Solution: Quiz 3

1. Prove or disprove the following statements.

(a) "*For all integers m and n, if $2m+n$ is odd then m and n are both odd.* "

(b) "*For all integers m, if m is even then $7m+3$ is odd.*"

Solution:

(a) This statement is false. We can **disprove** this by showing that its negation is true. The negation of this statement is given by
" there exist integers m and n such that $2m+n$ is odd but m and n are not both odd. "

Since the negation is in the form of existential statement, we can prove this by finding n and m such that $2m+n$ is odd, but n and m are not both odd (i.e. at least one value of n and m is even).

Consider $m=2$ and $n=1$ (i.e m and n are not both odd at the same time). We will see that

$$2m+n = 2(2)+1 = 5$$

is odd. That is, we found a counterexample that makes the negation true and therefore the given statement is false.

Remark: It is possible to use other values of m and n as counterexamples.
In fact, we can set m to be any even number of the form $m=2k$, for some integer k, and set n to be any odd number of the form $n=2a+1$, for some integer a. As a result,

$$2m+n = 2(2k)+2a+1 = 2(2k+a)+1 = 2c+1$$

is an odd number, since it is in the form of $2c+1$ where $c=2k+a$ is an integer (using $a, k \in \mathbb{Z}$). Hence the negation is true and the original statement is false. ■

(b) We can use the **direct proof** to show that this statement is true.

Suppose m is even. That is, $m=2a$ for some integer a. We want to show that $7m+3$ is odd. In particular, by substitution,

$$7m+3 = 7(2a)+3 = 14a+3 = 14a+2+1 = 2(7a+1)+1 = 2b+1$$

where $b=7a+1$ is an integer (because $a \in \mathbb{Z}$).

Therefore, $7m+3 = 2b+1$ is odd. ■