



B.E. International Program

Faculty of Economics, Thammasat University



EE 320 Introductory Mathematical Economics (Section 046402)

Semester 1/2013

Practice Problem 4 (Matrix Algebra and Applications)

1. Given

$$A = \begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix}; \quad B = \begin{bmatrix} 5 & 1 \\ -2 & 6 \end{bmatrix}$$

- Find AB and BA .
- Show that $(AB)' = B'A'$.

2. Given

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}; \quad B = \begin{bmatrix} 0 & 3 & 2 \\ 4 & 1 & 2 \end{bmatrix}; \quad C = \begin{bmatrix} 3 & 1 \\ -1 & 2 \\ 2 & 0 \end{bmatrix}$$

- Show that $(A+B)' = A'+B'$.
- Show that $(AC)B = A(CB)$.
- Show that $C(A+B) = CA + CB$.

3. Write the following systems as matrix equations:

a) $x_1 + 2x_2 + x_3 = 4$

$$x_1 - x_2 + x_3 = 5$$

$$x_1 - x_2 + 2x_3 = 15$$

b) $2x_1 - 3x_2 + x_3 = 0$

$$x_1 + x_2 - x_3 = 0$$

(Questions 4 – 12 are from Sydsaeter and Hammond, 2008)

4. For what values of a is $\begin{bmatrix} a & a^2 - 1 & -3 \\ a + 1 & 2 & a^2 + 4 \\ -3 & 4a & -1 \end{bmatrix}$ symmetric?

5. An $n \times n$ matrix \mathbf{P} is said to be orthogonal if $\mathbf{P}'\mathbf{P} = \mathbf{I}_n$.

a) For $\lambda = \pm 1/\sqrt{2}$, show that $\mathbf{P} = \begin{bmatrix} \lambda & 0 & \lambda \\ \lambda & 0 & -\lambda \\ 0 & 1 & 0 \end{bmatrix}$ is orthogonal.

b) Show that the 2×2 matrix $\begin{bmatrix} p & -q \\ q & p \end{bmatrix}$ is orthogonal if and only if $p^2 + q^2 = 1$.

6. Find the following determinants.

a) $\begin{vmatrix} 1 & -1 & 0 \\ 1 & 3 & 2 \\ 1 & 0 & 0 \end{vmatrix}$

b) $\begin{vmatrix} 1 & -1 & 0 \\ 1 & 3 & 2 \\ 1 & 2 & 1 \end{vmatrix}$

c) $\begin{vmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{vmatrix}$

d) $\begin{vmatrix} a & 0 & b \\ 0 & e & 0 \\ c & 0 & d \end{vmatrix}$

7. Let $A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 1 & -1 \end{bmatrix}$.

Calculate AB , $|A|$, $|B|$, $|A| \cdot |B|$ and $|AB|$.

8. Prove that the inverse of $\begin{bmatrix} 1 & 1 & -3 \\ 2 & 1 & -3 \\ 2 & 2 & 1 \end{bmatrix}$ is $\begin{bmatrix} -1 & 1 & 0 \\ 8/7 & -1 & 3/7 \\ -2/7 & 0 & 1/7 \end{bmatrix}$.

9. Use Cramer's rule to solve the following systems of equations. Check your answers.

a)
$$\begin{aligned}x_1 - x_2 + x_3 &= 2 \\x_1 + x_2 - x_3 &= 0 \\-x_1 - x_2 - x_3 &= -6\end{aligned}$$

b)
$$\begin{aligned}x_1 - x_2 &= 0 \\x_1 + 3x_2 + 2x_3 &= 0 \\x_1 + 2x_2 + x_3 &= 0\end{aligned}$$

10. Consider the simple macro model described by the three equations

(i) $Y = C + A_0$ (ii) $C = a + b(Y-T)$ (iii) $T = d + tY$

where Y is income, C is consumption, T is tax revenue, A_0 is the constant (exogenous) autonomous expenditure, and a , b , d , and t are all positive parameters. Find the equilibrium values of the endogenous variables Y , C , and T by writing the equations in matrix form and applying Cramer's rule.

11. Examine for what values of the constants a and b the system of equations

$$\begin{aligned}ax + y &= 3 \\x + z &= 2 \\y + az + bu &= 6 \\y + u &= 1\end{aligned}$$

has a unique solution in the unknown x , y , z , and u . Find the unique solution (expressed in terms of a and b).

12. Consider an economy divided into an agricultural sector (A) and an industrial sector (I). To produce one unit in sector A requires $1/6$ unit from A and $1/4$ unit from I. To produce in sector I requires $1/4$ unit from A and $1/4$ unit from I. Suppose final demands in each of the two sectors are 60 units.

- Write down the Leontief system for this economy.
- Find the number of units that has to be produced in each sector in order to meet the final demands.

(Questions 13 – 14 are from Dowling, 2012)

13. Use matrix inversion to solve for the unknowns in the system of linear equations given below.

$$2x_1 + 4x_2 - 3x_3 = 12$$

$$3x_1 - 5x_2 + 2x_3 = 13$$

$$-x_1 + 3x_2 + 2x_3 = 17$$

14. Given the IS equation $0.3Y + 100r - 252 = 0$ and the LM equation $0.25Y - 200r - 176 = 0$. Use matrix inversion to solve for the equilibrium of national income and rate of interest.