Sensitivity analysis (y; x)

how y is sensitive to x

· slope

Slope v.s. elasticity

General notation:

Let ε_x^y be the elasticity of "y" with respect to "x". The elasticity is then defined as follow

$0(x_0, y_0)$; $1(x_1, y_1)$

$\%\Delta y = \frac{y_1 - y_0}{y_0}$

$\%\Delta x = \frac{x_1 - x_0}{x_0}$

$$\frac{\%\Delta y}{\%\Delta x} = \frac{y_1 - y_0}{x_1 - x_0} * \frac{x_0}{y_0} = \frac{\Delta y}{\Delta x} * \frac{x_0}{y_0} = Slope * \frac{x_0}{y_0}$$

- Elasticity depends on the starting value. (starting point matters!)

---

mid-point Elasticity $= \frac{\%\Delta y}{\%\Delta x} = \dfrac{\frac{y_1 - y_0}{avg(y_1, y_0)}}{\frac{x_1 - x_0}{avg(x_1, x_0)}}$

Example 2.F: *Elasticity of demand*

Suppose we know any two points on the demand curve, namely A$(Q_0 = 4, P_0 = 1)$ and B$(Q_1 = 1, P_1 = 5)$ where A is the initial point and B is the terminal point

Price Elasticity of demand

Elasticity of demand ~~of demand~~ is then defined as

$$\frac{\%\Delta Q}{\%\Delta P} = \frac{Q_1 - Q_0}{P_1 - P_0} * \frac{P_0}{Q_0} = \frac{\Delta Q}{\Delta P} * \frac{P_0}{Q_0}$$

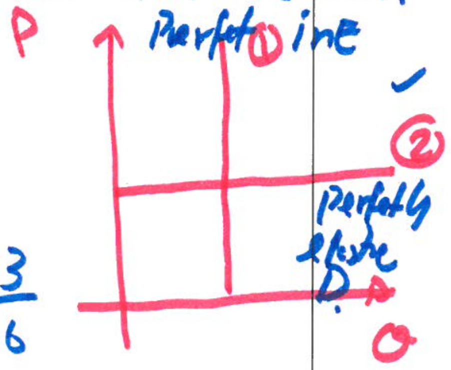
∞ (Perfectly) Elastic demand > 1

0 (Perfectly) Inelastic Demand < 1

· Unit Elastic Demand.

$A \rightarrow B$

$$\%\Delta Q = \frac{\frac{1-4}{4}}{\frac{5-1}{1}} = \frac{-\frac{3}{4}}{4} = -\frac{3}{16} \Rightarrow \frac{3}{16}$$



absolute value

More about the elasticity of demand

- Own-price elasticity of demand is always less than or equal to zero! (why?)
    - Negative sign only implies that negative relationship between price and quantity demand.
    - Absolute number of the figure tells us about the degree of sensitivity of quantity demand to price adjustment.

- Conventionally, we classify the case for elasticity of demand into three cases.

$$\left|\frac{\%\Delta Q}{\%\Delta P}\right| : \begin{array}{lcc} > 1 & ; & \textit{elastic} \\ = 1 & ; & \textit{unit elastic} \\ < 1 & ; & \textit{inelastic} \end{array}$$

Discuss about some important implications of elasticity for practical use.

P increase . ⇐ $|\varepsilon| < 1 \Rightarrow |\%\Delta Q| < |\%\Delta P|$

- Boosting revenue

Elastic Demand $(|\varepsilon| > 1)$ $\left|\frac{\%\Delta Q}{\%\Delta P}\right| > 1$

"P↑" P decrease

$\Rightarrow |\%\Delta Q| > |\%\Delta P|$

- Optimal commodity taxation

Tax high in the market w in Elasticity demand

Ramsey (1928) → lowest D.W.L.

