Assignment 3 & 4

3. a. Determine the level of revenue-maximizing output

Given that : Market demand function : $P = 40 + \frac{105}{Q} - \frac{3}{2}Q^2$

Cost function: $C(Q) = 6Q^3 - 81Q^2 - 175Q + 10$

$\Rightarrow$ Revenue function : $R(Q) = P(Q) \times Q$

$$= \left(40 + \frac{105}{Q} - \frac{3}{2}Q^2\right)Q = 40Q - \frac{3}{2}Q^3 + 105$$

Revenue maximization : $MR = 0$

$$MR = \frac{dR}{dQ} = 40 - \frac{9}{2}Q^2$$

$$MR = 0 \iff 40 - \frac{9}{2}Q^2 = 0$$

$$\frac{9}{2}Q^2 = 40$$

$$Q^2 = \frac{80}{9}$$

$$Q = \sqrt{\frac{80}{9}} = \frac{\sqrt{80}}{3}$$

$$MR' = \frac{d^2R}{dQ^2} = -9Q = -9\left(\frac{\sqrt{80}}{3}\right) = -3\cdot\sqrt{80} < 0 \ \forall x \in \mathbb{R}$$

$\therefore$ Revenue function has maximized value at $Q = \frac{\sqrt{80}}{3}$

Calculate elasticity of demand at that level of output

$$\varepsilon^d = \frac{dQ}{dP}\cdot\frac{P}{Q} = \frac{1}{\frac{dP}{dQ}}\cdot\frac{P}{Q}$$

$$\frac{dP}{dQ} = -105\frac{1}{Q^2} - 3Q$$

When $Q = \frac{\sqrt{80}}{3} \Rightarrow P\left(\frac{\sqrt{80}}{3}\right) = 40 + \frac{105}{\frac{\sqrt{80}}{3}} - \frac{3}{2}(\sqrt{80}/3)^2 = 40 + \frac{315}{\sqrt{80}} - \frac{40}{3} = 61.88$

$$\Rightarrow \varepsilon^d = \frac{1}{\frac{-105}{Q^2} - 3Q}\cdot\frac{P}{Q} = \frac{P}{\frac{-105}{Q} - 3Q^2} = \frac{61.88}{\frac{-105}{\frac{\sqrt{80}}{3}} - 3\left(\frac{\sqrt{80}}{3}\right)^2} = -1$$

b. Construct the profit function

$$\pi(Q) = R(Q) - C(Q)$$

$$\pi(Q) = 40Q - \frac{3}{2}Q^3 + 105 - (6Q^3 - 81Q^2 - 175Q + 10)$$

$$\pi(Q) = -\frac{15}{2}Q^3 + 81Q^2 + 215Q + 95 \quad \#$$

c. Determine the profit - maximizing level of output

$$\pi(Q) = -\frac{15}{2}Q^3 + 81Q^2 + 215Q + 95$$

First order condition : $\pi'(Q) = 0$

$$\pi'(Q) = -\frac{45}{2}Q^2 + 162Q + 215 = 0$$

$$\Delta = b^2 - 4ac$$

$$= 26244 - 4\left(-\frac{45}{2} \times 215\right) = 45594$$

$$\Rightarrow Q = \frac{-b \pm \sqrt{45594}}{2a} = \begin{cases} Q = \dfrac{-162 + \sqrt{45594}}{-45} < 0 \Rightarrow \text{not applicable} \\ Q^* = \dfrac{-162 - \sqrt{45594}}{-45} = \dfrac{162 + \sqrt{45594}}{45} \end{cases}$$

Second order condition

$$\pi''(Q) = -45Q + 162$$

$$\pi''\left(\frac{162 + \sqrt{45594}}{45}\right) = -45\left(\frac{162 + \sqrt{45594}}{45}\right) + 162 < 0$$

Therefore , $\pi(Q)$ is concave at $Q^* = \dfrac{162 + \sqrt{45594}}{45}$; the profit is maximized at Q^*.

D. If the government imposes the lump-sum tax on the monopolist, the total cost of the firm will increase. However, the marginal cost will not change, so the output remains the same. The price will be the same as well, but the profit will decrease due to the additional fixed cost which is lump-sum tax.

1. a. Show then domain of x where function is increasing.

$$Y = x^3 - 5x^2 + 7x - 5$$

$$\frac{dy}{dx} = 3x^2 - 10x + 7 = 0$$

Since $a + b + c = 0 \Leftrightarrow 3 + (-10) + 7 = 0$

$$\Rightarrow x_1 = 1 \; ; \; x_2 = \frac{c}{a} = \frac{7}{3}$$

Sign of $\dfrac{dy}{dx}$

x		1		7/3	
$\frac{dy}{dx}$	+	0	−	0	+

Therefore, function is increasing when $x \in (-\infty, 1) \cup (7/3, +\infty)$

b.. Define domain of the function

$D = \forall x \in \mathbb{R}$

- The function is not concave all over the domain.

