# Generalized Autoregressive Conditional Heteroscedastic (GARCH) Models

## Example Simulated Data

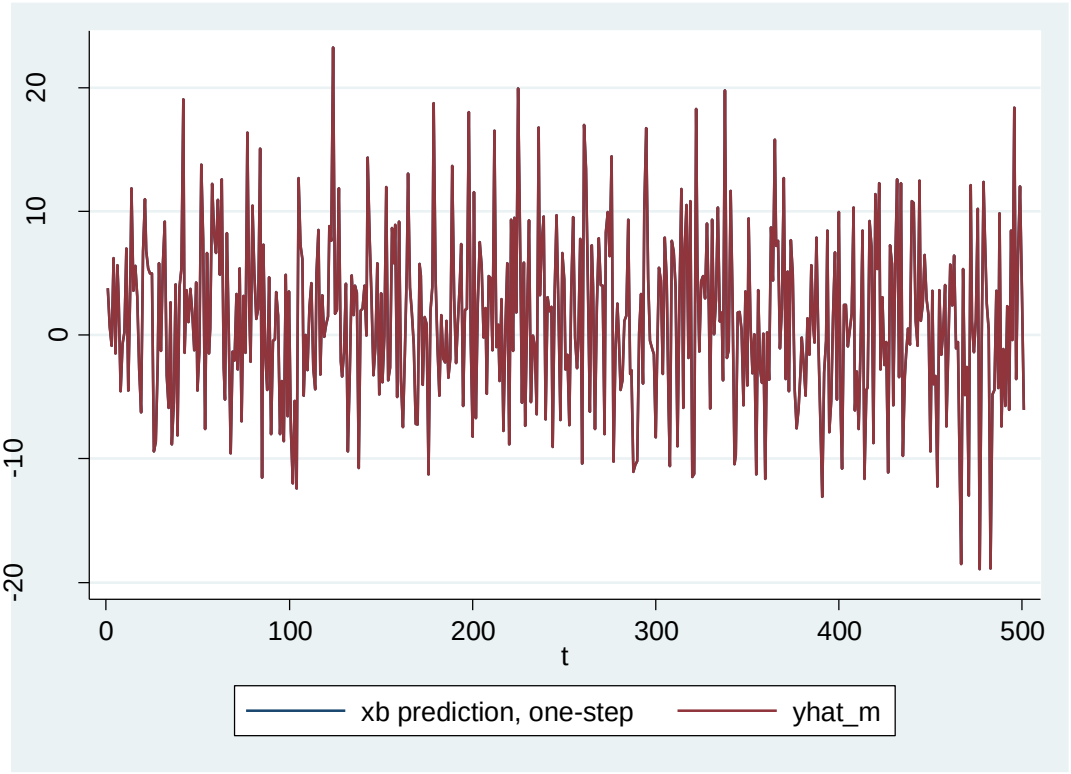
```
set obs 501
g t=_n
tsset t
g s2=1 in 1
g u=rnormal(0,s2) in 1
g u2=u^2 in 1
forvalue i=2(1)501 {
  replace s2=0.3+0.3*l.u2+0.3*l.s2 in `i'
  replace u=rnormal(0,s2) in `i'
  replace u2=u^2 in `i'
}
g x=rnormal(1,10)
g y=0.5+0.7*x+u
reg y x if t>1
estat archlm
arch y x  if t>1, arch(1) garch(1) nolog
predict s2hat, v
twoway (line s2hat t) (scatter s2 t)
```

```
. set obs 501
number of observations (_N) was 0, now 501

. g t=_n

. tsset t
        time variable:  t, 1 to 501
                delta:  1 unit

. g s2=1 in 1
(500 missing values generated)

. g u=rnormal(0,s2) in 1
(500 missing values generated)

. g u2=u^2 in 1
(500 missing values generated)

. forvalue i=2(1)501 {
  2.   replace s2=0.3+0.3*l.u2+0.3*l.s2 in `i'
  3.   replace u=rnormal(0,s2) in `i'
  4.   replace u2=u^2 in `i'
  5. }

. g x=rnormal(1,10)

. g y=0.5+0.7*x+u

. reg y x if t>1
      Source |       SS           df       MS      Number of obs   =       500
-------------+----------------------------------   F(1, 498)       =  63539.73
       Model |  23398.9523         1  23398.9523   Prob > F        =    0.0000
    Residual |  183.391992       498  .368257012   R-squared       =    0.9922
-------------+----------------------------------   Adj R-squared   =    0.9922
       Total |  23582.3443       499   47.259207   Root MSE        =    .60684

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           x |   .7013338   .0027823   252.07   0.000     .6958673    .7068002
       _cons |   .4743123   .0273053    17.37   0.000     .4206645    .5279601
------------------------------------------------------------------------------

. est store linear
```