Example 2.G: Elasticity of linear demand curve and some important properties

Suppose that $p = 10 - Q$, derive the formula for the elasticity of demand

$$\varepsilon_p^Q = \frac{\%\Delta Q}{\%\Delta P} = \frac{\frac{\Delta Q}{Q}}{\frac{\Delta P}{P}} = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q}$$

$$\frac{\Delta Q}{\Delta P} = \frac{1}{\frac{\Delta P}{\Delta Q}}$$

$$\frac{\Delta P}{\Delta Q} = -1 \rightarrow \frac{\Delta Q}{\Delta P} = \frac{1}{-1}$$

$$\varepsilon_p^Q = (-1) \cdot \frac{P}{Q} \quad ; \quad \cancel{q = 10 ; P} \quad q = 5 ; P = 15$$

$$|\varepsilon_p^Q|$$

$$P = 10 - Q$$

$$\varepsilon_p^Q = (-1)\left(\frac{10-Q}{Q}\right) = \frac{10-Q}{Q} = \frac{10}{Q} - 1$$

$Q\uparrow \rightarrow |\varepsilon_p^Q|$ lower

$|\varepsilon^Q_P| = \frac{10-Q}{Q} = \frac{10}{Q}-1$; $\boxed{Q\uparrow \to |\varepsilon^Q_P|\downarrow}$

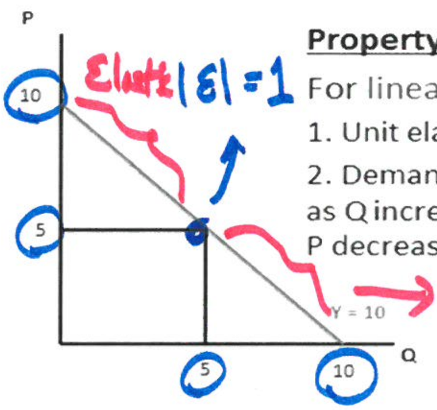
Linear function: slope v.s. elasticity

$\frac{10}{Q}-1=1$

$\frac{10}{Q}=2$

$Q=5$

$P=5$



Property:

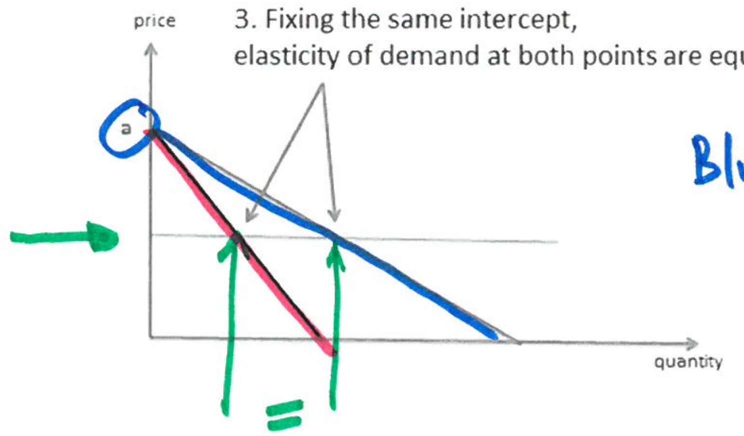
For linear demand curve:

1. Unit elasticity at the mid point.
2. Demand becomes more inelastic as Q increases, (correspondingly to P decreases)

InElastic demand.

Linear function: slope v.s. elasticity

3. Fixing the same intercept, elasticity of demand at both points are equal!!

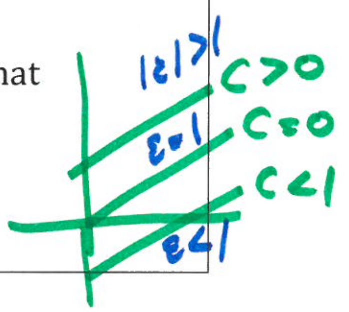


Blue flatter than red

Exercise 2.A:

2.A.1) Given a demand function by $p = a - bQ$, derive the formula for the elasticity of demand, and show that the third property holds

2.A.2) Given the market supply $p = c + dQ$ where $d \geq 0$, show that
(i) elasticity of supply is always greater than 1 if $c > 0$,
(ii) elasticity of supply is always eqaul to 1 if $c = 0$,
(iii) elasticity of supply is always less to 1 if $c < 0$.



