Appendix 1.1

$$\max \quad \ln\left(c_{1,t}\right) + \beta \ln\left(c_{2,t+1}\right)$$

s.t.

$c_{1,t} = y_t - v_t m_t = y_t - q_t$

and

$c_{2,t+1} = v_{t+1} m_t = \frac{v_{t+1}}{v_t} q_t$

$$\max_{q_t} \ln(y_t - q_t) + \beta \ln\left(\frac{v_{t+1}}{v_t} q_t\right)$$

We choose for q_t that set the derivative of the utility function, expressed in terms of q_t, equal to zero.

That is,

$$\frac{1}{y_t - q_t}(-1) + \beta \frac{1}{\frac{v_{t+1}}{v_t} q_t} \frac{v_{t+1}}{v_t} = 0$$

Optimal money holding (saving in the real-term) is then equal to

$$q_t^* = \frac{\beta}{1+\beta} y_t$$

Then, young consumption is equal to $c_{1,t}^* = y_t - q_t^* = y_t - \frac{\beta}{1+\beta} y_t = \frac{1}{1+\beta} y_t$

Old consumption is equal to $c_{2,t+1}^{*} = \frac{v_{t+1}}{v_t}\frac{\beta}{1+\beta}y_t$

Note from the three equations, we find that $\frac{\partial q_t^*}{\partial \beta} > 0$, $\frac{\partial c_{1,t}^*}{\partial \beta} < 0$ and $\frac{\partial c_{2,t+1}^*}{\partial \beta} > 0$. Intuitively, as "$\beta$" increases, household gives more weight of the benefit derived from consumption in the future. (He/she cares more about the level of utility derived from future-period consumption.) As a result, household would save more, and cut current consumption.