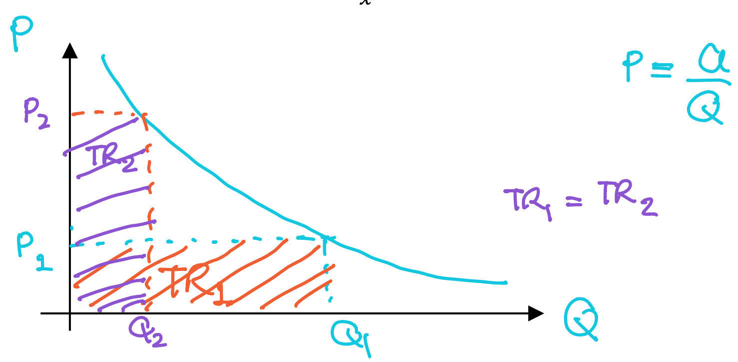
convex $b > 1$

$y = b^x$ exponential.

Power Function: $y = Ax^b$



A rectangular hyperbola: $y = \frac{a}{x}$, or $xy = a$



Example:………………………………