Example 2.H: Aggregate expenditure function

Consider a simple macroeconomic model where

$C = 100 + 0.7Y$ → MPC

$I = 200 + 0.1Y - 100 * r$

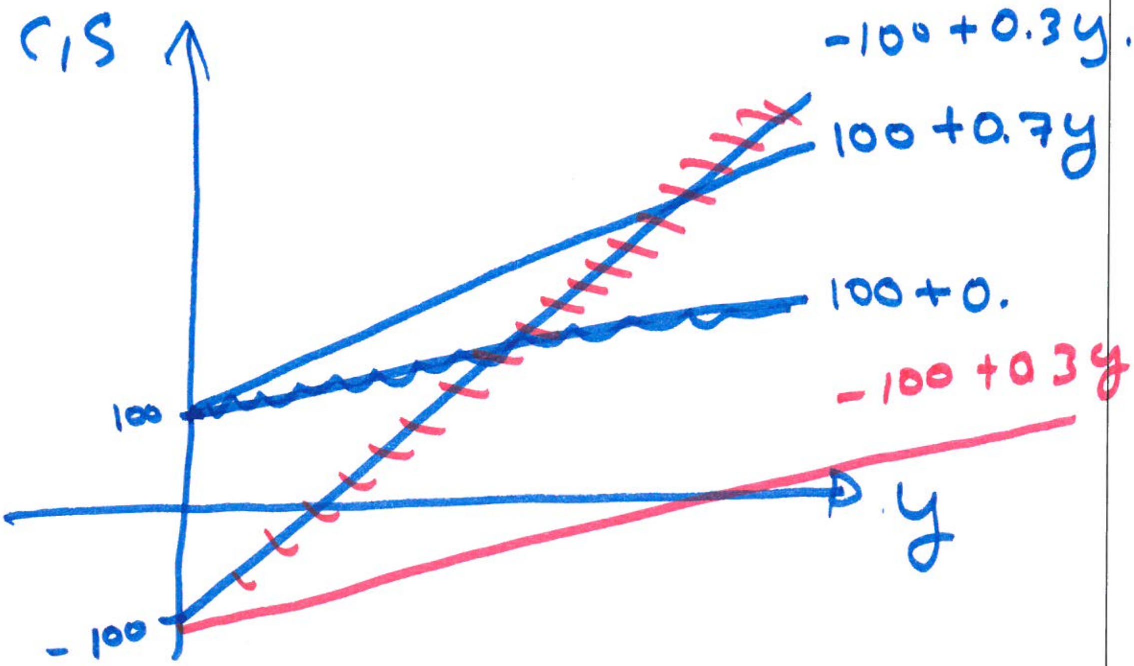
- Derive the saving function and plot both consumption and saving function in the same figure

$S \equiv y - C$

$\equiv y - (100 + 0.7y)$

$S = -100 + 0.3y$ → MPS

$\boxed{MPC + MPS = 1}$



- Derive the Aggregate Expenditure function. How does the aggregate expenditure behave over the course of changes in the interest rate?

$$AE \equiv C + I$$

$$= 100 + 0.7y + 200 + 0.1y - 100 \cdot r$$

$$= 300 + 0.8y - 100 \cdot r$$

$$AE(y, r)$$

$y\uparrow \rightarrow AE\uparrow \qquad \frac{\Delta AE}{\Delta y} = 0.8$ (r is fixed)

$r\uparrow \rightarrow AE\downarrow \qquad \frac{\Delta AE}{\Delta r} = -100$ (y is fixed.)

2.2.2 Non-linear functions

Drawbacks of the linear representation.

- Constant slope is a big deal of the problem.

- SES Panel survey shows that the rich tend to consume less out of each additional unit of income they have earned.
  - Thus, MPC should be decreasing as income increases.

- So, neither the slope of consumption function should be constant nor the form of consumption function could be truly represented by a linear equation. (In fact, the consumption function should be concave!)