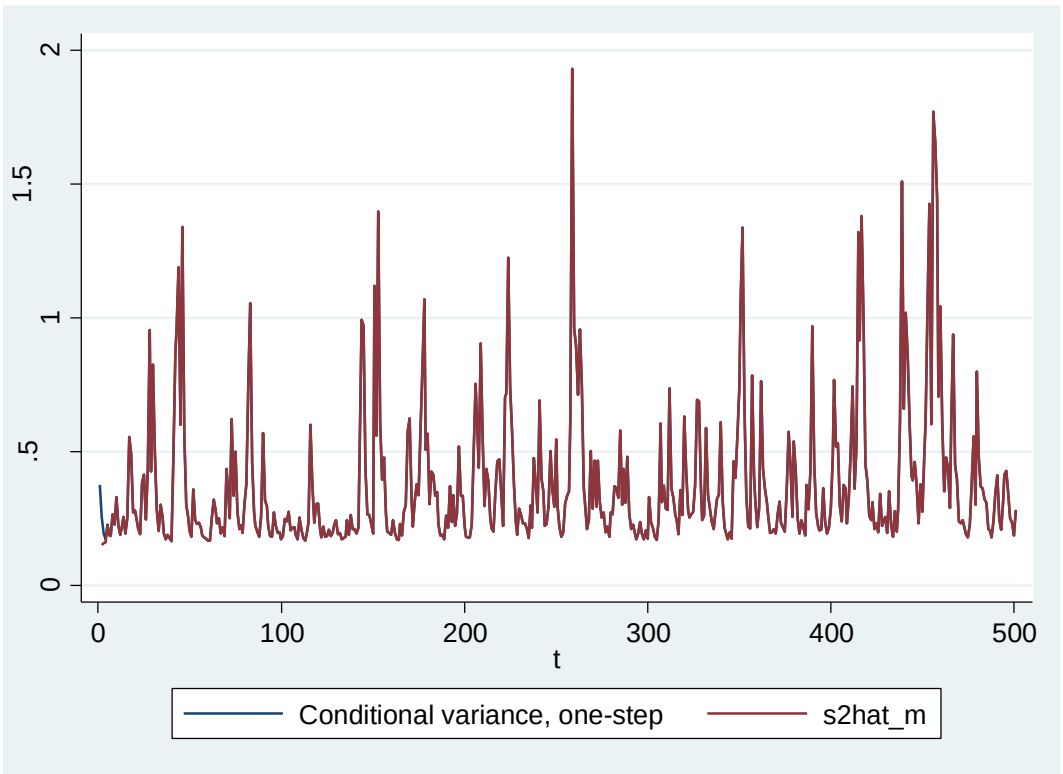
```
. sca logLr=e(ll)

. sca list logLr
      logLr = -458.72372

. estat archlm
LM test for autoregressive conditional heteroskedasticity (ARCH)
-------------------------------------------------------------------------------
    lags(p)  |          chi2               df                 Prob > chi2
-------------+-----------------------------------------------------------------
       1     |         35.821               1                   0.0000
-------------------------------------------------------------------------------
         H0: no ARCH effects      vs.  H1: ARCH(p) disturbance

. predict ehat, r

. g ehat2=ehat^2

. reg ehat2 l.ehat2 if t>2

      Source |       SS           df       MS      Number of obs   =       499
-------------+----------------------------------   F(1, 497)       =     38.44
       Model |  11.1919499         1  11.1919499   Prob > F        =    0.0000
    Residual |  144.716148       497  .291179372   R-squared       =    0.0718
-------------+----------------------------------   Adj R-squared   =    0.0699
       Total |  155.908098       498  .313068469   Root MSE        =    .53961

------------------------------------------------------------------------------
       ehat2 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       ehat2 |
         L1. |   .2679035   .0432121     6.20   0.000     .1830025    .3528045
             |
       _cons |   .2690816   .0289072     9.31   0.000     .2122862    .3258769
------------------------------------------------------------------------------

. sca r2=e(r2)

. sca list r2
        r2 =  .07178556

. sca N=e(N)

. sca list N
         N =        499

. sca archlm=r2*N

. sca list archlm
    archlm =  35.820994

. arch y x  if t>1, arch(1) garch(1) nolog

ARCH family regression

Sample: 2 - 501                                 Number of obs   =        500
Distribution: Gaussian                          Wald chi2(1)    =   84227.65
Log likelihood =  -426.751                      Prob > chi2     =     0.0000

------------------------------------------------------------------------------
             |                 OPG
           y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
y            |
           x |   .7028459   .0024218   290.22   0.000     .6980993    .7075925
       _cons |   .4758044    .023265    20.45   0.000     .4302058    .5214029
-------------+----------------------------------------------------------------
```

```
ARCH         |
        arch |
         L1. |   .3970325    .093721     4.24   0.000     .2133428    .5807223
             |
       garch |
         L1. |   .3355659    .112419     2.98   0.003     .1152287    .5559032
             |
       _cons |    .105968   .0289994     3.65   0.000     .0491302    .1628059
------------------------------------------------------------------------------

. est store garch

. lrtest linear garch, force

Likelihood-ratio test                                 LR chi2(3)  =     63.95
(Assumption: linear nested in garch)                  Prob > chi2 =    0.0000

. test [ARCH]l1.arch [ARCH]l1.garch

 ( 1)  [ARCH]L.arch = 0
 ( 2)  [ARCH]L.garch = 0

           chi2(  2) =   67.73
         Prob > chi2 =    0.0000

. predict s2hat, v

. predict yhat, xb

. mat beta=e(b)

. mat list beta

beta[1,5]
            y:          y:      ARCH:      ARCH:      ARCH:
                                   L.         L.
            x       _cons       arch      garch       _cons
y1  .70284588  .47580435  .39703251  .33556595  .10596802

. sca b0=el(beta,1,2)

. sca b1=el(beta,1,1)

. sca a0=el(beta,1,5)

. sca d1=el(beta,1,4)

. sca a1=el(beta,1,3)

. g yhat_m=b0+b1*x

. g uhat_m=y-yhat_m

. g uhat2_m=uhat_m^2

. predict uhat, r

. g uhat2=uhat^2

. g s2hat_m=a0+d1*l.uhat2_m+a1*l.uhat2_m in 2
(500 missing values generated)

. replace s2hat_m=a0+d1*l.s2hat_m+a1*l.uhat2_m if t>2
(499 real changes made)
```

```
. line yhat yhat_m t
```




```
. twoway (line s2hat t) (line s2hat_m t)
```



