10-2. The following table shows the one-year return distribution of Startup, Inc. Calculate

a. The expected return.

b. The standard deviation of the return.

Probability	40%	20%	20%	10%	10%
Return	–100%	–75%	–50%	–25%	1000%

a. $E[R] = -1(0.4) - 0.75(0.2) - 0.5(0.2) - 0.25(0.1) + 10(0.1) = 32.5\%$

b. $\text{Variance}[R] = (-1 - 0.325)^2\, 0.4 + (-0.75 - 0.325)^2\, 0.2 + (-0.5 - 0.325)^2\, 0.2$
$+ (-0.25 - 0.325)^2\, 0.1 + (10 - 0.325)^2\, 0.1$
$= 10.46$

Standard Deviation = $\sqrt{10.46} = 3.235 = 323.5\%$

10-4. You bought a stock one year ago for $50 per share and sold it today for $55 per share. It paid a $1 per share dividend today.

a. What was your realized return?

b. How much of the return came from dividend yield and how much came from capital gain?

Compute the realized return and dividend yield on this equity investment.

a. $R = \dfrac{1 + (55 - 50)}{50} = 0.12 = 12\%$

b. $R_{\text{div}} = \dfrac{1}{50} = 2\%$

$R_{\text{capital gain}} = \dfrac{55 - 50}{50} = 10\%$

The realized return on the equity investment is 12%. The dividend yield is 10%.

10-5. Repeat Problem 4 assuming that the stock fell $5 to $45 instead.

a. Is your capital gain different? Why or why not?

b. Is your dividend yield different? Why or why not?

Compute the capital gain and dividend yield under the assumption that the stock price has fallen to $45.

a. $R_{\text{capital gain}} = 45 - 50 / 50 = -10\%$. Yes, the capital gain is different, because the difference between the current price and the purchase price is different than in Problem 1.

b. The dividend yield does not change, because the dividend is the same as in Problem 1.

The capital gain changes with the new lower price; the dividend yield does not change.

10-7. The last four years of returns for a stock are as follows:

1	2	3	4
–4%	+28%	+12%	+4%

a. What is the average annual return?

b. What is the variance of the stock's returns?

c. What is the standard deviation of the stock's returns?

Given the data presented, make the calculations requested in the question.

a. $$\text{Average annual return} = \frac{-4\% + 28\% + 12\% + 4\%}{4} = 10\%$$

b. $$\text{Variance of returns} = \frac{(-4\% - 10\%)^2 + (28\% - 10\%)^2 + (12\% - 10\%)^2 + (4\% - 10\%)^2}{3}$$
$$= 0.01867$$

c. $$\text{Standard deviation of returns} = \sqrt{\text{variance}} = \sqrt{0.01867} = 13.66\%$$

The average annual return is 10%. The variance of return is 0.01867. The standard deviation of returns is 13.66%.

10-22. Consider the following two, completely separate, economies. The expected return and volatility of all stocks in both economies is the same. In the first economy, all stocks move together—in good times all prices rise together and in bad times they all fall together. In the second economy, stock returns are independent—one stock increasing in price has no effect on the prices of other stocks. Assuming you are risk-averse and you could choose one of the two economies in which to invest, which one would you choose? Explain.

A risk-averse investor would choose the economy in which stock returns are independent because this risk can be diversified away in a large portfolio.

11-1. You are considering how to invest part of your retirement savings. You have decided to put $200,000 into three stocks: 50% of the money in GoldFinger (currently $25/share), 25% of the money in Moosehead (currently $80/share), and the remainder in Venture Associates (currently $2/share). If GoldFinger stock goes up to $30/share, Moosehead stock drops to $60/share, and Venture Associates stock rises to $3 per share,

a. What is the new value of the portfolio?

b. What return did the portfolio earn?

c. If you don't buy or sell shares after the price change, what are your new portfolio weights?

a. Let n_i be the number of share in stock I, then

$$n_G = \frac{200{,}000 \times 0.5}{25} = 4{,}000$$

$$n_M = \frac{200{,}000 \times 0.25}{80} = 625$$

$$n_V = \frac{200,000 \times 0.25}{2} = 25,000.$$

The new value of the portfolio is

$$p = 30n_G + 60n_M + 3n_V$$

$$= \$232,500.$$

b. $\text{Return} = \frac{232,500}{200,000} - 1 = 16.25\%$

c. The portfolio weights are the fraction of value invested in each stock.