- Two broad classes of non-linear functions commonly used.
  - Polynomial function
  - Exponential/Logarithm function

2.2.2a) Polynomial function

Polynomial takes the form

$$y = a_0 + a_1 x + a_2 x^2 + \cdots + a_N x^N$$

N = 1 ➔ linear ➔ straight line

N = 2 ➔ Quadratic function ➔ Parabola

N = 3 ➔ Cubic function ➔ Don't know, but who care

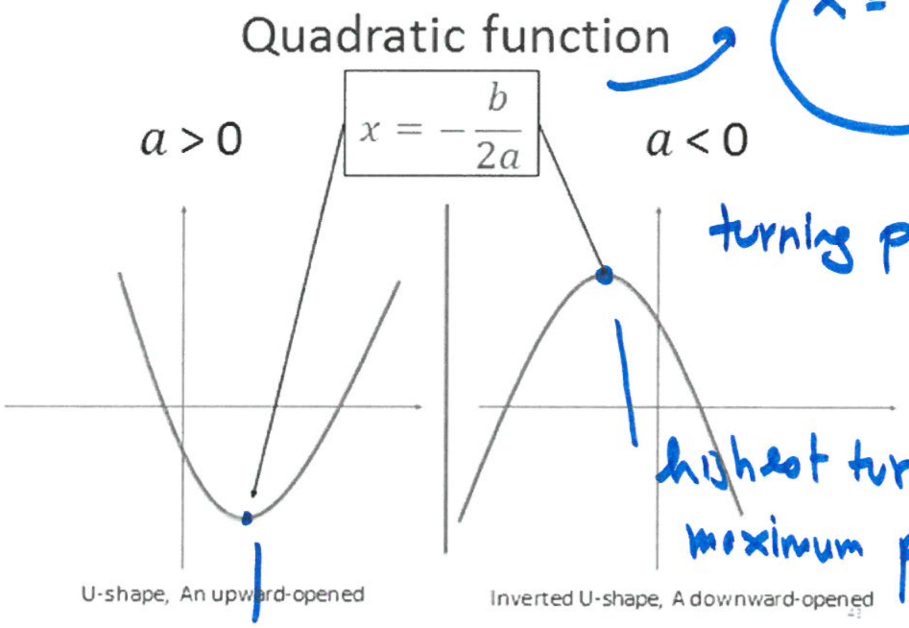
The Quadratic function

Consider the polynomial function with degree 2.

- Let's write the function in the form that you are used to.

$$y = ax^2 + bx + c$$

$x = -\frac{b}{2a}$



turning point

highest turning point

maximum point

minimum point

(lowest turning point

Example 2.I: A monopolist firm faces the market demand given by P = 10 - Q. Consider the following questions if the cost function C(Q) = 4Q.

- What is the revenue-maximizing level of output?

Output (Q) ⇒ maximizes the revenue.

$R = P \cdot Q$ $a=-1, b=10, c=0$ → inverted u-shape parabola

$R(Q) = (10-Q)\cdot Q = 10Q - Q^2$

Q ⇒ generate max/min value of R

$Q = -b/2a = \frac{-10}{2(-1)} = 5$ units ⇒ output maximize Revenue

- What is the break-even output?

$\pi = R - C$

$= (10Q - Q^2) - (4Q)$

$\pi(Q) := 6Q - Q^2$ → $6Q - Q^2 = 0$

$(6-Q)(Q) = 0$

Q such that $\pi = 0$ $Q = 0 ; Q = 6$

- What is the profit-maximizing level of output?

$\pi(Q) = 6Q - Q^2$ ⇒ Parabola inverted u

$P = 10 - Q = 10 - 6 = 4$

$R = 24$, $C = 24$ } 0

$Q \Rightarrow \frac{-b}{2a} = \frac{-6}{2(-1)}$

maximized profit $= 3 \rightarrow P = 7 : (10-3)$

$\Rightarrow \pi^* = 21 - 12 = 9$ $C = 4(3) = 12$