**Example**

$$SET_t = \beta_0 + \beta_1 IBR + \beta_2 GOLDB + \beta_3 USDS + \varepsilon_t$$

where: $SET_t$ = Return on Stock Exchange of Thailand (SET).

$IBR_t$ = Inter-bank rate.

$GOLDB_t$ = Gold price.

$USDS_t$ = Exchange rate ($US/฿Baht).

$\varepsilon_t$ = Residual which has GARCH(p,q) process:

$$\sigma_t^2 = \alpha_0 + \delta_1\sigma_{t-1}^2 + \delta_2\sigma_{t-2}^2 + \cdots + \delta_p\sigma_{t-p}^2 + \alpha_1\varepsilon_{t-1}^2 + \alpha_2\varepsilon_{t-2}^2 + \cdots + \alpha_q\varepsilon_{t-q}^2$$

**Testing GARCH Effect**

Firstly, estimate the model using OLS without ARCH(p) process.

```
. regress set ibr usds goldb

      Source |       SS       df       MS              Number of obs =     118
-------------+------------------------------           F(  3,   114) =  103.43
       Model |  215609.598     3  71869.8659           Prob > F      =  0.0000
    Residual |  79215.8193   114  694.875608           R-squared     =  0.7313
-------------+------------------------------           Adj R-squared =  0.7242
       Total |  294825.417   117  2519.87536           Root MSE      =   26.36

------------------------------------------------------------------------------
         set |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         ibr |  -142.0644   68.73617    -2.07   0.041    -278.2302   -5.898551
        usds |  -78.06484    5.30484   -14.72   0.000    -88.57369     -67.556
       goldb |    .029042   .0192431     1.51   0.134    -.0090784    .0671624
       _cons |   3698.659   332.8835    11.11   0.000     3039.219    4358.099
------------------------------------------------------------------------------

. estat archlm
LM test for autoregressive conditional heteroskedasticity (ARCH)
-----------------------------------------------------------------------------
    lags(p)  |          chi2               df                 Prob > chi2
-------------+---------------------------------------------------------------
       1     |         63.052               1                   0.0000
-----------------------------------------------------------------------------
         H0: no ARCH effects      vs.  H1: ARCH(p) disturbance
```

In this case, since p-value of the ARCH effect test (F-statistic or Chi-Square (Obs*R-squared = (117*0.538907) = 63.05211)) is less than level of significance 0.05, thus, null hypothesis that there is no ARCH effect is rejected, thus, there exists significant ARCH effect in this model with 0.05 significant level.