GoldFinger: $\frac{n_G \times 30}{232,500} = 51.61\%$

Moosehead: $\frac{n_M \times 60}{232,500} = 16.13\%$

Venture: $\frac{n_V \times 3}{232,500} = 32.26\%$

Lecture 10: Homework solution

11-2. You own three stocks: 600 shares of Apple Computer, 10,000 shares of Cisco Systems, and 5000 shares of Colgate-Palmolive. The current share prices and expected returns of Apple, Cisco, and Colgate-Palmolive are, respectively, $500, $20, $100 and 12%, 10%, 8%.

a. What are the portfolio weights of the three stocks in your portfolio?

b. What is the expected return of your portfolio?

c. Suppose the price of Apple stock goes up by $25, Cisco rises by $5, and Colgate-Palmolive falls by $13. What are the new portfolio weights?

d. Assuming the stocks' expected returns remain the same, what is the expected return of the portfolio at the new prices?

				Value	a.	b.	New Price	New Value	c.	d.
Apple	600	500	12	300000	0.30	3.6	525	315000	0.315	3.78
Cisco	10000	20	10	200000	0.20	2	25	250000	0.25	2.5
Colgate	5000	100	8	500000	0.50	4	87	435000	0.435	3.48
		Total		1000000		9.6				9.76

11-9. Suppose two stocks have a correlation of 1. If the first stock has an above average return this year, what is the probability that the second stock will have an above average return?

Because the correlation is perfect, they move together (always), so the probability is 1.

11-10. Arbor Systems and Gencore stocks both have a volatility of 40%. Compute the volatility of a portfolio with 50% invested in each stock if the correlation between the stocks is (a) + 1, (b) 0.50, (c) 0, (d) −0.50, and (e) −1.0. In which cases is the volatility lower than that of the original stocks?

stock vol	40%
corr	50-50 Port
1	40.0%
0.5	34.6%
0	28.3%
-0.5	20.0%
-1	0.0%

Volatility of portfolio is less if the correlation is < 1.

11-20. You currently hold a portfolio of three stocks, Delta, Gamma, and Omega. Delta has a volatility of 60%, Gamma has a volatility of 30%, and Omega has a volatility of 20%. Suppose you invest 50% of your money in Delta, and 25% each in Gamma and Omega.

a. What is the highest possible volatility of your portfolio?

b. If your portfolio has the volatility in (a), what can you conclude about the correlation between Delta and Omega?

a. Max vol = weighted average = 0.5(60%) + 0.25(30%) + 0.25(20%) = 42.5%

b. Correlation = 1 (otherwise there would be some diversification)

For Problems 23-26, suppose Johnson & Johnson and the Walgreen Company have expected returns and volatilities shown below, with a correlation of 22%.

	***E*[*R*]**	***SD*[*R*]**
Johnson & Johnson	7%	16%
Walgreen Company	10%	20%

11-27. A hedge fund has created a portfolio using just two stocks. It has shorted $35,000,000 worth of Oracle stock and has purchased $85,000,000 of Intel stock. The correlation between Oracle's and Intel's returns is 0.65. The expected returns and standard deviations of the two stocks are given in the table below:

	Expected Return	**Standard Deviation**
Oracle	12.00%	45.00%
Intel	14.50%	40.00%

a. What is the expected return of the hedge fund's portfolio?

b. What is the standard deviation of the hedge fund's portfolio?

a. The total value of the portfolio is $50m = (–$35 + $85). This means that the weight on Oracle is
–70% and the weight on Intel is 170%. The expected return is

$$\text{Expected return} = -0.7 \times 12\% + 1.7 \times 14.5\%$$
$$= 16.25\%.$$

$$\text{Variance} = (-0.7)^2 \times (0.45)^2 + (1.7)^2 \times (0.40)^2 + 2 \times (-0.7) \times (1.7) \times 0.65 \times 0.45 \times 0.40$$

b.

$$= 0.283165$$

Std dev = $(0.283165)^{0.5} = 53.2\%$

11-28. Consider the portfolio in Problem 27. Suppose the correlation between Intel and Oracle's stock increases, but nothing else changes. Would the portfolio be more or less risky with this change?

An increase in the correlation would increase the variance of the portfolio; meanwhile, the expected return of the portfolio would remain constant. The riskiness of the portfolio would increase.