Y concaves when $x \in (-\infty, \frac{7}{3})$, and

Y convexes when $x \in (\frac{7}{3} + \infty)$.

2. a. Derive AP_L and MP_L

$$AP_L = \frac{Q(L)}{L} = \frac{6L^2 - L^3}{L} = 6L - L^2$$

$$MP_L = \frac{dQ}{dL} = 12L - 3L^2$$

b. Find L that maximizes AP_L

$$\frac{dAP_L}{dL} = 6 - 2L = 0$$

$$L = \frac{6}{2} = 3 \text{ units } \#$$

c.

AP_L is maximized when $L = 3$

When AP_L is maximized, $MP_L = AP_L$

. $MP_L(L=3) = 12(3) - 3(3^2) = 9$

. $AP_L(L=3) = 6(3) - 3^2 = 9$

D derive MRP

$MRP = MP \times MR$

$$MP = \frac{dQ}{dL} = 12L - 3L^2$$

$TR = P \times Q$

Since $Q^d = 100 - 2P$

$$P = 50 - \frac{Q}{2}$$

$$\Rightarrow TR = \left(50 - \frac{Q}{2}\right)Q = 50Q - \frac{Q^2}{2}$$

$MR = 50 - Q$

$$\Rightarrow MRP = (12L - 3L^2) \times (50 - Q)$$

4. a. Short-run supply curve for each firm

From first order condition of profit maximization

$$\pi = P \times q - TC$$

$$= Pq - 0.5q^2 - 10q - 5$$

$$\frac{d\pi}{dq} = P - q - 10 = 0$$

$$\Rightarrow \quad q_i = P - 10$$

From second order condition

$$\frac{d^2\pi}{dq^2} = -1 < 0 \ \forall q \in \mathbb{R} \Rightarrow \pi \text{ concaves and is maximized at } q = P+10$$

Since $STC = 0.5q^2 + 10q + 5$

$$STVC = 0.5q^2 + 10q$$

$$SAVC = 0.5q + 10$$

if $q = 0 \Rightarrow$ min of $SAVC = 10$

$\therefore$ Therefore, each firm supply curve is $q_i^s = \begin{cases} P-10 & ; P > 10 \\ 0 & ; P < 10 \end{cases}$

Short-run supply curve for market as a whole $= Q^s = \begin{cases} 100(P-10) & ; P > 10 \\ 0 & ; P < 10 \end{cases}$

b. Find the market equilibrium

$$Q^s = Q^d = Q^*$$

$$1100 - 50P = 100(P-10)$$

$$1100 - 50P = 100P - 1000$$

$$2100 = 150P$$

$$\Rightarrow P^* = \frac{2100}{150} = 14 \text{ units } \#$$

$$\Rightarrow Q^* = 100(14-10) = 400 \text{ units } \#$$

Find each firm total profit

Each firm supplies : $q_i^* = \frac{Q^*}{100} = \frac{400}{100} = 4$ units

$\pi = P \cdot q - 0.5q^2 - 10q - 5 = 14 \times 4 - 0.5 \cdot 4^2 - 10 \cdot 4 - 5 = 56 - 53 = 3$ units #

c. If government imposed a €3 tax on each producer

$P^s = P^d - 3$

New equilibrium : $Q^* = Q^s = Q^d$

$$100(P^s - 10) = 1100 - 50P^d$$

$$100(P^d - 3 - 10) = 1100 - 50P^d$$

$$100P^d - 1300 = 1100 - 50P^d$$

$$150P^d = 1100 + 1300$$

$$P^d = \frac{2400}{150} = 16$$

$$P^s = P^d - 3 = 16 - 3 = 13$$

$$Q^* = 100(P^s - 10)$$

$$= 100(13 - 10) = 300$$

Therefore, the new market equilibrium after tax is $P^s = €13$; $P^d = €16$ and $Q^* = 300$ units

d. The tax burden that the consumers (pools' owners) have to bear is

$P^d_{\text{after tax}} - P_{\text{before tax}} = 16 - 14 = €2/\text{unit}$ #

The tax burden that the firms have to bear is

$P_{\text{before tax}} - P^s_{\text{after tax}} = 14 - 13 = €1/\text{unit}$ #

e. Calculate the loss in producer surplus

Loss of producer's surplus $= (14 - 13) \times 300 = 300$

There is 100 identical firms in this industry, and each firm has profit $= €3$

Therefore total profit in this industry is $3 \times 100 = €300$ which is equal to producer surplus that is lost after tax is imposed.