EE 320 Introductory Mathematical Economics (Section 046402)

Semester 1/2013

Homework 2

Due 17 September 2013

There are four questions in total. Each of them is worth 5 points.

1. a) (1 point each) Find the following determinants.

i) $\begin{vmatrix} 1 & -2 & 1 \\ 1 & 3 & 2 \\ 2 & 2 & 3 \end{vmatrix}$ = -1

ii) $\begin{vmatrix} 2 & -4 & 2 \\ 3 & 9 & 6 \\ 2 & 2 & 3 \end{vmatrix}$ = -6

iii) $\begin{vmatrix} 2 & 5 & 1 \\ 0 & 3 & 8 \\ 0 & 0 & 4 \end{vmatrix}$ = 24

b) (2 points) Find the inverse of the following matrix

$$A = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 2 & 3 \\ 3 & 0 & 2 \end{bmatrix}.$$

Ans. |A| = -9; $A^{-1} = -\frac{1}{9}\begin{bmatrix} 4 & -2 & -3 \\ 13 & 2 & -3 \\ -6 & -3 & 0 \end{bmatrix}$

2. Given the following supply and demand functions:

$$Q_D = 100 - 3P$$

$$Q_S = 80 + 2P$$

a) (1 point) Write the equilibrium condition for this market, and translate the system of equations into matrix notation.

Ans. Eq'm condition: $Q_D = Q_S$.

System of equations:

$$\begin{aligned} Q_D - Q_S &= 0 \\ Q_D \qquad + 3P &= 100 \\ Q_S - 2P &= 80 \end{aligned}$$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & 3 \\ 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} Q_D \\ Q_S \\ P \end{bmatrix} = \begin{bmatrix} 0 \\ 100 \\ 80 \end{bmatrix}$$

b) (1 points) Use matrix inversion to solve for the equilibrium quantity and equilibrium price.

Ans. $A^{-1} = \frac{1}{5}\begin{bmatrix} 3 & 2 & 3 \\ -2 & 2 & 3 \\ -1 & 1 & -1 \end{bmatrix}$

$$\Rightarrow \rightarrow \begin{bmatrix} Q_D \\ Q_S \\ P \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 3 & 2 & 3 \\ -2 & 2 & 3 \\ -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 100 \\ 80 \end{bmatrix} = \begin{bmatrix} 88 \\ 88 \\ 4 \end{bmatrix}$$

c) (3 points) Suppose that the government subsidizes the consumption of this good by giving the consumer \$5 per unit of the goods consumed. Use Cramer's rule to solve for (i) the equilibrium price paid by the consumer, (ii) the price received by the producer, and (iii) the amount of money the government needs for this subsidization.

Ans. (i) $P_d^* = P_s^* - 5 = \$7 - \$5 = \$2$

(ii) $P_s^* = \$7$

(iii) $Q^* = 94;\ S^* = 94x\$5 = \$470$

3. Consider the following IS-LM model:

Commodity market:

$Y = C + I + G_0 + X_0 - M$

$C = bY_d, \qquad (0 < b < 1)$

$Y_d = Y - T, \quad T \text{ is a constant.}$

$M = mY, \qquad (0 < m < 1)$

$I = I_0 - ar, \quad (I_0 > 0, a > 0)$

Money market:

$M_S = M_0$

$M_D = mY - hr, \quad (m > 0, h > 0)$

a) (2 points) Write out the explicit IS-LM system of equations, and write a matrix form of these IS-LM equations.

Ans. IS: $(1-b+m)Y + ar = -bT + I_0 + G_0 + X_0$

$$\Rightarrow Y = \frac{-bT+I_0+G_0+X_0-ar}{(1-b+m)}$$

LM: $mY - hr = M_0$

$$\Rightarrow Y = \frac{M_0+hr}{m}$$

Matrix form: $\begin{bmatrix} 1-b+m & a \\ m & -h \end{bmatrix}\begin{bmatrix} Y \\ r \end{bmatrix} = \begin{bmatrix} -bT+I_0+G_0+X_0 \\ M_0 \end{bmatrix}$

b) (3 points) Solve for the equilibrium level of national income and rate of interest by using Cramer's rule.

Ans.

$$Y^* = \frac{\begin{vmatrix} -bT+I_0+G_0+X_0 & a \\ M_0 & -h \end{vmatrix}}{\begin{vmatrix} 1-b+m & a \\ m & -h \end{vmatrix}} = \frac{h(-bT+I_0+G_0+X_0)+aM_0}{am+h(1-b+m)}$$

$$r^* = \frac{\begin{vmatrix} 1-b+m & -bT+I_0+G_0+X_0 \\ m & M_0 \end{vmatrix}}{\begin{vmatrix} 1-b+m & a \\ m & -h \end{vmatrix}} = \frac{m(-bT+I_0+G_0+X_0)-(1-b+m)M_0}{am+h(1-b+m)}$$

4. Consider an input-output model with two sectors: agriculture and manufactures, and labor is the primary input. The input-output table is given below.

Industry	**Input Demand Requirement**		**Final Demand ($)**
	Agriculture (X1)	**Manufacture (X2)**	
Agriculture	0.2	0.3	500
Manufacture	0.25	0.1	1400

a) (2 points) Write the input-coefficient matrix and the Leontif system in the matrix form.

Ans. $A = \begin{bmatrix} 0.2 & 0.3 \\ 0.25 & 0.1 \end{bmatrix}$

b) (2 points) What level of output should each sector produce in order to meet the final demands?

Ans. $\begin{bmatrix} X_1^* \\ X_2^* \end{bmatrix} = \begin{bmatrix} 0.8 & -0.3 \\ -0.25 & 0.9 \end{bmatrix}^{-1}\begin{bmatrix} 500 \\ 1400 \end{bmatrix} = \frac{1}{0.645}\begin{bmatrix} 0.9 & 0.3 \\ 0.25 & 0.8 \end{bmatrix}\begin{bmatrix} 500 \\ 1400 \end{bmatrix} = \begin{bmatrix} 1348.837 \\ 1930.233 \end{bmatrix}$

c) (1 point) Determine the level of primary input required for this economy.

Ans. a_{01} = 0.55; a_{02}= 0.6

Labor requirement: a0 = (0.55*1348.837)+(0.6*1930.233) = 1900