**Identify Order (p,q) and Estimation**

The next step is to identify order of GARCH(p,q) by estimating GARCH models in several orders and choose the model with the lowest AIC or SIC.

**Estimate GARCH(p,q) using MLE**

```
. arch set ibr usds goldb, arch(1) nolog
BFGS stepping has contracted, resetting BFGS Hessian (0)
      ...
BFGS stepping has contracted, resetting BFGS Hessian (4)

ARCH family regression

Sample:  02dec2003 to 28mar2004                 Number of obs      =       118
                                                Wald chi2(3)       =   1122.15
Log likelihood = -517.3025                      Prob > chi2        =    0.0000
------------------------------------------------------------------------------
             |                 OPG
         set |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
set          |
         ibr |  -64.13751   41.80188    -1.53   0.125    -146.0677    17.79268
        usds |  -72.48378    2.63951   -27.46   0.000    -77.65712   -67.31043
       goldb |   .0590191   .0138355     4.27   0.000      .031902    .0861362
       _cons |   3178.165    197.789    16.07   0.000     2790.506    3565.825
-------------+----------------------------------------------------------------
ARCH         |
        arch |
         L1. |   .9245129   .3233346     2.86   0.004     .2907888    1.558237
       _cons |   93.59632   38.20728     2.45   0.014     18.71142    168.4812
------------------------------------------------------------------------------

. estat ic
-----------------------------------------------------------------------------
       Model |    Obs    ll(null)   ll(model)     df          AIC          BIC
-------------+---------------------------------------------------------------
           . |    118           .   -517.3025      6     1046.605    1063.229
-----------------------------------------------------------------------------

. arch set ibr usds goldb, arch(1/2) nolog
BFGS stepping has contracted, resetting BFGS Hessian (17)

ARCH family regression

Sample:  02dec2003 to 28mar2004                 Number of obs      =       118
                                                Wald chi2(3)       =   1158.49
Log likelihood = -516.9879                      Prob > chi2        =    0.0000
------------------------------------------------------------------------------
             |                 OPG
         set |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
set          |
         ibr |  -61.07753   33.81901    -1.81   0.071    -127.3616    5.206507
        usds |  -72.49558   2.831238   -25.61   0.000     -78.0447   -66.94645
       goldb |   .0599955   .0140527     4.27   0.000     .0324528    .0875382
       _cons |    3167.73   207.2899    15.28   0.000      2761.45    3574.011
-------------+----------------------------------------------------------------
ARCH         |
        arch |
         L1. |   .9952712   .3383216     2.94   0.003      .332173    1.658369
         L2. |  -.1255426   .1038731    -1.21   0.227    -.3291301     .078045
       _cons |    117.519   40.07261     2.93   0.003     38.97815    196.0599
------------------------------------------------------------------------------

. estat ic
-----------------------------------------------------------------------------
       Model |    Obs    ll(null)   ll(model)     df          AIC          BIC
-------------+---------------------------------------------------------------
           . |    118           .   -516.9879      7     1047.976    1067.371
-----------------------------------------------------------------------------
```

**Estimate GARCH(1,1)**

```
. arch set usds, arch(1) garch(1) nolog

ARCH family regression

Sample:  02dec2003 to 28mar2004                 Number of obs      =       118
                                                Wald chi2(1)       =    882.55
Log likelihood = -518.5556                      Prob > chi2        =    0.0000

------------------------------------------------------------------------------
             |                 OPG
         set |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
set          |
        usds |  -74.04275   2.492366   -29.71   0.000    -78.92769    -69.1578
       _cons |   3607.771   97.79026    36.89   0.000     3416.106    3799.437
-------------+----------------------------------------------------------------
ARCH         |
        arch |
         L1. |   1.063916    .297246     3.58   0.000     .4813242    1.646507
       garch |
         L1. |  -.1115611   .1054421    -1.06   0.290    -.3182239     .0951017
       _cons |   112.9776   46.69068     2.42   0.016     21.46551    204.4896
------------------------------------------------------------------------------
```